Measure Theory and Functional Analysis

Exercise 6

1. Let $H = L^2(0,1)$ and M be the set

$$M = \{x \in L^2(0,1) : \int_0^1 x(t)dt = 0\}$$

   (a) Prove that M is a closed subspace of H.

   (b) Describe $M^{\perp}$.

   (c) Find the distance of $x(t) = t^2$ from M.

2. Solve No 4 from the Kreyszig book, Section 3.1.

3. Solve No 7 from the Kreyszig book, Section 3.2.

4. Solve No 9,10 from the Kreyszig book, Section 3.4.