Combinatorial Analyis	**Fall 2010**

Lecture 1: August 18

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Texts: Flajolet and Sedgwick Analytical Combinatoriccs
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Sorts of questions we will be addressing:

Questions regarding "Combinatorial objects" generally finite objects constructed by putting together simpler pieces.

Example:

The set of all finite sets of objects, each of which is a finite subset of the positive integers.

$\{\{1,2,7\},\{3,15,92\},\{3,15,92,93\},\{5\}\}$

Typical questions for this class of objects:

How many? Infinitely many.

Can we split them up so that we get infinitely many natural classes, naturally parametrized so that ech class is finite?

We can begin by thinking about what is interesting in this example? The largest element appearing is 93.

How many examples are there of objects with largest element ≤ 93?

Every subset which appears is a subset of $\{1,2,\ldots,93\}$. There are 2^{93}such possible subsets. Therefore, there are $2^{2^{93}}$ objects.

How many examples are there of objects with largest element $= 93$?

There are $2^{2^{93}} - 2^{2^{92}}$ objects.

Counting often involves counting solutions to equations.

Examples:

"Compositions of n into k parts" = ways of writing n as sum of k natural numbers.

Compositions of 5 into 2 parts are

$$\begin{array}{c} 5+0 \\ 4+1 \\ 3+2 \\ 2+3 \\ 1+4 \\ 0+5 \end{array}$$

Can also be described as counting solutions to

$$x_1 + x_2 + \cdots + x_k = n \text{ where } x_1, x_2, \ldots, x_k \in \mathbb{N}.$$

Note: Here $3+2$ is regarded as distinct from $2+3$.

"Partitions of n into k parts" = ways of writing $x_1 + x_2 + \cdots + x_k = n$ where $x_1 \geq x_2 \geq \cdots \geq x_k \geq 1$.

Example:

$$\begin{aligned} 7 &= 4+1+1+1 \\ &= 3+2+1+1 \\ &= 2+2+2+1 \end{aligned}$$

are all partitions of 7 into exactly 4 parts.

Exercise:

How many ways are there to parenthesize the product $a_1 a_2 \cdots a_n$?

Hint:

a_1: Not interesting

$a_1 a_2$: Not interesting

$(a_1 a_2)\, a_3$ and $a_1\, (a_2 a_3)$: There are 2 ways.

$((a_1 a_2)\, a_3)\, a_4$, $(a_1\, (a_2 a_3))\, a_4$,

$a_1\, (a_2\, (a_3 a_4))$, $a_1\, ((a_2 a_3)\, a_4)$

+ how many others for $n = 4$?

How do we list all of these in a good fashion?

Monday:

How to obtain $f(n)$ = numbers of parenthesise of the product

$$a_1 a_2 \cdots a_n \text{for } n \leq 20.$$

Do obtain!