South**E**ast **R**egional **M**eeting **O**n **N**umbers

SCHEDULE OF ACTIVITIES

Talks will take place in Owens Hall G01.
Refreshments will be in Owens Hall on the first floor in the Breezeway between Owens Hall and Bancroft. Coffee and refreshments will be available during all breaks in addition to the times listed.

Saturday, March 28, 2015

9:00 - 9:45	Coffee and snacks
9:45 - 10:35	**Holly Krieger**, MIT, *A case of the dynamical André-Oort conjecture*
10:45 - 11:05	**Paul Young**, College of Charleston, *The p-adic Arakawa-Kaneko zeta functions and p-adic Lerch transcendent*
11:05-11:20	Break
11:20 - 11:40	**Mckenzie West**, Emory University, *The Brauer Manin Obstruction for a Family of Cubic Surfaces*
11:50 - 12:10	**Jesse Thorner**, Emory University, *Applications of an Effective Log-Free Zero Density Estimate for Automorphic L-functions*
12:10 - 2:10	Lunch
2:10 - 3:00	**Ila Varma**, Princeton University, *The mean number of 3-torsion elements in the class groups and ideal groups of quadratic orders*
3:10 - 3:30	**Brian Sinclair** UNC - Greensboro, *An Invariant for Extensions of p-adic Fields based on Residual Polynomials*
3:30-3:45	Break
3:45 - 4:05	**Abbey Bourdon**, University of Georgia, *Torsion Points on CM Elliptic Curves over Number Fields of Odd Degree*
4:15 - 4:35	**Daniel Kamenetsky**, University of South Carolina, *Binary quartic forms over* $\mathbb{F}_p$
4:45 - 5:05	**David Zureick-Brown**, Emory University, *Tropical geometry and uniformity*

The organizers thank the National Science Foundation, the National Security Agency, and the Mathematics Department at Winthrop University for their support.

Sunday, March 29, 2015

9:00-9:30	Coffee and other refreshments
9:30-9:50	**Stevo Bozinovski**, South Carolina State University, *Should algorithm complexity be a chapter in a Number Theory book?*
10:00-11:00	**Ameya Pitale**, University of Oklahoma, *Special values of L-functions*
11:00 - 11:15	Break
11:15-11:35	**Frank Thorne**, University of South Carolina, *Counting non-S_5-quintic fields*
11:45 - 12:05	**Thomas Wright**, Wofford College, *Carmichael numbers with prime numbers of prime factors*
12:05 - 12:25	**Michael Bush**, Washington and Lee University, *Non-abelian generalizations of the Cohen-Lenstra Heuristics*
12:25	END OF CONFERENCE

Abstracts

ABBEY BOURDON, University of Georgia, *Torsion Points on CM Elliptic Curves over Number Fields of Odd Degree*

Let E be an elliptic curve with complex multiplication defined over a number field F. We give a complete classification of the torsion subgroups $E(F)_{\mathrm{tors}}$ when F has odd degree. In the case where F is prime, we show that in fact only finitely many torsion subgroups appear. This is joint work with Pete L. Clark and James Stankewicz.

STEVO BOZINOVSKI, South Carolina State University, *Should algorithm complexity be a chapter in a Number Theory book?*

The books in Number Theory deal with well areas such as primes, congruences, sums, etc. In general, all areas which deal with integers are of interest, but not all areas are considered well established. Algorithm complexity is an example of an area which deals with integers, moreover with natural numbers. This talk will present generation of N-queens patterns, and the space complexity of a backtracking algorithm for that problem for finite N. The discussion is related to Combinatorics and Number Theory points of view.

MICHAEL BUSH, Washington and Lee University, Non-abelian generalizations of the Cohen-Lenstra Heuristics

In the context of quadratic fields, the Cohen-Lenstra Heuristics make precise conjectures about how often one should expect a finite abelian group to appear as the odd part of the class group as one considers fields ordered by discriminant. Over the last several years, Nigel Boston, Farshid Hajir and I have formulated analogous non-abelian heuristics in which the p-class group (p an odd prime) is replaced with the Galois group of the maximal unramified p-extension of the field. I'll discuss the formulation of our conjectures and give an update on the numerical evidence collected so far.

DANIEL KAMENETSKY, University of South Carolina, *Binary quartic forms over* $\mathbb{F}_p$

Finding explicit formulas for the finite Fourier transform of specific functions on binary cubic forms over certain finite fields plays an important role in Taniguchi and Thorne's analytic approach to counting cubic fields of bounded discriminant. As a follow-up to their work, and with the connection between binary quartic forms and 2-Selmer groups of elliptic curves in mind, we would like to find explicit formulas for finite Fourier transforms of functions on binary quartic forms over certain finite fields. I will discuss some challenges that arise in this new setting and the progress we have made in understanding the Fourier transform of a particular function on binary quartic forms over $\mathbb{F}_p$.

HOLLY KRIEGER, MIT, *A case of the dynamical André-Oort conjecture*

In the past five years, Baker and DeMarco have proposed versions of unlikely intersection questions, originally arising in arithmetic geometry, to the setting of complex dynamics. I will discuss their dynamical generalization of the famous André-Oort conjecture on CM points in moduli spaces of abelian varieties. Ghioca, Nguyen, Ye, and myself recently proved the first complete case of this conjecture, for pairs of unicritical polynomials, and I will discuss our result and the connection to the structure of the Mandelbrot set.

AMEYA PITALE, University of Oklahoma, *Special values of L-functions*

In this talk, I want to address two kinds of special value results for L-functions. The first kind are related to Deligne's conjecture, which predicts algebraicity of the value of L-functions at certain "critical" points. An example is the well known fact that the value of the Riemann zeta function at positive even integers is a rational times a power of pi. The second kind are the central critical values. One might be interested in various aspects of this- non vanishing, positivity, sub-convexity etc. I want to give an incomplete survey of the ideas and results and finally tie in my joint work with various authors on these topics.

BRIAN SINCLAIR, UNC - Greensboro, *An Invariant for Extensions of p-Adic Fields based on Residual Polynomials*

An invariant for totally ramified extensions of p-adic fields based on the residual polynomials of segments of the ramification polygon of generating polynomials has proven useful in the enumeration of all extensions of p-adic fields. We investigate the invariant further and give some applications.

FRANK THORNE, University of South Carolina, *Counting non-S_5-quintic fields*

We will give a nontrivial bound on the number of quintic fields K with $|\mathrm{Disc}(K)| < X$, and such that the Galois closure of K has Galois group other than S_5. We will then describe four different proofs of it!
This is joint work with Manjul Bhargava and Alina Cojocaru.

JESSE THORNER, Emory University, *Applications of an Effective Log-Free Zero Density Estimate for Automorphic L-functions*

The first proofs of nontrivial bounds on the least prime in an arithmetic progression or the prime number theorem for short intervals required deep information about the density of zeros of L-functions close to the line $\Re(s) = 1$. For automorphic L-functions satisfying the Ramanujan-Petersson conjecture, we obtain zero density results of comparable strength that allow us to prove various automorphic analogues of these classical problems. We discuss applications to the Sato-Tate conjecture for a non-CM elliptic curve over a totally real field. This is joint work with Robert Lemke Oliver.

ILA VARMA, Princeton University, *The mean number of 3-torsion elements in the class groups and ideal groups of quadratic orders*

In 1971, Davenport-Heilbronn determined the average size of the 3-torsion subgroup in the class groups of quadratic fields ordered by discriminant. Cohen-Lenstra used this result as evidence towards their heuristics in which they predicted the increase in the average size of 3-torsion subgroups of class groups of all quadratic orders rather than quadratic fields. In joint work with Manjul Bhargava, we prove Cohen-Lenstra's prediction by determining the mean number of 3-torsion elements in the class groups of quadratic orders when ordered by discriminant. One reason for this increase can be seen through the asymptotic size of 3-torsion elements in the *ideal group* of quadratic orders (of which the class group is a quotient). I will explain asymptotic properties of torsion in the ideal group of quadratic orders, and if time permits, I will discuss generalizations to cubic and higher-degree fields.

MCKENZIE WEST, Emory University, *The Brauer Manin Obstruction for a Family of Cubic Surfaces*

The Hasse principle asks whether solutions to an equation in a local field extend to those in a global field. This does not always happen, with the Brauer Manin (BM) Obstruction being a common explanation. It is conjectured by Colliot-Thélène and Sansuc that for cubic surfaces the BM Obstruction explains every instance where the Hasse principle fails.
In 1975, Birch and Swinnerton-Dyer gave some of the first examples of the failure of the Hasse principle for cubic surfaces $m\mathrm{Norm}_{L/\mathbb{Q}}(ax+by+\phi z+\psi w) = (cx+dy)\mathrm{Norm}_{k/\mathbb{Q}}(x+\theta y)$ for given fields L, k over $\mathbb{Q}$ and fixed $m, a, b, c, d, \phi, \psi, \theta$. They make a rough number theoretic argument for the BM obstruction in the case that the Hasse principle fails, focusing on particular fields and constants. We make use of advancements in geometry and class field theory, taking a geometric look at this object and utilizing the correspondence between the Brauer group and the Picard group of a surface.

THOMAS WRIGHT, Wofford College, *Carmichael numbers with prime numbers of prime factors*

Assuming a conjecture of Heath-Brown about the first prime in an arithmetic progression, we prove that there are infinitely many Carmichael numbers that have a prime number of prime factors. This (conditionally) resolves a question posed by Steven J. Miller at the previous SERMON meeting.

PAUL YOUNG, College of Charleston, *The p-adic Arakawa-Kaneko zeta functions and p-adic Lerch transcendent*

The Arakawa-Kaneko zeta functions interpolate the poly-Bernoulli polynomials at the negative integers, while their values at the positive integers are connected to multiple zeta values and harmonic number sums. Here we construct p-adic analogues of these functions and show that these complex formulas have good p-adic counterparts. The method of construction, summing a series of forward differences of p-adic power functions, is a p-adic adaptation of an everywhere-convergent series for the Hurwitz zeta function due to Hasse. We also apply this method to construct a p-adic Lerch transcendent. Expressions for zeta values and related constants as harmonic number series which converge in both complex and p-adic senses are a prominent feature of this approach.

DAVID ZUREICK-BROWN, Emory University, *Tropical geometry and uniformity*

I will discuss recent progress on the uniformity conjecture – the existence of a universal bound on the number of rational points on curves of a fixed genus – and explain new ideas from tropical and non-archimedian analytic geometry which lead to a partial proof of the uniformity conjecture.
