MAT 3190
Binary Operations
Exercises

(1) Decide whether the given set B is closed iwth respect to the binary operation defined on the set of integers $\mathbb{Z}$. If B is not closed, exhibit elements $x, y \in B$ such that $x * y \notin B$.
a.) $x * y = xy, \quad B = \{n \in \mathbb{Z} \ : \ n \geq -3\}$.
b.) $x * y = x - y \quad B = \{n \in \mathbb{Z} \ : \ n > 0\}$.
c.) $x * y = x^2 + y^2 \quad B = \{n \in \mathbb{Z} \ : \ n > 0\}$.

(2) In each part below, a rule is given that determines a binary operation $*$ on $\mathbb{Z}$. Determine whether the operation is commutative or associative and whether there is an identity element. Also, find the inverse of each invertible element.
a.) $x * y = x - 6$.
b.) $x * y = 3(x + y)$.
c.) $x * y = x + xy + y + 2$.
d.) $x * y = |x - y|$.

(3) Prove or disprove that the set of even integers is closed under addition. Do the same for multiplication.

(4) Assume that $*$ is a binary operation on a nonempty set A which is commutative and associative. Show that

$$[(a * b) * c] * d = (d * c) * (a * b).$$

for all $a, b, c, c \in A$.

(5) Assume that $*$ is an associative binary operation on A with an identity element. Prove that the inverse of an element is unique when it exists.