Investigating Secant Lines with the TI-83

Matthew J. Saltzman

October 15, 2003

Here are some techniques that can be used with the TI-83 calculator to study secant lines and limits.

Recall that the slope of the secant line between the points $(x_0, f(x_0))$ and $(x_0 + h, f(x_0 + h))$ is given by

$$s = \frac{f(x_0 + h) - f(x_0)}{h}.$$

Note that for a value of x_0 fixed in advance, s is a function of the distance h between the two x-values.

Since we know that the secant line passes through the point $(x_0, f(x_0))$, we can use the point-slope form of a linear equation to find the equation for the actual secant line:

$$y = s(x - x_0) + f(x_0).$$

In the $Y =$ screen of your calculator, enter in the Y_2 field the formula for s as a function of h:

$$Y_2 = (Y_1(A + X) - Y_1(A))/X.$$

Here, the value of x_0 should be stored in the calculator memory cell A, and the value of h is represented by the function input variable X.

In the Y_3 field, enter the point-slope formula for the secant line:

$$Y_3 = Y_2(H) * (X - A) + Y_1(A).$$

Here, the input variable is the same x that is input to our original function $f(x)$. To draw the secant line, we need to fix in advance the value of h, and store this in the calculator memory cell H.

Now enter the formula for $f(x)$ in the Y_1 field, with input variable X representing x.

Turn on the graphs of Y_1 and Y_3, and turn off the graph of Y_2. If you graph around the region containing the points $(x_0, f(x_0))$ and $(x_0 + h, f(x_0 + h))$, you will see the curve $f(x)$ and the secant line connecting $(x_0, f(x_0))$ and $(x_0+h, f(x_0+h))$ together on the screen. Note that for points close together, it will be hard to distinguish the two graphs in the neighborhood of the secant points.

Now turn off the graphs for Y_1 and Y_3, and turn on the graph for Y_2. In addition, turn off the display of the axes (using WINDOW/FORMAT/AxesOff). The new graph (in the region of $X = 0$) will show the slope of the secant line as a function of h. Notice the missing pixel in the graph at the point $X = 0$. This appears because when $h = 0$, the secant slope formula requires computing $0/0$, which is invalid. If we zoom in on this point, we can get secant slopes for values of h close to zero, and we can see that they approach some value. This value (where the $X = 0$ point "should be") is the slope of the tangent line at $(x_0, f(x_0))$.