On the Performance Evaluation of Query-Based Wireless Sensor Networks

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Abstract

We present a queueing-theoretic framework to evaluate the performance of large-scale, query-based wireless sensor networks whose nodes detect and advertise significant events that are useful for only a limited time; queries generated by sensor nodes are also time-limited. The main performance parameter is the steady state proportion of generated queries that fail to be answered on time. Using an infinite transmission range model, we first provide an approximation for this parameter that is insensitive to the size of the network. Subsequently, we approximate the proportion of failed queries when the transmission range is limited and show that this proportion converges to its infinite range counterpart as the sensor transmission range tends to infinity. The analytical approximations are shown to be remarkably accurate when compared with benchmark values obtained using a commercial network simulator.

1 Introduction

This paper presents a framework to evaluate the performance of large-scale, query-based wireless sensor networks (WSNs) whose sensors detect and advertise significant events that are useful for only a limited time before they expire (e.g., detecting hazardous biological agents, military surveillance, environmental monitoring, etc.). Event lifetimes are established to ensure that sensor nodes have the most up-to-date information to share with other nodes in the network. Query-based WSNs derive their name from the fact that communication between nodes is either event- or query-driven. That is, either the witnessing of an event (e.g., a sudden increase in temperature), or the generation of a query (e.g., a request for the temperature reading at a distant region of the network) triggers communication between nodes which must act as routers for other nodes'

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packets due to a limited sensor transmission range. A query, which itself has a limited lifetime, traverses the network according to a two-dimensional random walk until it either locates the desired information or expires. For this type of network, the total proportion of generated queries that are not answered within their useful lifetime is a critical performance parameter. We devise simple analytical approximations for this proportion, along with other network quality-of-service measures, within a queueing framework. Our analytical approach is unique in that it explicitly accounts for the realism of limited event and query lifetimes which are generally distributed.

Large-scale wireless sensor networks are emerging in such diverse applications as ecological and environmental monitoring, structural health monitoring of aging infrastructure, industrial process control and military surveillance. The ever-increasing interest in WSNs stems from their ability to sense and convey critical information about objects, their surroundings, and the interaction between the two, autonomously. Large-scale WSNs are composed of thousands, or hundreds of thousands, of low-cost sensing devices, typically linked via a wireless channel, that cooperate to perform specific network tasks in a distributed manner. Due to their small physical dimensions, the sensing nodes have very limited energy reserves, local memory, and computational capabilities. Moreover, to conserve power and alleviate contention for access to the transmission medium, each node's transmission range may be limited to that required to ensure a connected network.

Wireless sensor networks have been analyzed from a variety of perspectives including design considerations, routing protocols, and resource management strategies, to name only a few. Some useful survey papers related to WSN sensing tasks, applications, design issues, and communications architectures include Akyildiz et al. [3], Yick et al. [44], and Dietrich and Dressler [19]. Owing to the fact that sensor nodes are energy-constrained, defining WSN lifetime and operating policies has emerged as a critical issue. Dietrich and Dressler [19] surveyed many definitions of WSN lifetime including the number of "alive" nodes, network coverage, network connectivity, and quality-of-service considerations (e.g., event detection rates). Other authors (cf. Anastasi et al. [7]) have classified energy conservation approaches (e.g., sensor sleep/wake protocols, data acquisition schemes, mobile sink-based approaches, etc.). WSN lifetime and energy conservation strategies have further been discussed in [6, 16, 34, 35, 45, 40].

Routing protocols constitute the largest area of research related to the performance of wireless sensor networks. A variety of techniques are reviewed in a cogent survey by Al-Karaki and Kamal [4]. Most protocols aim to minimize the energy expended by the network while satisfying quality-of-service guarantees. Routing protocols are broadly labeled as flat-based, hierarchical (see [36]), and location-based. Flat-based (or data-centric) protocols assume all sensor nodes have equal capabilities and similar roles whereas hierarchical protocols assign different roles to the nodes. Location-based protocols use sensor node position information to make routing decisions. The model we analyze here falls into the category of flat routing and, more specifically, query-based flat routing. Classical data-centric approaches include flooding and gossiping (see [23]) which are known to be energy- and bandwidth-inefficient. Alternatively, rumor-routing protocols (see [2, 14, 10, 20, 37, 41, 43]) can be used. Rumor routing uses packets called *agents* with relatively long lifetimes. When a node detects an event, it adds information pertaining to the event in a local *event table* and immediately creates a time-limited agent that "advertises" the local information to distant nodes via subsequent packet transmissions. These packets are referred to as *event agents*. Consequently, if any node in the network generates a query, another node with the information stored in its local event table can respond, if it receives the query. This approach obviates the need for flooding, thereby reducing energy expenditure. Rumor routing is effective (relative to flooding) when the arrival rate of events is relatively small but generally requires significant overhead. Specifically, witnessed events are assigned a time-to-live (TTL) counter, or resource replication level, that is tracked while query lifetimes must also be tracked.

The TTL counter (a hop counter) is the number of times a witnessed event is replicated in the network, and studies related to this parameter are relatively sparse. Bellavista et al. [11] developed a simulation model (REDMAN) to explore resource replication levels and related network settings. Krishnamachari and Ahn [26] derived cost expressions as a function of the resource replication level for unstructured networks in which the source node is unknown. They used expanding ring queries to search for the information and formulated a nonlinear programming (NLP) model to determine the optimal number of resource replicates, subject to a network storage capacity constraint. Ahn and Krishnamachari [1] extended the results of [26] to structured networks in $d-$dimensional space, and studied structured and unstructured two-dimensional grid and random topology networks. The authors also presented a model to obtain the optimal resource replication level that minimizes the total expected cost of replication and searching, subject to a storage capacity constraint. An algorithm for dissemination and retrieval of information that ensures an even geographical distribution of the informed nodes is proposed for unstructured wireless ad-hoc networks by Miranda et al. [32]. Antoniou et al. [8] presented a nature-inspired data flow model for WSNs that considers congestion regions and dead zones (regions with failing nodes) in a sensor field. Most relevant to our work here, Mann et al. [31] used a queueing framework to obtain the optimal replication level that minimizes a proxy for energy expenditure, subject to a performance guarantee on the steady state proportion of failed queries. Their approach is unique in that it considers time-limited event agents and queries but is limited to memoryless (exponentially-distributed) lifetimes. Bisnik and Abouzeid [13] used a queueing network model to analyze random access, multi-hop wireless networks and derived the average end-to-end delay. Niyato and Hossain [33] developed a queueing model to investigate the performance of different sleep and wake-up strategies. Chiasserini et al. [17] proposed a fluid queueing model that accounts for energy consumption, active/sleep dynamics, and traffic routing. Jiang et al. [24] proposed a queueing-theoretic, power-saving scheme to address non-uniform node power consumption patterns. Ata [9] considered the problem of dynamically choosing the transmission rate in a general wireless communications network such that the average energy consumption per time unit is minimized, subject to a quality-of-service constraint. In that work, the transmission queue was modeled as a finite-buffer, $M/M/1$ system. With the exception of Mann et al. [31], none of the analytical models described herein account explicitly for limited event agent and query lifetimes.

This paper provides a queueing-theoretic framework for evaluating the steady state proportion of query failures (i.e., the limiting proportion of generated queries that fail to be answered on time) in a large-scale WSN with time-critical data. While the network model itself is similar to the one described in [31], it has several important distinguishing attributes. Specifically, Mann et al. [31] consider only exponentially-distributed event agent and query lifetimes, whereas our model allows both types of lifetimes to be generally distributed. Second, the model of [31] is only an infinite-range (single-hop) model that ignores network topology and the limitations of a finite transmission range. Our approach explicitly models the dynamics of query movement over time using a temporally-nonhomogeneous stochastic model that depends explicitly on the transmission range. Derived are analytical approximations that explicitly account for (1) time-limited event agents and queries, (2) the limited transmission range of sensor nodes, and (3) generally-distributed resource and query lifetimes. The first approximation, derived using a single-hop model, is shown to be insensitive to the network's size. The second approximation, derived from a finite-range (or multi-hop) model, is shown to be asymptotically valid, and extensive numerical comparisons with simulated networks verify the exceptional accuracy of the approximations, even when key model assumptions are violated. It is well known that energy efficiency is a critical issue for WSNs; however, there exists a delicate tradeoff between satisfying quality-of-service guarantees and minimizing energy consumption (or maximizing the network's lifetime). Our proposed framework provides easy-to-implement approximations that can be used to devise optimal design or operating strategies for WSNs (e.g., optimizing the transmission range and/or TTL counter) to minimize energy expenditure or maximize network lifetime while limiting the proportion of failed queries to a fixed threshold.

The remainder of the paper is organized as follows. Section 2 provides a description of the network model, queueing models of sensor node elements, and the most relevant attributes. In Section 3, we derive the steady state proportion of query failures assuming an unlimited sensor transmission range, while Section 4 presents an approximation that explicitly accounts for the limited transmission range of sensors. Section 5 presents extensive numerical results that validate our analytical approximations, while Section 6 provides conclusions and directions for future work.

2 Model Description

Consider a multi-hop wireless sensor network (WSN) represented by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the node set (or set of vertices), N is the number of sensor nodes in the network, and $\mathcal{A}$ is the arc set of the sensor network. An arc (i, j) is an element of $\mathcal{A}$ if and only if nodes i and j are within transmission range of each other. Once deployed, the sensor nodes are spatially stationary (i.e., they are not mobile). In this research, we consider only networks with sensor nodes deployed in a rectangular sensor field R, a subset of Euclidean 2-space. The nodes are assumed to be spatially randomly distributed in R, i.e., the node locations are uniformly distributed. The node density of the network, ψ, is the average number of nodes per unit area (in nodes/m^2) given by $\psi = N/A$ where A is the area of sensor field R. For each $i \in \mathcal{N}$, denote by $\boldsymbol{x}_i$ the position of sensor node i in Euclidean 2-space. Then for $j \in \mathcal{N}$, $j \neq i$, the Euclidean distance between $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ is $\rho(i, j) \equiv \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ where $\|\cdot\|$ denotes the Euclidean norm. Assuming each sensor node has a transmission range r (in meters), the *degree* of node $i \in \mathcal{N}$ is the number of nodes within transmission range of i given by

$$d_i(r) \equiv \sum_{j \in \mathcal{N} \setminus \{i\}} \mathbf{1}(\rho(i, j) \leq r),$$

where $\mathbf{1}(x)$ is an indicator function that assumes the value 1 if condition x holds and 0 otherwise. Obviously, $d_i(r)$ depends on the deployment of nodes in R, the network topology, and the transmission range of individual sensor nodes. Finally, the average degree of the network is

$$\bar{d}(r) \equiv \frac{1}{N} \sum_{i=1}^{N} d_i(r).$$

A node $i \in \mathcal{N}$ for which $d_i(r) = 0$ is said to be *isolated*. Isolated nodes are essentially useless to the WSN since they cannot exchange information with other nodes. The WSN is said to be *disconnected* if there is a non-empty subset of isolated nodes in $\mathcal{N}$ but is *completely connected* if there exists at least one path between nodes i and j for every $i, j \in \mathcal{N}$. Obviously, it is undesirable for the network to be disconnected, particularly when the information relayed by nodes is time sensitive. When the nodes are uniformly distributed in R with homogeneous node density ψ, the minimum transmission range needed to ensure the network is completely connected with probability p is (see Theorem 1 of [12])

$$\widehat{r} \geq \sqrt{\frac{-\ln\left(1 - p^{1/N}\right)}{\pi\,\psi}}. \tag{1}$$

The lower bound in (1) can be used, for example, to create discrete-event simulation models of wireless sensor networks that ensure connectivity with high probability.

Next, we describe individual sensor nodes in greater detail. (This discussion is similar to that of Mann [31].) It is assumed that sensor nodes are identical, i.e., they have identical resource requirements, physical limitations, and performance limitations. They are also similar with respect to their information requirements and the rates at which they observe and report relevant phenomena. Each sensor node is equipped with processing, transmitting, and sensing capabilities, as well as a limited power supply (in the form of an on-board battery) that cannot be recharged and is generally difficult, if not impossible, to replace.

In query-based WSNs, sensor nodes serve as both producers and consumers of network resources, and the transmission of data is triggered when an event occurs or a query is generated. A node produces a *resource* when (1) it monitors the environment and gathers data on the occurrence of pertinent events; or (2) it offers a particular service to the network. In addition to data gathering, nodes are also required to execute specific applications in support of the network's goals. When a node requires access to a resource that is not available locally, the node is forced to traverse the network to locate the necessary information and/or services. Next, we describe node activities triggered by the occurrence of an event or a request for information.

When a node witnesses a relevant phenomenon, or offers a particular service to the network, it broadcasts this information to a subset of the network by means of an *event agent* – a packet that describes the resource available, the location of the resource (or, alternatively, the data itself), and the duration of time the resource is available or valid. In this research, we assume that agents are transmitted from node-to-node via a random walk until either the witnessed event expires (i.e., it reaches its deadline), or it exhausts its *time-to-live* (TTL) counter – an integer hop counter representing the maximum number of times the resource may be replicated in the network. It is worth mentioning that a variety of routing protocols can be assumed (cf. [30]), but the results herein assume transmission to a randomly-selected neighbor. This type of random-walk-based routing protocol is useful for maintaining load balancing in a statistical sense (see [4]). Additionally, it is simple to implement, requires nodes to store very little state information, and is a pragmatic choice for large-scale networks with limited node mobility. Each sensor node is equipped with an on-board *event table*. Whenever an event agent is received, or an event is witnessed by the node, the contents of the event agent are added to the event table, and the node is labeled as *informed*, as long as the event agent's lifetime has not expired. On the other hand, if a node's event table does not contain the information witnessed or delivered by an event agent, then the node is said to be *uninformed*.

In addition to witnessing and forwarding events, nodes generate *queries* to request data or resources from the network. A query contains at least three pieces of information: the identifier and/or location of the node originating the request, the resource sought, and the maximum amount of time the query is permitted to traverse the network in search of an informed node. Only informed nodes are capable of answering the queries of uninformed nodes. Similar to event agents, queries are forwarded from node-to-node via a random walk. If a query is received by an informed node, the query is terminated and the informed node generates a *response* that is returned to the query origin node via the shortest path (least number of hops). We assume responses follow the shortest path because, whatever protocol is used to determine the response route, the best currently available route will be discovered first since those routing packets will reach the route-requesting node first. The query response packet contains the information stored in the informed node's event table and, if available, the desired data. If a query cannot locate an informed node within its lifetime, the query fails. It is worth noting that we assume there are known data elements, and each query requests a particular data element; so there is a one-to-one correspondence between a query and a satisfying data element. Moreover, while it is conceivable that nodes receive redundant queries and/or event agents from multiple sources, we assume the receiving node neither aggregates nor generalizes the data in any way. Finally, it is assumed that all transmitted data are accurate, and there are no packet collisions.

Our main objective is to assess a critical quality-of-service measure for query-based WSNs, namely the long-run proportion of queries that fail to be answered on time. To this end, we create a queueing network model that leads to simple analytical expressions and accommodates easy computational implementation.

2.1 Queueing Models of Node Elements

For each $i \in \mathcal{N}$, events are assumed to arrive according to a Poisson process with rate λ. Each witnessed event is time sensitive, i.e., it is useful for only a limited time before it expires. Therefore, once an event is witnessed by a node, it is added to the node's event table and assigned a *lifetime*, Z, a non-negative, non-defective random variable. Event lifetimes (across all nodes) are independent and identically distributed (i.i.d.) random variables with common cumulative distribution function (c.d.f.) $G(w) \equiv \mathbb{P}(Z \leq w)$, $w \geq 0$, and mean $\mathbb{E}(Z) = 1/\delta < \infty$. As long as the event agent has not expired in the event table, the node is informed and can answer queries arriving from other nodes.

Because event agents are mutually independent, and do not necessarily expire in their order of arrival, the event table can be modeled as an $M/G/\infty$ queueing system whose input is a Poisson process with aggregate arrival rate Λ and whose service time is generally distributed with c.d.f. G (see Figure 1). The event arrival rate Λ depends on many factors, not the least of which is the transmission range r. We pause here to remark that, in general, the superposition of locally-

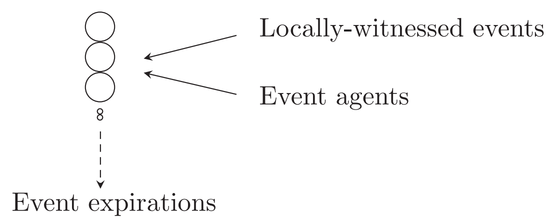

Figure 1: Graphical depiction of a sensor node's event table as an $M/G/\infty$ queue.

witnessed events and externally-generated event advertisements does not necessarily form a Poisson process since the latter do not (in general) originate from a Poisson stream. Furthermore, the event table may not realistically have infinite capacity. Therefore, the evolution of the number of busy servers in the $M/G/\infty$ model must be viewed as an approximation of the evolution of event table content. However, it will be shown in Section 5 that this assumption is not overly restrictive and that the proportion of query failures is surprisingly insensitive to the Poisson assumption. We choose the $M/G/\infty$ model for its tractability and generality with respect to event lifetimes. Specifically, it provides a simple expression for the steady state proportion of time an arbitrary node in the WSN is uninformed given by

$$\pi_0 \equiv \mathbb{P}(E = 0) = \exp\left(-\Lambda/\delta\right),$$

where E is the steady state number of events in the event table. Once a node witnesses an event, the information is forwarded until its TTL counter is exhausted. Henceforth, we denote the TTL counter by $\ell \in \mathbb{N}$.

Each sensor node contains a transmitter along with an (assumed) infinite buffer for storing data packets (queries, event agents, or responses). When a non-expired event agent arrives to a node, either because an event was witnessed, or because the agent is received from another node, the agent joins the transmission queue after a copy has been added to the node's event table. Moreover, when a node receives a query, either the query or the response is sent to the transmission queue, depending on whether the node is informed or uninformed. In either case, the query fails if the time elapsed from the moment of its inception until it locates an informed node exceeds its lifetime. While responses also join the transmission queue, this traffic stream is assumed to be negligible (relative to event agent and query traffic) since responses follow the shortest path and make far fewer hops than event agent agents or queries. Hence, we do not include response traffic in the total arrival rate calculation. The node's transmission queue is modeled as a single-server queueing system as depicted in Figure 2.

Specifically, we assume that each node's transmission queue operates as a non-prioritized, multi-class $M/M/1$ queueing system with a first-come-first-served (FCFS) queueing discipline. Arrivals are assumed to originate from a Poisson process with rate λ_q. As depicted in Figure 2, the aggregate arrival process is comprised of locally-witnessed events, agents from other nodes, locally-generated queries, and queries arriving from other nodes. When a query is generated or received by a node, it joins the transmission queue only if the node is uninformed. The service time is the time needed to transmit a query or an event agent (either a locally-witnessed event or an advertisement from another node). Irrespective of the packet type, we assume the transmission time τ is an

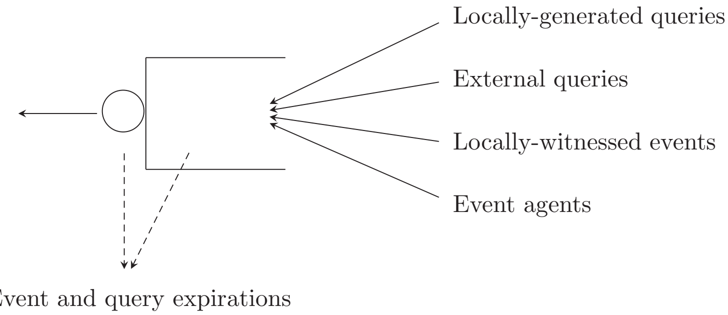

Figure 2: Graphical depiction of the sensor node's transmission queue.

exponential random variable with parameter μ, c.d.f. $F(x) \equiv \mathbb{P}(\tau \le x) = 1 - \exp(-\mu x)$, and finite mean $\mathbb{E}(\tau) = 1/\mu$. This assumption not only facilitates analytical tractability, but allows for larger variance in the transmission time. The transmission queue is stable if and only if $\mu > \lambda_q$. This condition is usually met in practice since transmission rates are generally very high. It is important to note that the total arrival rate of traffic to the transmission queue serves as a proxy for energy expenditure at a node since transmitting is the primary energy consuming activity (cf. [38]).

2.2 Network Performance Parameters

The primary concern of this research is the assessment of the steady state probability that a generated query fails to be answered on time. We refer to this performance parameter as the proportion of query failures. A query is said to fail if it expires awaiting transmission or while being transmitted. Our main aim is to provide easy-to-use analytical expressions for this parameter that allow us to circumvent costly, time-consuming simulation experiments for large-scale networks. To this end, let T be a non-negative random variable denoting the total time needed for a query to locate an informed node as measured from the time the query is generated at a node $n \in \mathcal{N}$. This random time depends on the status of node n at the time of creation. Define the indicator variable

$$I_n = \begin{cases} 1, & \text{if node } n \text{ is informed,} \\ 0, & \text{if node } n \text{ is uninformed.} \end{cases}$$

The c.d.f. of $[T|I_n = 0]$ is denoted by $B(t) \equiv \mathbb{P}(T \le t | I_n = 0)$, $t \ge 0$ for any $n \in \mathcal{N}$. At the time of generation, the query is assigned a lifetime, X, so that if T exceeds X, the query does not locate the desired information before expiring, and it fails. The c.d.f. of X is $H(x) \equiv \mathbb{P}(X \le x)$, $x \ge 0$, and its mean is $\mathbb{E}(X) = 1/\beta < \infty$. Recall that π_0 is the proportion of time an arbitrary node is uninformed. Proposition 1 characterizes the primary performance parameter.

Proposition 1 *The unconditional proportion of query failures is*

$$\Delta \equiv \mathbb{P}(T > X) = \pi_0 \int_0^\infty [1 - B(x)]\, \mathrm{d}H(x). \tag{2}$$

Proof. Since (in steady state) a node is uninformed with probability π_0, we can use a conditioning argument to obtain

$$
\begin{aligned}
\Delta \equiv \mathbb{P}(T > X) &= \sum_{i=0}^{1} \int_0^\infty \mathbb{P}(T > X | X = x, I_n = i)\mathbb{P}(I_n = i)\mathrm{d}H(x) \\
&= \int_0^\infty \mathbb{P}(T > x | I_n = 0)\mathbb{P}(I_n = 0)\mathrm{d}H(x) \\
&= \pi_0 \int_0^\infty \mathbb{P}(T > x | I_n = 0)\mathrm{d}H(x) \\
&= \pi_0 \int_0^\infty [1 - B(x)]\, \mathrm{d}H(x).
\end{aligned}
$$

■

The expression for the proportion of query failures is straightforward except that the distribution function B is difficult to characterize in all but a few cases. That is, the time to locate an informed node is influenced by many factors including (but not limited to) the transmission range, availability of the requested data, the query's lifetime, and network traffic, all of which are interrelated. The next section considers the case when the sensors all use an infinite transmission range.

3 Unlimited Sensor Transmission Range

In this section, we provide an approximation for Δ when $r = \infty$, i.e., when each node's transmission range is large enough to ensure that any other node in the network can be reached with a single hop. Although several assumptions are employed, the primary purpose of this model is to provide a framework for a more realistic limited transmission range model.

3.1 Approximating Network Traffic

Here we establish approximations for event agent and query arrival rates at an arbitrary node of the network. The first result bounds the aggregate event agent arrival rate to the sensor node's event table. This bound sets the stage for an approximation of the steady state proportion of time that any node is uninformed.

Proposition 2 *Assume events arrive locally to each $n \in \mathcal{N}$ according to a Poisson process with rate λ. Then the aggregate event arrival rate Λ to an arbitrary $n \in \mathcal{N}$ is bounded above by $\lambda\,(1+\ell)$ where ℓ is the time-to-live value.*

Proof. The aggregate event arrival rate Λ consists of the Poisson rate of locally-witnessed events, and the aggregate rate of witnessed events arriving from the other $N-1$ nodes in the network. Therefore, $\Lambda = \lambda + \Lambda_x$ where Λ_x denotes the rate of external event arrivals. An event agent can be forwarded to, at most, ℓ nodes. Since $r = \infty$, each node can be reached with a single hop; hence, each event advertisement is equally likely to be received by one of the other $N-1$ nodes. That is, a particular node receives one of the (potential) ℓ advertisements with probability $\ell/(N-1)$, and since $N-1$ other nodes transmit event agents,

$$\Lambda_x \leq \lambda\,(N-1)\left(\frac{\ell}{N-1}\right) = \lambda\,\ell.$$

Therefore, $\Lambda \le \lambda + \lambda\,\ell = \lambda\,(1+\ell)$. ■

Next, we provide a lower bound for the steady state proportion of time a node is uninformed. An event agent is assigned a lifetime Z once it enters the event table. The random variable Z has c.d.f. G and finite mean $\mathbb{E}(Z) = 1/\delta$. As noted in Section 2, the event table is approximated by an $M/G/\infty$ queue with (Poisson) arrival rate Λ and service time distribution G, since the limiting probability of an empty system depends on G only through its mean. Using the well-known steady state distribution of the $M/G/\infty$ system (see [27]), the limiting proportion of time a node is uninformed is

$$\pi_0 = \exp(-\Lambda/\delta), \quad 0 < \delta < \infty. \tag{3}$$

By Proposition 2, the event agent arrival rate to a node is bounded above by $\lambda\,(1+\ell)$. Therefore,

$$\pi_0 \ge \exp\left[\frac{-\lambda(1+\ell)}{\delta}\right]. \tag{4}$$

Similarly, the event agent arrival rate to the node's *transmission queue*, λ_e, is also bounded above. Although an event agent is transmitted at most ℓ times, the node which receives it at the ℓth transmission does not add the agent to its transmission queue since the agent's time-to-live counter will have expired. Therefore,

$$\lambda_e \le \lambda + \lambda(N-1)\sum_{i=1}^{\ell-1}\frac{1}{N-1} = \lambda\ell. \tag{5}$$

While the bounds of (4) and (5) are legitimate, they may not be tight since they do not explicitly account for the expiration of event agents waiting in the transmission queue, or those that expire during transmission. Proposition 3 provides an improved bound for λ_e (by considering the effect of event expirations) that leads to an improved approximation for π_0. In what follows, let α_j denote the probability that an event agent expires at the jth visited node, $j = 0, 1, 2, \ldots$, where the 0th visited node is the event witnessing node. For simplicity, define the expiration probability at the witnessing node by $\alpha \equiv \alpha_0$. Assuming the event lifetime c.d.f. G has an increasing failure rate (IFR), then $0 < \alpha \le \alpha_1 \le \alpha_2 \le \cdots$. This assumption asserts that event agents age over time, i.e., given that an event agent is alive at time t, the likelihood that it expires in $(t, t+a)$ for some $a > 0$ is increasing in t.

Proposition 3 *Suppose G is an IFR distribution function so that $0 < \alpha \le \alpha_1 \le \alpha_2 \le \cdots$. Then for a fixed time-to-live value ℓ,*

$$\lambda_e \le \lambda\left[\frac{1-(1-\alpha)^{\ell}}{\alpha}\right] \le \lambda\ell.$$

Proof. Since each of the $N-1$ nodes is equally likely to receive an advertised event agent, an individual node receives the kth transmission with probability

$$\frac{1}{N-1}\prod_{j=0}^{k-1}(1-\alpha_j), \quad k = 1, 2, \ldots, \ell-1.$$

That is, the event agent is forwarded at most ℓ times; however, the last node that receives the agent does not add it to its transmission queue since the agent's time-to-live counter will have expired. Therefore, the approximate total rate of event arrivals to a node's transmission queue is given by

$$\begin{aligned}
\lambda_e &= \lambda + \lambda(N-1) \cdot \sum_{k=1}^{\ell-1} \frac{1}{N-1} \prod_{j=0}^{k-1} (1-\alpha_j) \\
&\leq \lambda + \lambda \cdot \sum_{k=1}^{\ell-1} (1-\alpha)^k \\
&= \lambda \sum_{k=0}^{\ell-1} (1-\alpha)^k \\
&\leq \lambda \left[\frac{1-(1-\alpha)^\ell}{\alpha} \right] \leq \lambda \ell,
\end{aligned}$$

where the last inequality follows from $\alpha \in (0,1)$. ■

For the results that follow, we use the approximation,

$$\lambda_e \approx \lambda \left[\frac{1-(1-\alpha)^\ell}{\alpha} \right],$$

to improve the approximation of π_0. Similarly, it can be shown that the event agent arrival rate to the event table is approximated by

$$\Lambda \approx \widehat{\Lambda} = \lambda \left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha} \right]. \tag{6}$$

Therefore, by equation (3), when $r = \infty$, the approximate steady state proportion of time a node is uninformed is given by

$$\pi_0 \approx \exp\left[-\frac{\lambda}{\delta} \left(\frac{1-(1-\alpha)^{\ell+1}}{\alpha} \right) \right]. \tag{7}$$

Approximations (6) and (7) essentially ignore the probability that an event agent revisits a node because, in the single-hop model with N large and ℓ moderately small, the likelihood of revisiting any node is negligible.

Next, we examine the total traffic experienced at the transmission queue. Let λ_q be the total arrival rate of event agents and queries to a sensor node's transmission queue. Each node generates local queries according to a Poisson process with rate γ. When a query is generated locally, or received from another node, it is added to the transmission queue only if the subject node is uninformed. The arrival rate of locally-generated queries to the transmission queue, q_l, is $q_l = \pi_0 \, \gamma$. Let q_x denote the rate at which external queries arrive at a node. In steady state, the query visits an informed node with probability $1-\pi_0$ and an uninformed node with probability π_0, independent of the number of hops prior to the current visit. Consequently, the number of hops needed to first locate an informed node follows a geometric distribution with success probability $1-\pi_0$ and mean $1/(1-\pi_0)$. Since any node is equally likely to receive a query, the probability of receiving an external query is $1/[(1-\pi_0)(N-1)]$. Because all other $N-1$ nodes generate queries identically, q_x is approximately

$$q_x = \gamma \, \pi_0 \, (N-1) \, \frac{1}{(1-\pi_0)(N-1)} = \frac{\gamma \pi_0}{1-\pi_0}. \tag{8}$$

Finally, we approximate the total arrival rate of traffic to a node's transmission queue by

$$\lambda_q \approx \widehat{\lambda}_e + q_l + q_x = \lambda \left[\frac{1-(1-\alpha)^{\ell}}{\alpha} \right] + \pi_0 \gamma \left(\frac{2-\pi_0}{1-\pi_0} \right). \tag{9}$$

By (7) and (9), we see that π_0 and, consequently, λ_q are explicit functions of α. Therefore, the approximation of λ_q is written as

$$\lambda_q \approx c(\alpha) \equiv \lambda \left[\frac{1-(1-\alpha)^{\ell}}{\alpha} \right] + \gamma e^{-g(\alpha)} \left[\frac{2-e^{-g(\alpha)}}{1-e^{-g(\alpha)}} \right], \tag{10}$$

where

$$g(\alpha) = \frac{\lambda}{\delta} \left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha} \right].$$

We are now prepared to provide an expression for α, the probability that an event agent expires in the first transmission queue. The result is approximate since the input to the transmission queue is assumed to be the superposition of independent Poisson arrival streams. The equilibrium random variable Z_e associated to the lifetime Z with c.d.f. G and mean $\mathbb{E}(Z)$ has c.d.f.

$$G_e(z) \equiv \mathbb{P}(Z_e \le z) = \frac{1}{\mathbb{E}(Z)} \int_0^z [1-G(u)] \mathrm{d}u.$$

We make use of the equilibrium distribution of the event agent lifetime in the following proposition that characterizes α.

Proposition 4 *Assume $\mu > c(\alpha)$ for each λ and δ such that $0 < \lambda < \infty$ and $0 < \delta < \infty$. Let W be the total time spent at a node's transmission queue (delay plus transmission time) by an arbitrary arrival in steady state. Then α satisfies the fixed point equation*

$$\alpha = \mathbb{P}(W > Z_e) = \widetilde{G}_e(\mu - c(\alpha)) = \mathbb{E}\left[e^{-[\mu - c(\alpha)] Z_e} \right], \tag{11}$$

where $\widetilde{G}_e(\mu - c(\alpha))$ denotes the Laplace-Stieltjes transform (LST) of G_e evaluated at $\mu - c(\alpha)$.

Proof. The transmission queue is modeled as an $M/M/1$ queueing system with mean transmission time $1/\mu$ and aggregate arrival rate $c(\alpha)$. Let W_n be the total time spent in the transmission queue (i.e., the delay time plus the transmission time) by the nth arrival to the queue, either an event agent or a query. It is well known (see [21]) that if $\mu > c(\alpha)$, then $W_n \Rightarrow W$ as $n \to \infty$ where W is exponentially distributed with mean $1/(\mu - c(\alpha))$ and $(\Rightarrow)$ is convergence in distribution (or weak convergence). Suppose an event agent arrives at time t so that $Z - t$ is the residual lifetime of the event. Using basic results from renewal theory (cf. [27]), $Z - t \Rightarrow Z_e$ as $t \to \infty$. Therefore, by conditioning on Z_e, we obtain

$$\alpha = \mathbb{P}(W > Z_e) = \int_0^{\infty} e^{-(\mu - c(\alpha))z} \mathrm{d}G_e(z) = \widetilde{G}_e(\mu - c(\alpha)) = \mathbb{E}\left[e^{-(\mu - c(\alpha)) Z_e} \right].$$

■

To illustrate (11), suppose the event lifetime is exponentially distributed with mean $1/\delta$. Then, $G_e(z) = G(z) = 1 - \exp(-\delta z)$ for all $z \ge 0$, and the unique probability α solves the fixed point problem

$$\alpha = \frac{\delta}{\mu - c(\alpha) + \delta},$$

where $c(\alpha)$ is given by (10). For an arbitrary equilibrium distribution G_e, we need to solve (11) numerically to obtain α. As seen by (7) and (9), the approximations of π_0 and λ_q depend explicitly on α. Therefore, we use the following fixed point iteration algorithm, which is standard in most numerical analysis textbooks (cf. [15]), to solve for α. Let $\pi_0^{(k)}$, $\lambda_q^{(k)}$ and $\alpha^{(k)}$ be the approximated values of π_0, λ_q and α at the kth iteration of the algorithm, respectively. The algorithm first obtains an initial guess of α using bounds (4) and (5). Each subsequent iteration uses approximations (7) and (9) to update these values until a convergence criterion is satisfied.

Algorithm to Compute α:

Step 0: *Initialization via the bounds of* (4) *and* (5).

$$\begin{aligned}
&k := 0;\\
&\pi_0^{(k)} := \exp\left[-\lambda(1+\ell)/\delta\right];\\
&\lambda_q^{(k)} := \lambda\,\ell + \gamma\pi_0^{(k)}\left(\frac{2-\pi_0^{(k)}}{1-\pi_0^{(k)}}\right);\\
&\alpha^{(k)} := \widetilde{G}_e\left(\mu - \lambda_q^{(k)}\right).
\end{aligned}$$

Step 1: *Update the approximations.*

$$\begin{aligned}
&k := k+1;\\
&\pi_0^{(k)} := \exp\left[-\frac{\lambda}{\delta}\left(\frac{1-\left(1-\alpha^{(k-1)}\right)^{\ell+1}}{\alpha^{(k-1)}}\right)\right];\\
&\lambda_q^{(k)} := \lambda\left[\frac{1-\left(1-\alpha^{(k-1)}\right)^{\ell}}{\alpha^{(k-1)}}\right] + \gamma\pi_0^{(k)}\left[\frac{2-\pi_0^{(k)}}{1-\pi_0^{(k)}}\right];\\
&\alpha^{(k)} := \widetilde{G}_e\left(\mu - \lambda_q^{(k)}\right).
\end{aligned}$$

Step 2: *Check convergence criterion.*

If $\left|\alpha^{(k)} - \alpha^{(k-1)}\right| > \epsilon$, return to Step 1;

Else $\alpha := \alpha^{(k)}$;

Stop.

Recall that our aim is to approximate Δ of equation (2) by assuming $r = \infty$. To this end, let $\widetilde{T}$ denote the time to locate an informed node when $r = \infty$ and let

$$\Delta_\infty \equiv \mathbb{P}(\widetilde{T} > X) = \pi_0 \int_0^\infty [1 - B(x)]\mathrm{d}H(x).$$

The c.d.f. of $\widetilde{T}$ is a function of both λ_q ($c(\alpha)$) and π_0, both of which are determined by α. The next section shows how to obtain Δ_∞.

3.2 Approximate Query Failure Rate

Queries, which can be generated at any node $n \in \mathcal{N}$, are forwarded via a random walk to one-hop neighbors until either an informed node is located, or the query expires while awaiting transmission in some node's transmission queue (or while being transmitted). Once generated, a query is assigned a lifetime X having c.d.f. $H(x) \equiv \mathbb{P}(X \leq x)$, $x \geq 0$. Let us assume for the moment that a query generated at an uninformed node can be forwarded indefinitely (i.e., $X = \infty$ w.p. 1), and let M be the (integer) number of hops needed to first locate an informed node.

Let T_k denote the time spent by a query at its kth location. That is, T_0 denotes the time spent at the query origin node (which is uninformed), T_1 is the time spent at the first visited node, which might be informed or uninformed, and so forth. To simplify notation, let $\widetilde{T}_u \equiv [\widetilde{T} | I_n = 0]$ be the elapsed time between creation of the query at an *uninformed* node and the time it first locates an informed node. It is easy to see that

$$\widetilde{T}_u = \sum_{k=0}^{M-1} T_k.$$

Because we assume $r = \infty$ and identical nodes, in steady state, a query visits an informed node with probability $1 - \pi_0$ and an uninformed node with probability π_0, independent of any prior visits. Thus, M is a geometric random variable with success probability $1 - \pi_0$ and mean $1/(1 - \pi_0)$, i.e., $\widetilde{T}_u$ is a geometric sum of i.i.d. exponential random variables.

Lemma 1 *Given that a query is generated at an uninformed node, the time to locate an informed node is exponentially distributed with parameter* $(1 - \pi_0)(\mu - \lambda_q)$*, i.e.,*

$$B(x) \equiv \mathbb{P}(\widetilde{T}_u \leq x) = 1 - \exp\left[-(1 - \pi_0)(\mu - \lambda_q)x\right], \quad x \geq 0, \tag{12}$$

where $\lambda_q \equiv c(\alpha)$ *is obtained using the value of* α *that solves the fixed point equation* (11)*.*

Using Lemma 1, we next provide our approximate expression for the steady state proportion of query failures when $r = \infty$.

Proposition 5 *Assuming Poisson event arrivals and query generation, the proportion of query failures in an infinite-range WSN is*

$$\Delta_\infty = \mathbb{P}(\widetilde{T} > X) = \pi_0\, \widetilde{H}[(1 - \pi_0)(\mu - \lambda_q)], \tag{13}$$

where $\widetilde{H}(s) = \mathbb{E}\left(e^{-sX}\right)$ *is the LST of the query lifetime distribution function* H*.*

Proof. The proof follows directly by conditioning on the lifetime X and utilizing Lemma 1. Specifically,

$$\begin{aligned}
\Delta_\infty = \mathbb{P}(\widetilde{T} > X) &= \int_0^\infty \mathbb{P}(\widetilde{T} > X | I_n = 0, X = x)\mathbb{P}(I_n = 0)\mathrm{d}H(x) \\
&= \pi_0 \int_0^\infty e^{-(1-\pi_0)(\mu-\lambda_q)x}\mathrm{d}H(x) \\
&= \pi_0\, \widetilde{H}\left[(1 - \pi_0)(\mu - \lambda_q)\right].
\end{aligned}$$

■

Proposition 5 provides the steady state proportion of generated queries that fail to be answered on time, and it holds for all query lifetime distributions that possess a LST. However, if the distribution function H is heavy-tailed and does not possess an LST, the transform approximation method (TAM) developed by Harris and Marchal [22], or its modification by Shortle et al. [42], can be used to approximate $\widetilde{H}$. It is noteworthy that (13) is insensitive to the size of the network N.

Scalability of the WSN is an important issue as realistic networks are envisioned to have thousands or even hundreds of thousands of sensor nodes. The infinite range approximations of this section are appealing due to their insensitivity to N. In this single-hop model, for large N, the likelihood that a given node is visited more than once by an event agent or query is negligible since a witnessing node forwards an event agent to, at most, ℓ *distinct* nodes. Similarly, queries are assumed to visit a distinct node at each hop, independently of all prior hops. However, to conserve energy, realistic sensor nodes use a limited transmission range, so the likelihood of revisiting neighbors when using a random-walk protocol can be significant, as highlighted by Rodero-Merino et al. [39]. Obviously, forwarding event agents to informed nodes, and/or repeatedly transmitting queries to uninformed nodes, wastes precious energy stores, prolongs the time needed to locate informed nodes and, ultimately, increases the query failure rate. This revisiting effect is even more pronounced for nodes located near the borders of the deployment region, as these nodes generally have a smaller node degree. In the next section, we present an approximation scheme that assumes a limited transmission range and explicitly accounts for the revisiting and border effects.

4 Limited Sensor Transmission Range

In this section, we present an approximation for the steady state proportion of query failures that explicitly accounts for the limited transmission range of wireless sensors (i.e., a multi-hop model). Specifically, we approximate Δ_r, the steady state proportion of query failures when the sensor nodes have transmission range of r ($r < \infty$). Additionally, we show that for large N, the approximation converges appropriately to Δ_∞ as $r \to \infty$.

4.1 Modeling Query Dynamics

Here we consider the status (and movement) of an individual query from its inception until it either locates an informed node or fails due to expiration. If a query is generated at an informed node, then it is answered immediately and never forwarded; therefore, we focus on the case when a query is generated at an uninformed node. At its inception the query is instantaneously assigned a lifetime X with c.d.f. $H(x)$ and mean $\mathbb{E}(X) = 1/\beta$ ($0 < \beta < \infty$). It is forwarded to a randomly selected node within the r-radius of the current node until either an informed node is located, or the query lifetime ends, in which case it is destroyed. In what follows, all random quantities are conditioned on the event $\{X = x\}$; therefore, we make the dependence on x explicit. Before proceeding with the formal model description, let us recall our node-labeling convention.

The query origin node is labeled as the 0th visited node, and if the query is successfully transmitted to an uninformed node next, it joins that node's transmission queue. This subsequent node is labeled as the *first* visited node at which the query awaits its second transmission, and so forth. More generally, a query awaits its kth transmission at the $(k-1)$st visited node. Now, for each integer k ($k \geq 0$), let Q_k be the status of the query just before *potentially* joining the transmission queue of the kth visited node. That is, following the $(k-1)$st visit, the query might not join the next transmission queue because its lifetime may have ended, or it may have been answered at the $(k-1)$st visited node. The query only joins the kth node's transmission queue if the query is alive and unanswered after the $(k-1)$st visit. Therefore, the query can be in one of three mutually exclusive and exhaustive states: *active* (state 0), *answered* (state 1), or *expired* (state 2). For each $k \geq 0$, $Q_k \in S \equiv \{0, 1, 2\}$ where $Q_k = 0$ means the query, having been successfully transmitted k times, has not expired but has not been answered; $Q_k = 1$ means the query, having been successfully transmitted k times is answered at the kth visited node (i.e., the kth visited node is informed); and $Q_k = 2$ means the query was successfully transmitted $k-1$ times but expired awaiting its kth transmission (or during its kth transmission) at the $(k-1)$st visited node. (Note that $\mathbb{P}(Q_0 = 2) = 0$.) The process $Q \equiv \{Q_k : k \geq 0\}$ is an S-valued discrete-time Markov chain (DTMC) with *temporally-nonhomogeneous* one-step transition probability matrix, $\mathbf{P}(k, x)$, given by

$$\mathbf{P}(k,x) = \begin{bmatrix} p_{00}(k,x) & p_{01}(k,x) & p_{02}(k,x) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad k \geq 0, \quad x \geq 0 \tag{14}$$

where for each $i, j \in S$,

$$p_{ij}(k,x) \equiv \mathbb{P}_x(Q_{k+1} = j | Q_k = i), \quad k \geq 0,$$

denotes the probability that the status of the query transitions from i to j at the $(k+1)$st step and $\mathbb{P}_x(A) \equiv \mathbb{P}(A | X = x)$ for any measurable event A. Once a query locates an informed node, it is no longer forwarded to a neighbor node, and if the query lifetime ends awaiting transmission (or during transmission), it is destroyed; therefore, states 1 and 2 are absorbing states of the DTMC. Row 0 of $\mathbf{P}(k, x)$ contains the critical transition probabilities. In particular, $p_{02}(k, x)$ is the probability that the query fails at the kth visited node, given it was active just before being added to the kth visited node's transmission queue. Likewise, $p_{00}(k, x)$ is the probability the query remains active just before being added to the $(k+1)$st visited node's transmission queue, given it was active just before being added to the kth node's transmission queue. Finally, $p_{01}(k, x)$ is the probability that a query is answered at the $(k+1)$st visited node, given it was active just before being added to the kth visited node's transmission queue.

Obviously, the DTMC Q is reducible with one transient state (state 0) and two closed communicating classes, namely $C_1 = \{1\}$ and $C_2 = \{2\}$; therefore, its limiting behavior is fairly easy to characterize. Before examining the limiting behavior, we characterize the distribution of $\{Q_k : k \geq 0\}$ at a particular step k. Let $v_j^k(x) = \mathbb{P}_x(Q_k = j)$ be the (unconditional) probability that the query is in state $j \in S$ just before joining the transmission queue of the kth node, and let $\boldsymbol{v}^k(x) = [v_j^k(x)]_{j \in S}$ be a (1×3) row vector comprised of these values. Because Q possesses a time-nonhomogeneous transition probability matrix, the vector $\boldsymbol{v}^k(x)$ can be obtained recursively (cf. [25]) by

$$\boldsymbol{v}^{k+1}(x) = \boldsymbol{v}^k(x)\,\mathbf{P}(k,x), \quad k \geq 0,$$

whose solution is given by

$$\boldsymbol{v}^{k+1}(x) = \boldsymbol{v}^0(x) \prod_{n=0}^{k} \mathbf{P}(n,x), \quad k \geq 0. \tag{15}$$

The square matrix on the right-hand side of (15) is the $(k+1)$-step transition probability matrix of Q. The transient analysis of Q facilitates an analysis of its limiting behavior which, in turn, is used to derive an expression for the steady state probability that a query fails to locate an informed node before its lifetime ends.

To this end, let us define the limiting probability vector

$$\boldsymbol{v}(x) \equiv \lim_{k\to\infty} \boldsymbol{v}^{k+1}(x) = \lim_{k\to\infty} \boldsymbol{v}^0(x) \prod_{n=0}^{k} \mathbf{P}(n,x) = \boldsymbol{v}^0(x) \lim_{k\to\infty} \prod_{n=0}^{k} \mathbf{P}(n,x). \tag{16}$$

Before approximating this vector, we first establish the existence and structure of the limit in the right-most term of (16) via Theorem 1.

Theorem 1 *For a fixed lifetime x $(x > 0)$, there exist real numbers $\alpha_1(x)$ and $\alpha_2(x)$ such that*

$$\mathbf{A}(x) \equiv \lim_{k\to\infty} \prod_{n=0}^{k} \mathbf{P}(n,x) = \begin{bmatrix} 0 & \alpha_1(x) & \alpha_2(x) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $\alpha_1(x), \alpha_2(x) \in (0,1)$ and $\alpha_1(x) + \alpha_2(x) = 1$.

Proof. Using induction, it can be shown that the $(k+1)$-step transition probability matrix is given by

$$\prod_{n=0}^{k} \mathbf{P}(n,x) = \begin{bmatrix} \prod_{n=0}^{k} a_n & \sum_{n=0}^{k} b_n \left(\prod_{j=0}^{n-1} a_j\right) & \sum_{n=0}^{k} c_n \left(\prod_{j=0}^{n-1} a_j\right) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{17}$$

where $a_n \equiv p_{00}(n,x)$, $b_n \equiv p_{01}(n,x)$, $c_n \equiv p_{02}(n,x)$, $n \ge 0$, and $a_{-1} \equiv 1$. First, note that rows 1 and 2 of $\prod_{n=0}^{k} \mathbf{P}(n,x)$ are as given in (17) for any $k \in \mathbb{N}$; hence, we need only concern ourselves with row 0. Allowing $k \to \infty$ on both sides of (17), and noting that $0 < a_n < 1$, we see immediately that

$$\lim_{k\to\infty} \prod_{n=0}^{k} a_n = 0,$$

$$\alpha_1(x) \equiv \lim_{k\to\infty} \sum_{n=0}^{k} b_n \prod_{j=0}^{n-1} a_j = b_0 + \sum_{n=1}^{\infty} b_n \prod_{j=0}^{n-1} a_j \ge b_0 > 0, \tag{18}$$

and

$$\alpha_2(x) \equiv \lim_{k\to\infty} \sum_{n=0}^{k} c_n \prod_{j=0}^{n-1} a_j = c_0 + \sum_{n=1}^{\infty} c_n \prod_{j=0}^{n-1} a_j \ge c_0 > 0. \tag{19}$$

Since each row of $\mathbf{A}(x)$ is comprised of nonnegative real numbers, and the row sums must be unity (cf. [18]), we conclude that $\alpha_1(x) + \alpha_2(x) = 1$ which, in light of (18) and (19), implies that $0 < \alpha_1(x) < 1$ and $0 < \alpha_2(x) < 1$. ∎

For computational purposes, we approximate $\boldsymbol{v}(x)$ by truncating the infinite product of (16) at an appropriate integer q. Specifically, for a sufficiently large $q \in \mathbb{N}$, the approximation for $\boldsymbol{v}(x)$ is given by

$$\boldsymbol{v}(x) \approx \boldsymbol{v}^{q+1}(x) = \boldsymbol{v}^0(x) \prod_{n=0}^{q} \mathbf{P}(n,x), \tag{20}$$

where q is chosen such that $\|\boldsymbol{v}^{q+1}(x) - \boldsymbol{v}^q(x)\|_\infty < \epsilon$ with $\|\cdot\|_\infty$ the usual ∞-norm and ϵ a convergence threshold.

4.2 Approximate Query Failure Rate

Let Δ_r be the limiting probability of query failure provided each sensor's transmission range is r ($r < \infty$) and let

$$v_2(x) \equiv \lim_{k\to\infty} v_2^k(x) = \lim_{k\to\infty} \mathbb{P}_x(Q_k = 2).$$

The unconditional proportion of query failures is approximately

$$\Delta_r = \int_0^\infty v_2(x)\mathrm{d}H(x). \tag{21}$$

Let $\pi_0(r)$ be the steady state proportion of time an arbitrary node is uninformed when the transmission range is r. Note that $v_2(x)$ depends implicitly on r through $\pi_0(r)$ since $\boldsymbol{v}^0(x) = (\pi_0(r),\, 1-\pi_0(r),\, 0)$, and $\mathbf{A}(x)$ depends on $\pi_0(r)$. However, we suppress this dependence on r for ease of notation. To compute $\boldsymbol{v}(x)$ (or its approximation $\boldsymbol{v}^q(x)$ via (20)), we now provide an expression for $p_{02}(k,x)$ and, subsequently, expressions for $p_{00}(k,x)$ and $p_{01}(k,x)$.

Lemma 2 *For a fixed lifetime x ($x > 0$), the transition probability $p_{02}(k,x)$ is*

$$p_{02}(k,x) = \frac{e^{-(\mu-\lambda_q)x}}{G(k,x)} \cdot \frac{[(\mu-\lambda_q)x]^k}{k!}, \quad k \geq 0, \tag{22}$$

where for each $k \geq 1$, $G(k,x)$ is the c.d.f. of a k-phase Erlang random variable with parameter $\mu - \lambda_q$ and $G(0,x) \equiv 1$.

Proof. If the query is transmitted to the kth node, then it had k successful prior transmissions without expiring. As before, let T_i denote the sojourn time at the ith visited node, $i \geq 0$. Because each node's transmission queue is modeled as a stable $M/M/1$ queue, $\{T_i : i \geq 0\}$ is an i.i.d. sequence of random variables with parameter $\mu - \lambda_q$. Denote by Y_k the total time elapsed from the moment a query is generated at an uninformed node up to and including its kth transmission, i.e.,

$$Y_k = \sum_{i=0}^{k-1} T_i,$$

where Y_k is a k-phase Erlang random variable with parameter $\mu - \lambda_q$. It is well-known (cf. [27]) that, for $k \geq 1$, the c.d.f. of Y_k is

$$G(k,x) \equiv \mathbb{P}(Y_k \leq x) = 1 - \sum_{n=0}^{k-1} e^{-(\mu-\lambda_q)x} \frac{[(\mu-\lambda_q)x]^n}{n!}.$$

We can express the conditional probability $p_{02}(k,x)$ in terms of the random variables Y_k and Y_{k+1} by noting that

$$p_{02}(k,x) = \mathbb{P}(Y_{k+1} > x | Y_k \leq x)$$

is the probability the query lifetime ends at the kth visited node while awaiting its $(k+1)$st transmission, given it had successfully made k prior transmissions and was active just before joining the kth node's transmission queue. When $k = 0$, $p_{02}(0, x)$ is the probability the query lifetime ends in the transmission queue of the query origin node given by

$$p_{02}(0, x) = \mathbb{P}(Y_1 > X | X = x) = \mathbb{P}(T_0 > x) = e^{-(\mu-\lambda_q)x}.$$

For $k \geq 1$, using basic conditional probability,

$$p_{02}(k, x) = \mathbb{P}(Y_{k+1} > x | Y_k \leq x) = \frac{G(k, x) - G(k+1, x)}{G(k, x)} = \frac{e^{-(\mu-\lambda_q)x}}{G(k, x)} \frac{[(\mu - \lambda_q)x]^k}{k!}.$$

■

The remaining probabilities in row 0 of $\mathbf{P}(k, x)$, $p_{00}(k, x)$ and $p_{01}(k, x)$, depend on whether or not the query revisits uninformed nodes during its lifetime when $r < \infty$. For this reason, it is necessary to first compute the probability that the query visits a particular node $n \in \mathcal{N}$ for the first time at its kth visit.

To this end, let U_k be the location of the query just after its kth hop and note that $\{U_k : k \geq 0\}$ is a time-homogeneous DTMC with state space $\mathcal{N} = \{1, \ldots, N\}$. Define its one-step transition probability matrix by $\boldsymbol{\theta}(r) = [\theta_{ij}(r)]_{i,j\in\mathcal{N}}$. As in section 2, for $j \neq i$, let $\rho(i, j) = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ and let $d_i(r)$ be the degree of node $i \in \mathcal{N}$. Assuming any neighbor of the current node is equally likely to receive a query transmission, for $i, j \in \mathcal{N}$ such that $j \neq i$, the transition probability $\theta_{ij}(r)$ is

$$\theta_{ij}(r) = \begin{cases} 1/d_i(r), & \text{if } \rho(i, j) \leq r, \\ 0, & \text{if } \rho(i, j) > r. \end{cases}$$

(Note that $\theta_{ii}(r) = 0$ for all $i \in \mathcal{N}$ as a query cannot be transmitted to the current node.)

Now, to account for revisiting effects, let $q(k, r)$ be the probability that a query (or event agent) visits a distinct (previously unvisited) node at the kth visit, and let $u_r(i, j, k)$ be the probability of visiting node j *at least once* before the $(k + 1)$st visit, given that the query (or agent) originates at node i. Let $w_r(i, j, k)$ denote the probability the query visits state j for the first time on the kth visit, given it originated at node i. We have the following lemma.

Lemma 3 *For each $k \in \mathbb{N}$ and $r \in (0, \infty)$,*

$$q(k, r) \approx \widehat{q}(k, r) = \frac{1}{N} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}\setminus\{i\}} [u_r(i, j, k) - u_r(i, j, k - 1)] \tag{23}$$

where

$$u_r(i, j, k) = \begin{cases} \theta_{ij}(r) + \sum_{m\in\mathcal{N}\setminus\{j\}} \theta_{im}(r)\, u_r(m, j, k - 1), & k \geq 1, \\ 0, & k = 0. \end{cases}$$

Proof. The lemma is proved using standard results for DTMCs. Specifically, define

$$T^r_{ij} = \inf\{k \geq 1 : U_k = j | U_0 = i\}$$

as the first passage time to node $j \in \mathcal{N}$, given that the query (or event agent) was generated at node $i \in \mathcal{N}$. Then,

$$u_r(i, j, k) = \mathbb{P}(T^r_{ij} \leq k),$$

and these probabilities can be obtained recursively by conditioning on the location of the query after its first transmission. The derivation is similar to that outlined in Theorem 4.1 of [27] and shows that for $k \geq 1$,

$$u_r(i,j,k) = \theta_{ij}(r) + \sum_{m \in \mathcal{N} \setminus \{j\}} \theta_{im}(r)\, u_r(m,j,k-1), \quad i,j \in \mathcal{N},$$

where $u_r(i,j,0) \equiv 0$ for each $i,j \in \mathcal{N}$. Using $u_r(i,j,k)$, the probability the query's first visit to node j is the kth visit, given the query originated at node i, is

$$w_r(i,j,k) \equiv \mathbb{P}(T_{ij}^r = k) = u_r(i,j,k) - u_r(i,j,k-1), \quad k \geq 1.$$

Assuming a query is generated at any $i \in \mathcal{N}$ with equal probability (i.e., $\mathbb{P}(U_0 = i) = 1/N$ for all $i \in \mathcal{N}$), via unconditioning, the approximate probability a query visits a distinct node at the kth visit is

$$q(k,r) \approx \widehat{q}(k,r) = \frac{1}{N} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} w_r(i,j,k), \quad k \geq 1.$$

■

Lemma 3 facilitates simple approximations for the transition probabilities $p_{00}(k,x)$ and $p_{01}(k,x)$, $k \geq 0$, which are provided in the next proposition.

Proposition 6 *The transition probabilities $p_{00}(k,x)$ and $p_{01}(k,x)$, $k \geq 0$, are respectively approximated by*

$$p_{00}(k,x) \approx [1 - \widehat{q}(k+1,r)(1-\pi_0(r))]\,[1 - p_{02}(k,x)], \tag{24}$$

$$p_{01}(k,x) \approx \widehat{q}(k+1,r)[1-\pi_0(r)]\,[1 - p_{02}(k,x)]. \tag{25}$$

Proof. This approximation assumes that if node i is uninformed when a query first visits the node, it remains uninformed during any subsequent visits to node i by the same query. We justify this assumption by noting that the mean recurrence time to node i is proportional to r. To approximate $p_{00}(k,x)$, condition on whether or not the $(k+1)$st visited node is distinct. First, given the query does not expire at the kth visited node, the $(k+1)$st visited node is not distinct with probability $1 - \widehat{q}(k+1,r)$. In the second case, given the query does not expire at the kth visited node, the $(k+1)$st node is distinct with probability $\widehat{q}(k+1,r)$, and it is uninformed with probability $\pi_0(r)$. Therefore, the probability of locating an uninformed node at the $(k+1)$st visit, given the query was active just before joining the transmission queue of kth node is, for $k \geq 0$,

$$\begin{aligned} p_{00}(k,x) &\approx [1-\widehat{q}(k+1,r)][1-p_{02}(k,x)] + \widehat{q}(k+1,r)\pi_0(r)[1-p_{02}(k,x)] \\ &= [1-\widehat{q}(k+1,r)(1-\pi_0(r))]\,[1-p_{02}(k,x)]. \end{aligned}$$

To approximate $p_{01}(k,x)$, note that the query moves from state 0 (active) to state 1 (answered) if it was successfully transmitted from the kth visited node to a distinct node that is informed. Therefore, for $k \geq 0$,

$$p_{01}(k,x) \approx \widehat{q}(k+1,r)[1-\pi_0(r)][1-p_{02}(k,x)].$$

■

Using the approximation of $\mathbf{P}(k,x)$, we now provide improved approximations for the WSN traffic rates, the steady state proportion of time nodes are uninformed, and the steady state proportion of failed queries. It was shown in Section 3 that, if each sensor's range is such that all $N-1$ other nodes belong to its neighborhood, the total arrival rate of witnessed events (both local and external) to the node's event table is

$$\Lambda \approx \widehat{\Lambda} = \lambda \left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha}\right].$$

The approximation $\widehat{\Lambda}$ does not account for the revisiting effects noted in this section. The following result uses $\widehat{q}(k,r)$ to correct for revisits and improve the approximate total arrival rate to the event table. To distinguish these values, let $\Lambda(r)$ be the total arrival rate of local and external events as a function of r. Then we can write

$$\begin{aligned}\Lambda(r) \approx \widehat{\Lambda}(r) &= \lambda + \lambda\, \bar{d}(r)\left(\frac{\widehat{q}(1,r)(1-\alpha)}{\bar{d}(r)} + \frac{\widehat{q}(2,r)(1-\alpha)^2}{\bar{d}(r)} + \cdots + \frac{\widehat{q}(\ell,r)(1-\alpha)^\ell}{\bar{d}(r)}\right) \\ &= \lambda\left[1+\sum_{i=1}^{\ell}\widehat{q}(i,r)(1-\alpha)^i\right],\end{aligned}$$

where $\bar{d}(r)$ is the network's average node degree. Using $\widehat{\Lambda}(r)$, the steady state proportion of time nodes are uninformed, $\pi_0(r)$, is

$$\pi_0(r) \approx \exp\left[-\frac{\lambda}{\delta}\left(1+\sum_{i=1}^{\ell}\widehat{q}(i,r)(1-\alpha)^i\right)\right]. \tag{26}$$

Equation (26) is used to compute the elements of $\mathbf{P}(k,x)$, namely $p_{00}(k,x)$ and $p_{01}(k,x)$ via (24) and (25), respectively. These lead to the limiting matrix $\mathbf{A}(x)$, from which we obtain the limiting probability $v_2(x)$ via (20). Finally, we obtain Δ_r via (21). The asymptotic validity of this approximation is discussed in the next subsection.

4.3 Asymptotic Validity of Approximation

In this subsection, we show that the finite transmission range approximation is asymptotically valid by proving that, for large N, the proportion of query failures converges to Δ_∞ as $r\to\infty$. To this end, we have the following important lemma.

Lemma 4 *For large N, as $r\to\infty$, $\widehat{q}(k,r)\to 1$ for each $k\in\mathbb{N}$.*

Proof. First note that

$$\lim_{r\to\infty} d_i(r) = \lim_{r\to\infty}\sum_{j\in\mathcal{N}\setminus\{i\}}\mathbf{1}(\rho(i,j)\le r) = N-1.$$

Therefore, for $i,j\in\mathcal{N}$ with $j\neq i$,

$$\theta_{ij}(r) = \frac{1}{d_i(r)} \to \frac{1}{N-1}$$

as $r \to \infty$. By induction on $k \in \mathbb{N}$, we now characterize the limiting behavior of $u_r(i,j,k)$ as $r \to \infty$. For $k = 1$, note that $u_r(i,j,1) = \theta_{ij}(r) \to 1/(N-1)$. For $k = 2$, it is easy to show that

$$\begin{aligned}
\lim_{r\to\infty} u_r(i,j,2) &= \lim_{r\to\infty} \left(\theta_{ij}(r) + \sum_{m \in \mathcal{N}\setminus\{j\}} \theta_{im}(r)\, u_r(m,j,1) \right) \\
&= \frac{1}{N-1} + \sum_{m\in\mathcal{N}\setminus\{i,j\}} \left(\frac{1}{N-1}\right)^2 \\
&= \frac{2}{N-1} + O(N^{-2}),
\end{aligned}$$

where $O(N^{-2}) \to 0$ as $N \to \infty$. For the inductive step, assume $u_r(i,j,n) \to n/(N-1) + O(N^{-2})$ for any $n \in \mathbb{N}$. With some simplification we obtain

$$\begin{aligned}
\lim_{r\to\infty} u_r(i,j,n+1) &= \lim_{r\to\infty} \left(\theta_{ij}(r) + \sum_{m\in\mathcal{N}\setminus\{j\}} \theta_{im}(r)\, u_r(m,j,n) \right) \\
&= \frac{1}{N-1} + \sum_{m\in\mathcal{N}\setminus\{i,j\}} \frac{1}{N-1}\left[\frac{n}{N-1} + O(N^{-2})\right] = \frac{n+1}{N-1} + O(N^{-2}),
\end{aligned}$$

which completes the induction proof. Therefore, for each $k \in \mathbb{N}$ and $i, j \in \mathcal{N}$ with $j \neq i$,

$$\lim_{r\to\infty} w_r(i,j,k) \equiv \lim_{r\to\infty} [u_r(i,j,k) - u_r(i,j,k-1)] = \frac{1}{N-1},$$

and consequently,

$$\lim_{r\to\infty} \widehat{q}(k,r) = \lim_{r\to\infty} \frac{1}{N} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}\setminus\{i\}} w_r(i,j,k) = \frac{1}{N} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}\setminus\{i\}} \frac{1}{N-1} = 1.$$

■

Lemma 4 is used to prove Theorem 2 which asserts that, as $r \to \infty$, the approximate event arrival rate, proportion of time uninformed, and the proportion of query failures all converge appropriately to their respective infinite-range counterparts for large networks.

Theorem 2 *For large N, as $r \to \infty$, $\widehat{\Lambda}(r) \to \Lambda$, $\pi_0(r) \to \pi_0$, and $\Delta_r \to \Delta_\infty$.*

Proof. By Lemma 4, $\widehat{q}(k,r) \to 1$ for each $k \in \mathbb{N}$ as $r \to \infty$. Therefore,

$$\begin{aligned}
\lim_{r\to\infty} \widehat{\Lambda}(r) = \lim_{r\to\infty} \left[\lambda + \lambda \sum_{i=1}^{\ell} \widehat{q}(i,r)(1-\alpha)^i \right] &= \lambda \lim_{r\to\infty} \sum_{i=0}^{\ell} \widehat{q}(i,r)(1-\alpha)^i \\
&= \lambda \left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha} \right] = \Lambda.
\end{aligned}$$

Consequently, by (26) we see that $\pi_0(r) \to \pi_0$ as $r \to \infty$. Next, recall that for $r < \infty$,

$$\Delta \approx \Delta_r = \int_0^\infty v_2(x) \mathrm{d}H(x)$$

where $v_2(x) = \lim_{k\to\infty} v_2^k(x)$. So as $r \to \infty$, we substitute $\boldsymbol{v}^0(x) = (\pi_0,\, 1-\pi_0,\, 0)$ in the expression

$$\boldsymbol{v}^k(x) = \boldsymbol{v}^0(x) \prod_{n=0}^{k-1} \mathbf{P}(n,x).$$

Using (14), (22), (24), and (25), we now show by induction on k that the elements of $\boldsymbol{v}^k(x)$ are

$$v_0^k(x) = \pi_0^{k+1} G(k,x), \tag{27}$$

$$v_1^k(x) = \sum_{n=1}^{k} [\pi_0^n (1-\pi_0) G(n,x)] + 1 - \pi_0, \tag{28}$$

$$v_2^k(x) = \pi_0 \left[1 - \pi_0^{k-1} G(k,x) - (1-\pi_0) \sum_{n=1}^{k-1} \pi_0^{n-1} G(n,x) \right]. \tag{29}$$

For $k = 1$, applying (15) with $\boldsymbol{v}^0(x) = (\pi_0,\, 1-\pi_0,\, 0)$, it is easy to see that

$$\begin{aligned} v_0^1(x) &= \pi_0^2 G(1,x), \\ v_1^1(x) &= \pi_0(1-\pi_0)G(1,x) + 1 - \pi_0, \\ v_2^1(x) &= \pi_0[1 - G(1,x)], \end{aligned}$$

where the summation in (29) is 0 when $k = 1$. Similarly, for $k = 2$,

$$\begin{aligned} v_0^2(x) &= \pi_0^3 G(2,x), \\ v_1^2(x) &= \pi_0^2(1-\pi_0)G(2,x) + \pi_0(1-\pi_0)G(1,x) + 1 - \pi_0, \\ v_2^2(x) &= \pi_0\left[1 - (1-\pi_0)G(1,x) - \pi_0 G(2,x)\right]; \end{aligned}$$

therefore, the result holds for $k = 1, 2$. For the inductive step, assume that (27)–(29) hold for an arbitrary $m \in \mathbb{N}$. Then, after some matrix algebra, we obtain

$$\begin{aligned} v_0^{m+1}(x) &= \pi_0^{m+2} G(m+1,x), \\ v_1^{m+1}(x) &= \sum_{n=1}^{m+1} [\pi_0^n (1-\pi_0) G(n,x)] + 1 - \pi_0, \\ v_2^{m+1}(x) &= \pi_0 \left[1 - \pi_0^{m} G(m+1,x) - (1-\pi_0) \sum_{n=1}^{m} \pi_0^{n-1} G(n,x) \right], \end{aligned}$$

and the induction proof is complete. Now, as $r \to \infty$,

$$\begin{aligned} v_2(x) = \lim_{k\to\infty} v_2^k(x) &= \lim_{k\to\infty} \left[\pi_0 \left(1 - \pi_0^{k-1} G(k,x) - (1-\pi_0) \sum_{n=1}^{k-1} \pi_0^{n-1} G(n,x) \right) \right] \\ &= \pi_0 \left[1 - (1-\pi_0) \sum_{n=1}^{\infty} \pi_0^{n-1} G(n,x) \right]. \end{aligned}$$

We obtain a closed-form expression for $v_2(x)$ via its Laplace-Stieltjes transform, $\tilde{v}_2(s)$, given by

$$
\begin{aligned}
\tilde{v}_2(s) \equiv \int_0^\infty e^{-sx}\mathrm{d}v_2(x) &= \pi_0\left[1-\frac{1-\pi_0}{\pi_0}\sum_{n=1}^{\infty}\left(\frac{\pi_0(\mu-\lambda_q)}{\mu-\lambda_q+s}\right)^n\right] \\
&= \pi_0\left[1-\frac{1-\pi_0}{\pi_0}\left(\sum_{n=0}^{\infty}\left(\frac{\pi_0(\mu-\lambda_q)}{\mu-\lambda_q+s}\right)^n-1\right)\right] \\
&= \pi_0\left[1-\frac{(1-\pi_0)(\mu-\lambda_q)}{(1-\pi_0)(\mu-\lambda_q)+s}\right].
\end{aligned}
$$

Now, $\tilde{v}_2(s)$ can be inverted analytically to obtain

$$
v_2(x) = \mathcal{L}^{-1}\left\{\frac{\tilde{v}_2(s)}{s}\right\} = \pi_0\, e^{-(1-\pi_0)(\mu-\lambda_q)x},
$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform operator. Finally, we obtain

$$
\begin{aligned}
\lim_{r\to\infty}\Delta_r &= \int_0^\infty \pi_0\, e^{-(1-\pi_0)(\mu-\lambda_q)x}\mathrm{d}H(x) \\
&= \int_0^\infty \mathbb{P}(\widetilde{T} > X | I_n = 0, X = x)\pi_0 \mathrm{d}H(x) \\
&= \mathbb{P}(\widetilde{T} > X) \\
&= \Delta_\infty.
\end{aligned}
$$

■

In this section, we have modeled query dynamics using a temporally-nonhomogeneous DTMC. The elements of the transition probability matrix (14) are provided by Lemma 2 and Proposition 6. We derived a new approximation for the proportion of query failures via (21) by examining the limiting behavior of the DTMC. This analysis explicitly accounts for the dependence of the network's performance on a limited transmission range and query revisiting by approximating the probability, $q(k, r)$, that a query visits a distinct node on its kth visit. This probability also captures the boundary effect – namely that nodes near the borders of the deployment region are likely to have fewer neighbors, and hence, an increased likelihood of transmitting to previously visited nodes. In Section 5, we illustrate and assess the quality of the finite- and infinite-range approximations by comparing the steady state proportion of time uninformed and proportion of query failures with results obtained by a commercial network simulator.

5 Numerical Examples and Validation

The analytical approximations of Sections 3 and 4 provide a relatively easy way to evaluate the behavior of query-based wireless sensor networks. In this section, we assess the quality of these approximations by comparing them with simulated values obtained using the OPNET commercial network simulator. Presented herein are summary tables and figures for uniform-topology networks with a variety of distributional assumptions and sensor transmission ranges. For each experiment, the minimum transmission range was chosen to ensure a connected network with probability $p =$

0.9999 using (1). Results for 1000- and 5000-node networks first are provided before presenting an extensive validation study that examines impact of our model assumptions.

For each scenario, we compute the maximum absolute deviation (MAD) between the approximated value and its simulated counterpart over a finite set of TTL values, $L \equiv \{1, 2, \ldots, 30\}$. We choose this set because, for many typical wireless applications, a TTL counter between 3 and 25 is suitable. For each $\ell \in L$, let π_0^ℓ be the approximate steady state proportion of time nodes are uninformed, assuming $r = \infty$, which is obtained via (7), i.e.,

$$\pi_0^\ell = \exp\left[-\frac{\lambda}{\delta}\left(\frac{1-(1-\alpha)^{\ell+1}}{\alpha}\right)\right].$$

Similarly, let $\pi_0^\ell(r)$ be the same value, assuming $r < \infty$, obtained by (26). That is,

$$\pi_0^\ell(r) = \exp\left[-\frac{\lambda}{\delta}\left(1 + \sum_{i=1}^{\ell} \widehat{q}(i, r)(1-\alpha)^i\right)\right].$$

For both cases, the probability α is approximated using the fixed point algorithm described in Section 3. To express the dependence of Δ on the TTL value ℓ, let Δ_∞^ℓ and Δ_r^ℓ denote the steady proportion of query failures when $r = \infty$ and $r < \infty$, respectively. Using (13) and (21), respectively, we compute

$$\Delta_\infty^\ell \equiv \pi_0^\ell \int_0^\infty \exp\left[-(1-\pi_0^\ell)(\mu - \lambda_q)x\right] \mathrm{d}H(x)$$

and

$$\Delta_r^\ell \equiv \int_0^\infty v_2(x)\mathrm{d}H(x),$$

where $v_2(x)$ is obtained via (20). In cases where the integrals cannot be evaluated in closed form, we perform numerical integration via the trapezoidal rule. Finally, we define $\pi_0^s(\ell)$ as the simulated steady state proportion of time nodes are uninformed, and $\Delta^s(\ell)$ as the simulated steady state proportion of query failures when the TTL counter is $\ell \in L$.

The MAD between the true (simulated) values and their corresponding analytical approximations are therefore

$$D_\pi \equiv \max_{\ell \in L} \left|\pi_0^s(\ell) - \widehat{\pi}_0(\ell)\right|, \tag{30}$$

where $\widehat{\pi}_0(\ell) = \pi_0^\ell$ if $r = \infty$, and $\widehat{\pi}_0(\ell) = \pi_0^\ell(r)$ if $r < \infty$. Similarly, let

$$D_\Delta \equiv \max_{\ell \in L} \left|\Delta^s(\ell) - \widehat{\Delta}_0(\ell)\right|, \tag{31}$$

where $\widehat{\Delta}(\ell) = \Delta_\infty^\ell$ if $r = \infty$, and $\widehat{\Delta}(\ell) = \Delta_r^\ell$ if $r < \infty$. For Examples 1 and 2 that follow, a few parameter values were held constant; these values are summarized in Table 1. Moreover, we assumed event lifetimes are exponentially distributed with mean $1/\delta$ in these two cases, but this assumption is relaxed in Example 3.

The analytical approximations were coded in the C programming language and executed in Microsoft® Visual Studio® 2008 on a personal computer equipped with an Intel® Core™ 2 Duo CPU operating at 3.00GHz with 2.00 GB of RAM. The simulated values were obtained via a discrete-event simulation model created in the OPNET Modeler® Wireless Suite v. 15. Ten

Table 1: Summary of parameter values for OPNET simulation: Examples 1 and 2.

Parameter	Parameter description	Value
μ	Transmitter's exponential transmission rate	5.000
λ	Poisson rate of locally-witnessed events (for all $n \in \mathcal{N}$)	0.005
γ	Poisson rate of locally-generated queries (for all $n \in \mathcal{N}$)	0.050
$1/\delta$	Mean event lifetime	10.000
$1/\beta$	Mean query lifetime (for all distributions)	5.000

(10) independent replications were performed for each $\ell \in L$ to ensure a standard error less than 5×10^{-4}. The plotted simulated values represent the average of the 10 replications. The run length was 3720s, including a 120s warm-up period for each replication. The simulation experiments were conducted on a personal computer equipped with an Intel® Core™ i7 CPU operating at 2.67GHz with 2.00 GB of RAM.

Example 1: 1000-Node Network: Here, we present results for a 1000-node wireless sensor network with nodes distributed randomly in a 3335m $\times$ 3335m sensor field. The node density is $\psi \approx 9.00 \times 10^{-5}$ nodes per square meter. To ensure a connected network with probability 0.9999, the minimum required sensor transmission range is $r = 239$m. Therefore, we considered the following transmission ranges: 350m, 500m, 1000m, 5000m. Table 2 summarizes the MAD in the proportion of time uninformed using each transmission range. The column labeled "$r = \infty$" corresponds to the infinite transmission range approximation, and the column labeled "$r < \infty$" is the finite range approximation.

Table 2: MAD in the proportion of time uninformed (D_π) when $N = 1000$.

Query lifetime	350m		500m		1000m		5000m	
	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
Exponential(0.2)	0.0523	0.0045	0.0289	0.0065	0.0202	0.0116	0.0024	0.0060
Triangular(0.1, 5.0, 9.9)	0.0529	0.0059	0.0296	0.0055	0.0173	0.0088	0.0173	0.0131
Uniform(0.1, 9.9)	0.0507	0.0068	0.0285	0.0041	0.0171	0.0093	0.0039	0.0070
Rayleigh(5.645)	0.0511	0.0050	0.0298	0.0055	0.0176	0.0088	0.0043	0.0069
Weibull(3.0, 5.6)	0.0511	0.0061	0.0302	0.0061	0.0173	0.0088	0.0035	0.0072

Table 2 indicates an order of magnitude improvement in the MAD by using the finite-range approximation, especially when the actual transmission range in the simulation model is small (350m). Because queries are more likely to revisit neighbors when the transmission range is small, the difference between the two approximations is quite pronounced. Table 2 also illustrates consistency in the performance of the approximations when the query lifetime distribution is not exponential. Specifically, the magnitudes of the MAD values for the non-exponential cases are generally consistent with those of the exponential case. In the worst case, the MAD of the triangular lifetime distribution exceeds the MAD of the exponential by 0.01494 (5000m range assuming $r = \infty$); however, on average, the increase in the MAD over all the non-exponential cases is 0.0022, or roughly 0.2%. Figure 3 depicts the performance of the approximations and reveals that the finite range approximation is superior to the infinite range approximation for all TTL values when r is small. Indeed, the gap between the latter approximation and OPNET simulation values increases with ℓ since the revisiting effect is more pronounced when the TTL value is large. For larger ranges, the approximations nearly coincide and both closely track the simulated values.

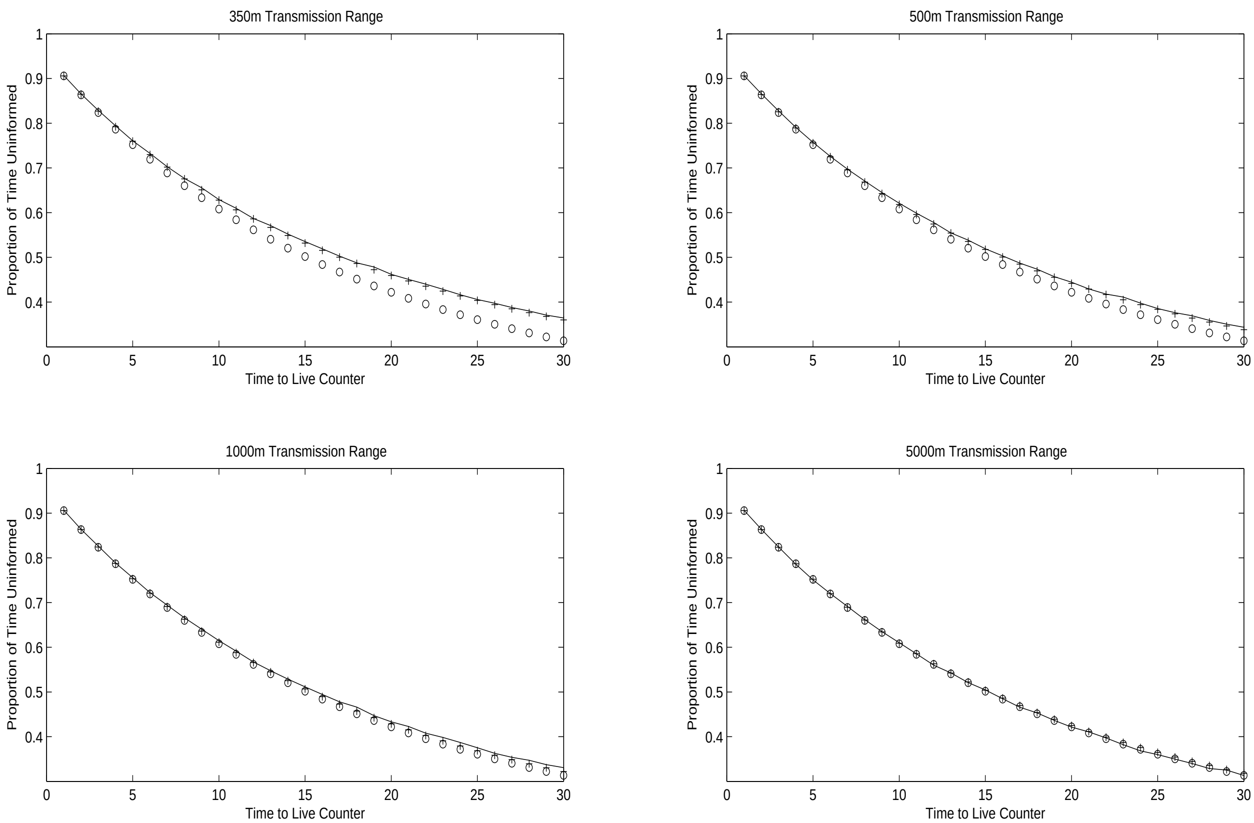

Figure 3: Comparison of π_0 values with Weibull query lifetimes ($N = 1000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

Results for the steady state proportion of query failures are summarized in Table 3. Both approximation schemes perform extremely well (the maximum absolute deviation over all cases is less than 0.049). It is also worth noting that the finite range approximation outperforms the infinite range approximation, particularly when r is relatively small. The results here are also consistent for non-exponential query lifetimes. In the worst case, the MAD of the Rayleigh lifetime distribution exceeds the MAD of the exponential by 0.0215 (1000m range assuming $r < \infty$); on average, the increase in the MAD over all the non-exponential cases is 0.0065, or roughly 0.65%. Figure 4 graphically depicts the four cases.

Example 2: 5000-Node Network: Here, we consider a 5000-node wireless sensor network with nodes deployed in the same region as the 1000-node case but with node density $\psi \approx 4.50 \times 10^{-4}$ nodes per square meter. To ensure a connected network with probability 0.9999, the minimum required sensor transmission range is $r = 112$m. Therefore, we considered the following transmission ranges: 115m, 350m, 500m, and 5000m. Table 4 illustrates the quality of both approximations for the

Table 3: MAD in the proportion of failed queries (D_Δ) when $N = 1000$.

Query lifetime	350m		500m		1000m		5000m	
	$r=\infty$	$r<\infty$	$r=\infty$	$r<\infty$	$r=\infty$	$r<\infty$	$r=\infty$	$r<\infty$
Exponential(0.2)	0.0371	0.0246	0.0168	0.0103	0.0047	0.0055	0.0060	0.0052
Triangular(0.1, 5.0, 9.9)	0.0459	0.0301	0.0170	0.0128	0.0030	0.0008	0.0082	0.0024
Uniform(0.1, 9.9)	0.0383	0.0258	0.0178	0.0121	0.0035	0.0015	0.0051	0.0016
Rayleigh(5.645)	0.0237	0.0127	0.0065	0.0158	0.0256	0.0270	0.0255	0.0247
Weibull(3.0, 5.6)	0.0485	0.0306	0.0230	0.0149	0.0031	0.0014	0.0022	0.0014

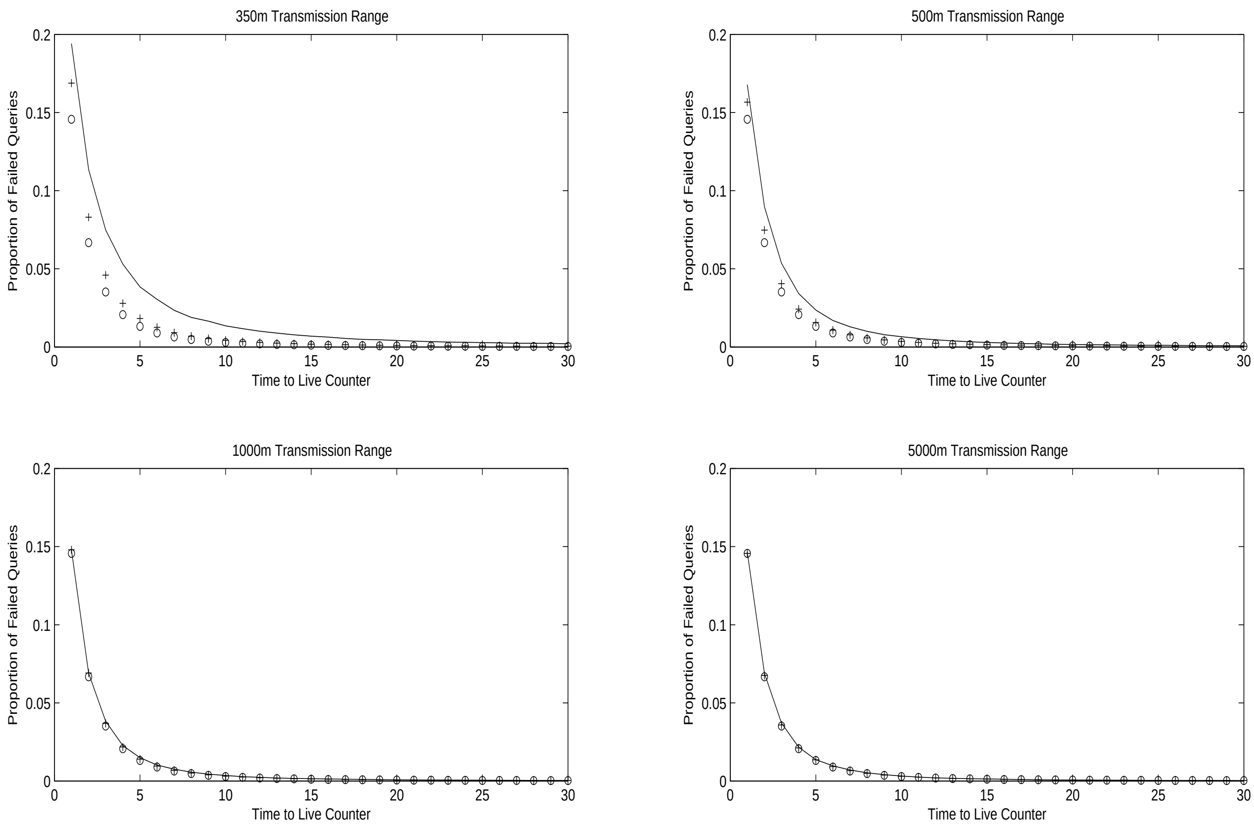

Figure 4: Comparison of Δ values with Weibull query lifetimes ($N = 1000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

5000-node network. The maximum absolute deviation for the proportion of time uninformed is less than 0.082 for $r = \infty$, and it is reduced to, at most, 0.0174 when the revisiting effect is included. As before, the superiority of the finite range approximation is generally more pronounced for smaller transmission ranges. Figure 5 depicts the simulated and approximated values of π_0 when the query lifetime follows a triangular distribution. When the transmission range is small (115m), we see some discrepancy between the two approximation schemes. However, for the other three cases, the approximations nearly coincide and are very similar to the simulated results ($D_\pi < 0.011$).

Next, we compare the maximum absolute deviation of the proportion of query failures. Table

Table 4: MAD in the proportion of time uninformed (D_π) when $N = 5000$.

Query lifetime	115m		350m		500m		5000m	
	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
Exponential(0.2)	0.0605	0.0172	0.0107	0.0019	0.0082	0.0031	0.0040	0.0049
Triangular(0.1, 5.0, 9.9)	0.0612	0.0174	0.0100	0.0015	0.0068	0.0026	0.0053	0.0062
Uniform(0.1, 9.9)	0.0819	0.0054	0.0105	0.0013	0.0074	0.0025	0.0048	0.0058
Rayleigh(5.645)	0.0595	0.0172	0.0101	0.0016	0.0074	0.0026	0.0051	0.0060
Weibull(3.0, 5.6)	0.0611	0.0174	0.0099	0.0014	0.0068	0.0028	0.0073	0.0083

5 shows that the maximum deviation values are bounded above by 0.0725. Again, the finite range approximation outperforms the infinite range version when the transmission range is small. However, for larger ranges, the results nearly coincide and closely track the simulated values.

Table 5: MAD in the proportion of query failures (D_Δ) when $N = 5000$.

Query lifetime	115m		350m		500m		5000m	
	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
Exponential(0.2)	0.0493	0.0283	0.0046	0.0022	0.0045	0.0043	0.0061	0.0044
Triangular(0.1, 5.0, 9.9)	0.0588	0.0333	0.0052	0.0026	0.0013	0.0024	0.0044	0.0045
Uniform(0.1, 9.9)	0.0724	0.0493	0.0044	0.0047	0.0051	0.0021	0.0078	0.0023
Rayleigh(5.645)	0.0371	0.0170	0.0214	0.0192	0.0265	0.0225	0.0286	0.0228
Weibull(3.0, 5.6)	0.0619	0.0351	0.0071	0.0064	0.0025	0.0041	0.0039	0.0021

Figure 6 graphically depicts the simulated and approximated values of Δ and illustrates the high quality of the approximations. In the worst case (115m), the MAD is less than 0.0725 and 0.05 for $r = \infty$ and $r < \infty$, respectively.

Example 3: Model Validation: Finally, we conducted an experiment to validate the approximations when some of the model assumptions are violated. For the benchmark simulation experiments presented here, events arrive according to a renewal process with a specified (non-exponential) interarrival time distribution (i.e., the event arrival process is not Poisson). This experiment also employs non-exponential event agent and query lifetimes, both of which are used in the approximations of Sections 3 and 4.

Table 6 provides a summary of the numerical results for 45 distinct test cases using a 1000-node wireless sensor network with nodes distributed randomly in a 3335m $\times$ 3335m sensor field. The node density is $\psi \approx 9.00 \times 10^{-5}$ nodes per square meter. To ensure a connected network with probability 0.9999, the minimum required sensor transmission range is $r = 239$m; therefore, we set $r = 350$m.

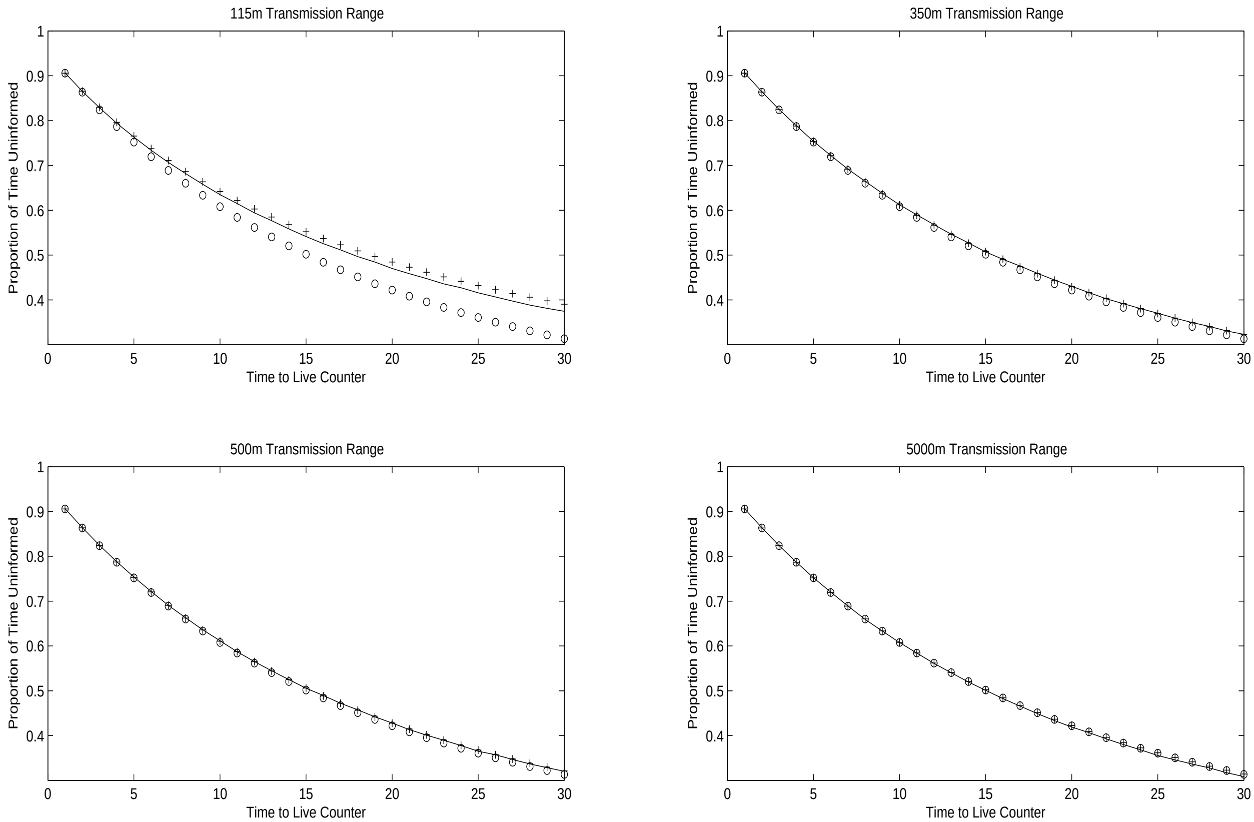

Figure 5: Comparison of π_0 values with triangular query lifetimes ($N = 5000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

Table 6 reveals some very interesting results. First, we note that the performance of the finite-range approximation is similar to that reported in Example 1 which assumed Poisson-generated events. Specifically, despite the fact that the event arrival process is distinctly non-Poisson, and the query and event lifetimes are not exponential, the benchmark proportion of failed queries is approximated very closely using the finite-range approximation. Over all 45 test cases, the observed maximum absolute deviation between the simulated proportion of query failures and the approximated values (using the finite-range model) is about 0.0319, and the average absolute deviation is about 0.0237. Considering the complexity of event agent and query dynamics, and the random nature of arrivals and transmissions, we consider these discrepancies to be quite acceptable. For example, if an engineer is interested in selecting the optimal TTL value that minimizes energy expenditure while satisfying a quality-of-service constraint based on the proportion of query failures, then our approximation can be used to quickly assess the query failure rate using alternative TTL values. Alternatively, one might consider jointly optimizing the TTL value and the transmission range of the sensors in order to maximize network lifetime, subject to an upper limit on the proportion of query failures. Here too, our approximations can be used, in lieu of a simulation model, to quickly evaluate alternative solutions. For such purposes, an average deviation on the order of 0.0237 is tolerable. The results of this section are significant because they provide empirical evidence that the approximations are not heavily influenced by the Poisson arrival assumption imposed at the event tables and the transmission queues. This hypothesis is also supported theoretically in that the arrival streams at the event tables and transmission queues are superpositions of multiple in-

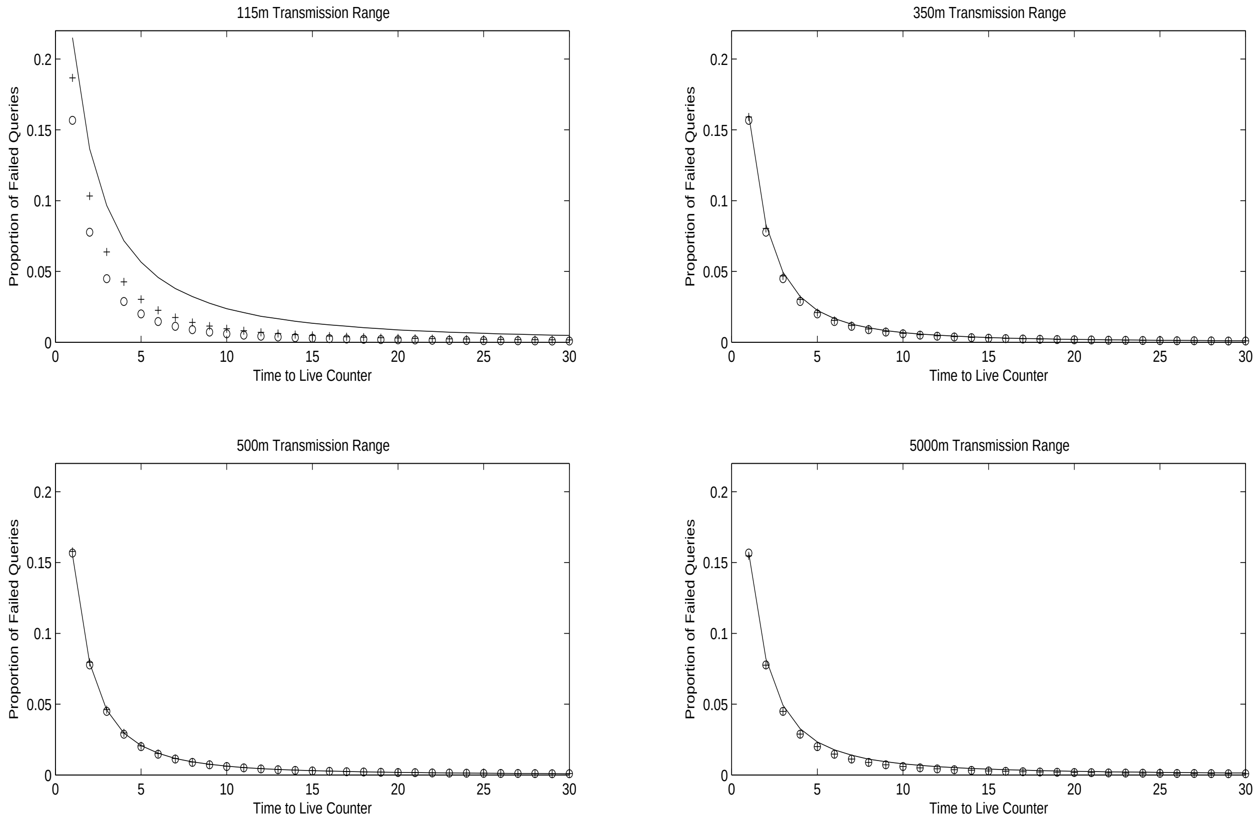

Figure 6: Comparison of Δ values with triangular query lifetimes ($N = 5000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

dependent sources. Albin [5] argued that such superpositions are well approximated by a Poisson process if the number of sources is large (say 10 or more), and the traffic intensity (the traffic arrival rate multiplied by the expected service time) at the node is light or moderate. However, the approximation can be poor if the traffic intensity is high, even if the number of sources is large. We conjecture that the Poisson assumption is adequate here because N is large, and the rates at which events are witnessed and/or queries are generated are moderate.

This section has demonstrated that the approximations of Sections 3 and 4 are remarkably accurate, even when some key model assumptions are violated. Moreover, for each instance, the approximated proportion of time uninformed and proportion of query failures were computed in less than 20 minutes as compared to the OPNET simulation results, which required a minimum of 2 hours.

Table 6: MAD in the proportion of time uninformed (D_π) and proportion of failed queries (D_Δ).

Trial	Event interarrival time	Event lifetime	Query lifetime	D_π		D_Δ	
				$r=\infty$	$r<\infty$	$r=\infty$	$r<\infty$
1	Erlang(5, 40.0)	Erlang(4, 2.5)	Erlang(5, 1.0)	0.0711	0.0233	0.0421	0.0265
2	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Erlang(5, 1.0)	0.0693	0.0215	0.0428	0.0273
3	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Erlang(5, 1.0)	0.0596	0.0124	0.0424	0.0273
4	Erlang(5, 40.0)	Erlang(4, 2.5)	Rayleigh(5.645)	0.0696	0.0213	0.0201	0.0106
5	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Rayleigh(5.645)	0.0699	0.0220	0.0203	0.0106
6	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Rayleigh(5.645)	0.0594	0.0130	0.0213	0.0106
7	Erlang(5, 40.0)	Erlang(4, 2.5)	Triangular(0.1, 5.0, 9.9)	0.0687	0.0213	0.0399	0.0262
8	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0682	0.0206	0.0407	0.0267
9	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0584	0.0100	0.0400	0.0259
10	Erlang(5, 40.0)	Erlang(4, 2.5)	Uniform(0.1, 9.9)	0.0691	0.0218	0.0340	0.0209
11	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Uniform(0.1, 9.9)	0.0690	0.0211	0.0342	0.0205
12	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Uniform(0.1, 9.9)	0.0589	0.0118	0.0344	0.0211
13	Erlang(5, 40.0)	Erlang(4, 2.5)	Weibull(3.0, 5.6)	0.0699	0.0229	0.0424	0.0278
14	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Weibull(3.0, 5.6)	0.0677	0.0201	0.0427	0.0282
15	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Weibull(3.0, 5.6)	0.0584	0.0096	0.0428	0.0278
16	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Erlang(5, 1.0)	0.0739	0.0261	0.0438	0.0282
17	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Erlang(5, 1.0)	0.0713	0.0235	0.0429	0.0274
18	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Erlang(5, 1.0)	0.0615	0.0145	0.0433	0.0282
19	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Rayleigh(5.645)	0.0718	0.0235	0.0213	0.0106
20	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Rayleigh(5.645)	0.0728	0.0250	0.0215	0.0106
21	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Rayleigh(5.645)	0.0614	0.0150	0.0217	0.0114
22	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Triangular(0.1, 5.0, 9.9)	0.0713	0.0239	0.0410	0.0273
23	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0725	0.0249	0.0405	0.0265
24	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0523	0.0052	0.0418	0.0277
25	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Uniform(0.1, 9.9)	0.0709	0.0237	0.0343	0.0212
26	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Uniform(0.1, 9.9)	0.0717	0.0238	0.0356	0.0219
27	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Uniform(0.1, 9.9)	0.0618	0.0146	0.0296	0.0163
28	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Weibull(3.0, 5.6)	0.0718	0.0247	0.0437	0.0290
29	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Weibull(3.0, 5.6)	0.0721	0.0246	0.0431	0.0286
30	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Weibull(3.0, 5.6)	0.0519	0.0037	0.0438	0.0288
31	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Erlang(5, 1.0)	0.0737	0.0259	0.0453	0.0297
32	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Erlang(5, 1.0)	0.0675	0.0201	0.0459	0.0304
33	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Erlang(5, 1.0)	0.0630	0.0158	0.0457	0.0306
34	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Rayleigh(5.645)	0.0742	0.0260	0.0243	0.0127
35	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Rayleigh(5.645)	0.0737	0.0258	0.0234	0.0123
36	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Rayleigh(5.645)	0.0637	0.0167	0.0233	0.0133
37	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Triangular(0.1, 5.0, 9.9)	0.0751	0.0276	0.0435	0.0298
38	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0745	0.0269	0.0440	0.0300
39	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0634	0.0155	0.0444	0.0300
40	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Uniform(0.1, 9.9)	0.0749	0.0277	0.0369	0.0255
41	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Uniform(0.1, 9.9)	0.0749	0.0270	0.0356	0.0219
42	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Uniform(0.1, 9.9)	0.0634	0.0165	0.0379	0.0252
43	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Weibull(3.0, 5.6)	0.0728	0.0257	0.0458	0.0311
44	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Weibull(3.0, 5.6)	0.0746	0.0270	0.0464	0.0319
45	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Weibull(3.0, 5.6)	0.0648	0.0161	0.0469	0.0319

6 Conclusions

In this paper we have presented both single- and multi-hop models for evaluating the performance of large-scale WSNs with time-limited events and queries using a queueing-theoretic approach. The former model leads to an approximation for the steady state proportion of query failures that is insensitive to the network's size, while the latter model captures the realistic effects of a limited transmission range and is asymptotically valid. Both models can accommodate generally distributed (non-exponential) event agent and query lifetimes. The numerical results indicate that the approximations perform very well (as compared to results obtained via a commercial network simulator), even when several of the key model assumptions are violated; the maximum absolute deviation between the benchmark and approximated values is about 0.0319.

The main results can be used for optimally designing and/or operating large-scale query-based WSNs. Specifically, our models provide a proxy for energy expenditure (in the form of traffic rates), and the approximations can be used to optimize other operating parameters including (but not limited to) the transmission range and/or the TTL value so that a quality-of-service constraint is satisfied. For instance, one might be interested in optimally selecting ℓ and r to minimize energy expenditure while ensuring that the proportion of failed queries does not exceed a specified threshold. For this purpose, our procedures can be used to quickly evaluate and rank alternative operating policies without the need for costly and time-consuming simulation runs.

Although our models are mathematically valid, and the approximations are easy to compute, they currently lack the flexibility to account for some realistic features of WSNs. First, in the present framework, we assume that all transmissions are perfect (i.e., there are no fading effects or packet collisions) so that retransmissions are not necessary. In future work, it may be possible to model each transmission queue as a single-server retrial queueing station to account for event agents and queries that require retransmission. Second, it was assumed that event agents and queries are transmitted in the order in which they are received. However, it is more realistic to incorporate the deadlines of packets in the transmission queue so as to prioritize transmissions (e.g., giving preference to those queries with the smallest remaining lifetime). One approach is to consider real-time queueing network theory (see Lehoczky [28, 29]). Third, and finally, event agents and queries were assumed to use a random-walk protocol that does not exploit additional state information that can be used to improve routing and potentially reduce the overall proportion of query failures. In the future, it will be instructive to develop similar approximations for WSNs that use other common routing protocols.

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