

# A Quantized Stochastic Modeling Approach for Fault Diagnosis of Lithium-ion Batteries

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**Abstract:** Safety and reliability are still key concerns for the Lithium-ion (Li-ion) battery systems in spite of their current popularity as energy storage solutions for transportation and other applications. To improve the overall reliability of the Li-ion batteries, the Battery Management Systems (BMS) should have the capabilities to detect different types of faults. Some of these faults can lead to catastrophic scenarios if they are not diagnosed early. In this paper, a stochastic approach of quantized systems is proposed for fault detection in Li-ion batteries. The scheme uses a quantized stochastic model derived from the equivalent circuit model of the battery to predict the most probable future states/outputs from the measured inputs and quantized outputs. Fault detection is achieved via comparison of the expected event and the actual event. To illustrate the effectiveness of the approach, the model parameters for commercial Li-ion battery cell have been extracted from experiments, and then faults are injected in simulation studies.

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**Keywords:** Stochastic Modeling, Quantized Systems, Lithium-ion Batteries, Fault Diagnosis.

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## 1. INTRODUCTION

Lithium-ion (Li-ion) battery technology is the leading candidate for energy storage in electrified automotive applications especially in Hybrid and Electric Vehicles (H/EVs). Several advantages such as high specific energy and power, low self-discharge, no memory effect, negligible environmental impact have made Li-ion batteries an attractive energy storage solution as compared to other battery chemistries. However, Li-ion batteries still suffer from the shortcomings of safety and reliability, which necessitates the need for advanced Battery Management Systems (BMSs) with diagnostic capabilities. These BMSs should be able to detect the faults in the batteries to help improve the overall reliability of the system. Different types of faults can happen in a Li-ion battery such as internal faults, sensor faults and actuation faults, which lead to battery degradation and potentially unsafe situations including many reported fires. This paper deals with one approach to the detection of these faults.

In battery literature, the real-time diagnostic problem in Li-ion batteries is much less discussed as compared to estimation and control problems. Diagnostics related challenges for Li-ion batteries are discussed in Alavi et al. (2013a). An observer-based fault detection scheme for sensor and actuation faults is presented in Markicki et al. (2010). A multiple model based adaptive estimation scheme is proposed in Singh et al. (2013) for over charge and over discharge faults. In Mukhopadhyay and Zhang (2012), terminal voltage collapses has been diagnosed using Universal adaptive stabilization techniques. A bank of observers has been used for simultaneous fault isolation and estimation in a battery string in Chen et al. (2014). In Alavi et al. (2013b), a particle filter based approach is used for detecting Li-plating.

Diagnostic algorithms for the detection and isolation of sensor faults are presented in Lombardi et al. (2014). In Dey et al. (2014), the authors of the present paper developed a sliding mode observer based approach for detection and estimation of sensor faults.

In this paper, we propose an alternative approach for Li-ion battery fault detection based on a stochastic quantized modeling approach to detect sensor as well as internal faults in the Li-ion battery. Specifically, the stochastic modeling approach proposed by Mohon and Pisu (2013, 2014) for a quantized system is adopted for the present battery application. First, the continuous equivalent circuit model of the Li-ion battery is quantized into discrete states in order to simplify diagnostic efforts as explained in Blanke, et al (2006). This quantized stochastic model is then used to predict the most probable future state after a small time  $\Delta t$  using the measured inputs and quantized outputs. Here future states mean the states the system can transition into with probability greater than zero. One of these future states will have the largest probability and will be the most probable future state. Then, fault is detected through comparison of the expected future event and the measured state at  $\Delta t$ .

The advantages of this approach are less computational burden and ease of implementation. First, this approach eliminates the need of multiple observers for fault detection (Dey et al. 2014). Only the system state space equations are needed. Also, the nature of the quantization approach allows measurements to be uncertain within a quantized region while still achieving fault diagnosis. The use of uncertain measurements means the approach is robust to noise as well. The new method of calculating state transition probabilities is less computationally burdensome than previous methods explored for general quantized systems such as Monte Carlo simulations and Generalized Cell Mapping method (Lunze, et

al 2004 and Blanke, et al 2006). Therefore, by decreasing the accuracy of measurements needed and the amount of computations necessary to find preferred state transitions, the approach is less computationally intensive. However, a trade off exists between the memory requirement to store the necessary probability transition matrix and the minimal detectable fault size with this new approach.

Another practical motivation for using this quantization approach is the inherent nature of sensor measurements. The measured value from a sensor and the actual value will always differ by some small amount depending on the accuracy of the sensor used. Therefore, the sensor measurement range is always quantized. As long as the sensor measures within a certain small range, it will observe the same value. The quantized system diagnostics approach allows for cheaper, less accurate sensors to be used to solve a system diagnostic problem.

The rest of the paper is organized as follows. In Section 2, modeling and the diagnostics problem is formulated for Li-ion batteries. Section 3 details the stochastic quantized modeling approach for diagnostics. Section 4 presents simulation results that validate the effectiveness of the approach. Section 5 concludes the paper.

## 2. MODELING AND DIAGNOSTIC PROBLEM FORMULATION

In the current literature, different kinds of models can be found for Li-ion batteries. Arguably, the most accurate type of model is based on the electrochemical principles (Doyle et al. 1993). However, due to its complex mathematical structure and requirement of high computation burden, it is seldom used in real-time designs. There are other types of models that try to mimic the phenomenological behaviour of batteries. One such popular kind of model is the equivalent circuit model where the battery cell is represented as an electrical circuit (Liaw et al. 2004 and Dubarry et al. 2009). This kind of model is simple for real-time design and is convenient from computational cost perspective. In this paper, an electrical circuit model is considered for a Li-ion battery cell along with its lumped thermal dynamics. The electrical equivalent circuit model is shown in Figure 1.

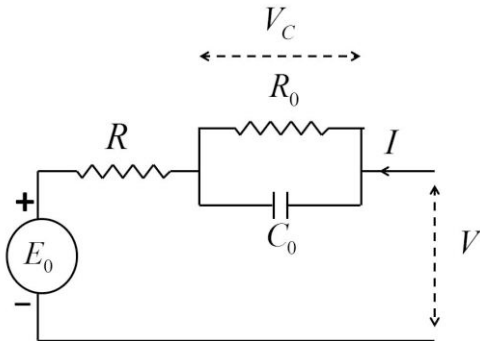


Figure 1. Electrical equivalent circuit model

Considering Figure 1 and using Kirchoff's law, the electrical dynamics of the battery cell can be given as:

$$\frac{dV_c}{dt} = -\frac{V_c}{R_0 C_0} + \frac{1}{C_0} \quad (1)$$

$$V = E_0 - IR - V_c$$

where  $V$  is the terminal voltage,  $I$  is the input current,  $R$ ,  $R_0$  and  $C_0$  are the resistors and capacitors of the electrical circuit,  $V_c$  is the voltage across the capacitor  $C_0$ ,  $E_0$  is the open-circuit voltage. The State-of-Charge (SOC) evolution equation can be written as:

$$\frac{dSOC}{dt} = -\frac{I}{Q} \quad (2)$$

where  $Q$  is the capacity of the battery cell. The lumped thermal model of the battery cell can be derived from the energy balance and is given by:

$$mc \frac{dT}{dt} = I^2(R + R_0) - hA(T - T_{amb}) \quad (3)$$

where  $T$  is the lumped battery cell temperature,  $m$  is the mass,  $c$  is the specific heat capacity of the battery cell,  $hA$  is the effective heat transfer coefficient and  $T_{amb}$  is the ambient temperature.

In general, all the circuit elements of the battery cell are functions of  $SOC$  and  $T$ . However, in this paper, we restrict the discussion to a Hybrid Electric Vehicle (HEV) type application, where we can reasonably assume those circuit elements to be constant within the operating range (narrow SOC windows). Furthermore, the open circuit voltage is taken as a linear function of SOC within the operating range:

$$E_0 = \alpha_1 + \alpha_2 SOC \quad (4)$$

where  $\alpha_1$  and  $\alpha_2$  are known constants which can be determined from the experimental data of the battery cell.

The faulty scenario in a Li-ion battery can be modelled as:

$$mc \frac{dT}{dt} = I^2(R + R_0) - hA(T - T_{amb}) + f_T \quad (5)$$

$$\frac{dV_c}{dt} = -\frac{V_c}{R_0 C_0} + \frac{1}{C_0} + f_V$$

where  $f_V$  and  $f_T$  are the additive faults to the voltage and temperature dynamics, respectively.

The states of the battery model can be written in the following form so that, apart from input current  $I$ , the two states are clearly decoupled from each other.

$$\begin{aligned} \dot{T} &= f_1(T, I) \\ \dot{V}_c &= f_2(V_c, I) \end{aligned} \quad (6)$$

## 3. STOCHASTIC MODELING APPROACH FOR QUANTIZED SYSTEMS

In this section, we describe the idea of stochastic modeling approach for quantized systems. The goal is to diagnose a

continuous system shown in Figure 2 (Blanke, et al 2006) where  $u(t)$  is the input at time  $t$ ,  $f(t)$  is the amount of fault at time  $t$ , and  $z(t)$  is the output at time  $t$ .

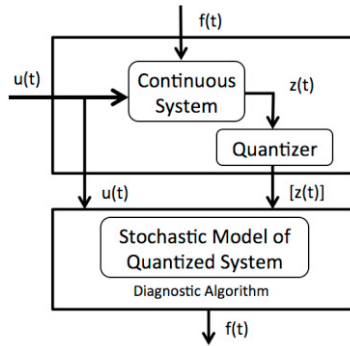


Figure 2. Diagnostics of quantized system

Referring to Figure 2, the output  $z(t)$  can be quantized by passing it through the quantizer where we denote the quantized output as  $[z(t)]$ . Now, we will illustrate the approach by considering a case with two outputs. Denoting the outputs as  $z_1(t)$  and  $z_2(t)$ , a plot of quantized outputs  $z_2$  vs  $z_1$  is shown in Figure 3. Note that, output  $z_1(t)$  and  $z_2(t)$  are quantized into intervals denoted by ‘1’ through ‘9.’ The grey section in Figure 3 essentially represents the current quantized state for the system. Based on the behavior of the input signal  $u(t)$ , the quantized state will transition from the grey quantized state to another quantized state. As time evolves, the quantized state will transform to a new location according to the system equations that could possibly overlap many quantized states. Essentially, the quantization of the state space introduces stochastic behavior. From our sensor measurements, we only know which quantized state is occupied, not where inside the quantized state. Quantization also shows that one quantized state can transition to many quantized states. The evolution of the system will not be deterministic now. Of course some states will be more favorable for the system to transition into (like state 6) and the probabilities of these transitions are of interest for diagnostics. The transition probabilities from one quantized state to another can be arranged in a probability transition matrix. If a transition occurs that has very low probability, then the presence of a fault can be inferred. Next, we will detail the method to develop probability transition matrices.

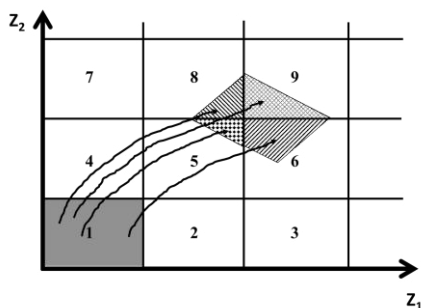


Figure 3: Example of quantizing two outputs

The example given in Figure 3 will be simplified for ease of explanation. Suppose  $z_2$  is unobservable output from the continuous system. Only the maximum and minimum values

of  $z_2$  are known. Output  $z_2$  is still quantized into intervals or states numbered ‘1’ through ‘5’ as before. The main difference between Figure 3 and Figure 4 is that now only vertical transitions are allowed in Figure 4. Horizontal transitions are not considered in this first work.

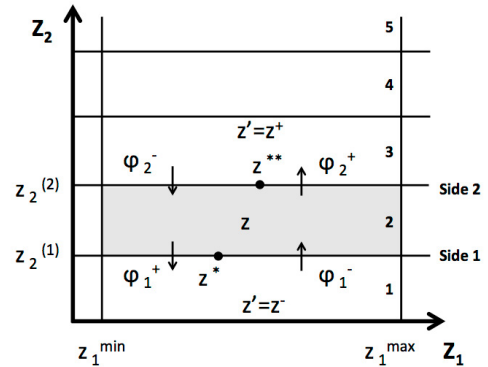


Figure 4. Graph of quantized system with flow definitions

In this paper, some assumptions about transitions are taken. We assume that with each event, which in our case is one time step, the current state  $z$  can only transition to adjacent states  $z'$  or remain in the same state  $z$ . The state may not transition to nonadjacent states. Proper selection of quantized states depends on the system. We choose the quantized states such that a healthy system only transitions to adjacent states.

The objective of the proposed method is to calculate the probability of transitioning from one state to another allowable state. A two-dimensional form of the divergence theorem can be used to calculate transition probabilities. The theorem is stated in Eq. (7).

$$\iint_A (\nabla \cdot \vec{F}) dA = \int_C (\vec{F} \cdot \vec{n}) dr \tag{7}$$

where  $C$  is a closed curve,  $A$  is the 2D region in the plane enclosed by  $C$ ,  $\vec{n}$  is the outward pointing normal vector of the closed curve  $C$ , and  $\vec{F}$  is a continuously differentiable vector field in region  $A$ . A graph of the 2D divergence theorem is shown in Figure 5. This yields a 2D space with desired upward and downward flows consistent with the system in Figure 4. As stated above, for the results in this paper, the flow through the left and right sides of the area  $A$  in Figure 5 will be assumed to be zero.

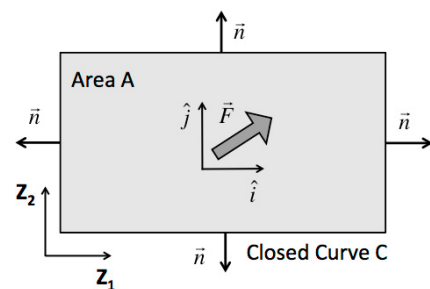


Figure 5. Illustration of the divergence theorem in 2D

Vector field  $\vec{F}$  describes the flow in and out of the current state  $z$  along the boundaries of the quantized regions. For a system with two states  $z_1$  and  $z_2$  and one input  $u$ , vector field  $\vec{F}$  is defined as Eq. (8) where  $\hat{i}$  and  $\hat{j}$  are coordinates of vector

field  $\bar{F}$  and functions  $f_1$  and  $f_2$  are defined by states  $z_1$  and  $z_2$  from the system state space model.

$$\begin{aligned}\bar{F} &= f_1 \hat{i} + f_2 \hat{j} \\ \dot{z}_1 &= f_1(z_1, z_2, u) \\ \dot{z}_2 &= f_2(z_1, z_2, u)\end{aligned}\quad (8)$$

Calculating the line integrals over side 1 and side 2 boundaries in Figure 4 will determine flow into and out of state  $z$ . We define outward flow  $\varphi^+$  as a positive value and inward flow  $\varphi^-$  as a negative value. Each side of the quantized state may have outward and inward flows with a transition point  $z^{**}$  or  $z^*$  where flow is zero as shown in Figure 4. The transition point is necessary for correct limits of integration in line integral calculations for flow in and flow out on each side. Without loss of generality, assume  $f_2 < 0$  if  $z_1 < z^*, z^{**}$  and  $f_2 > 0$  if  $z_1 > z^*, z^{**}$  such that Eq. (9) holds. The inward and upward flow through each component of the state  $z$  is shown in Eq. (10).

$$f_2(z^*, z_2^{(1)}, u) = 0 \quad (9)$$

$$f_2(z^{**}, z_2^{(2)}, u) = 0$$

$$\varphi_1^+ = - \int_{z_1^{\min}}^{z^*} f_2(z_1, z_2^{(1)}, u) dz_1 > 0$$

$$\varphi_1^- = - \int_{z^*}^{z_1^{\max}} f_2(z_1, z_2^{(1)}, u) dz_1 < 0$$

$$\varphi_2^- = \int_{z_1^{\min}}^{z^{**}} f_2(z_1, z_2^{(1)}, u) dz_1 < 0$$

$$\varphi_2^+ = \int_{z^{**}}^{z_1^{\max}} f_2(z_1, z_2^{(1)}, u) dz_1 > 0$$

We can gather the inward and outward flows to define  $\varphi_{in}$ ,  $\varphi_{out}$ , and  $\varphi_{total}$  in Eq. (11). These will be used to build probabilities.

$$\begin{aligned}\varphi_{in} &= |\varphi_1^- + \varphi_2^-| \\ \varphi_{out} &= \varphi_1^+ + \varphi_2^+ \\ \varphi_{total} &= \varphi_1^+ + |\varphi_1^-| + |\varphi_2^-| + \varphi_2^+\end{aligned}\quad (11)$$

We can interpret the concept of probability as counting the number of a certain type of occurrence and then normalizing by total number of occurrences of all types. Suppose the occurrences of outward and inward flow are normalized by the total flow. For example, the probability to transition down will be defined as the outward flow through side 1,  $\varphi_1^+$ , divided by the total flow  $\varphi_{total}$ . Now define  $z^-$  as the state below current state  $z$  and  $z^+$  as the state above state  $z$ . The probability to remain in state  $z$ , transition up, or transition down in the next time step is given in Eq. (12).

$$\begin{aligned}1 &= \frac{\varphi_{in}}{\varphi_{total}} + \frac{\varphi_{out}}{\varphi_{total}} \\ 1 &= \frac{|\varphi_1^- + \varphi_2^-|}{\varphi_{total}} + \frac{\varphi_2^+}{\varphi_{total}} + \frac{\varphi_1^+}{\varphi_{total}} \\ 1 &= \Pr(z' = z^- | z) + \Pr(z' = z^+ | z) \\ &\quad + \Pr(z' = z^- | z)\end{aligned}\quad (12)$$

These probabilities are calculated at each time step using the current quantized state and current input. With this information, a time-varying probability transition matrix named  $L$  can be organized as shown in Table 1.

Table 1. Example of probability transition matrix  $L$  for current state  $z=2$  at a time  $t$

		Future State $z'$				
		1	2	3	4	5
Current State $z$	1	0	0	0	0	0
	2	$\Pr(z' = z^-   z)$	$\Pr(z' = z   z)$	$\Pr(z' = z^+   z)$	0	0
	3	0	0	0	0	0
	4	0	0	0	0	0
	5	0	0	0	0	0

When the output state from data transitions to a state other than the predicted state from matrix  $L$ , then a fault is probably present. We can therefore detect faults using the residual in Eq. (13) where  $[y]$  is the quantized output state from data and  $[y_{predicted}]$  is the predicted state with largest probability from probability transition matrix  $L$ . When the residual is nonzero and the probability from matrix  $L$  is high, a fault is most likely occurring in the system. This residual should be computed for every dimension that is chosen to be quantized. In this example, only  $z_2$  is quantized and therefore only a residual in the  $z_2$  direction is used.

$$r = [y] - [y_{predicted}] \quad (13)$$

While the above discussion detailed how to use the stochastic modeling approach for a quantized two-dimensional system, it can be simplified for a quantized one-dimensional system as well. However, the quantized boundaries no longer have lengths in a second dimension. Therefore, the line integrals in Eq. (10) collapse to an antiderivative at a point and no flow transition points exist on the quantized boundaries.

Finally, we extend the aforementioned method to fault detection in Li-ion batteries. This requires quantization of the electrical and thermal dynamics given in Eq. (1) and Eq. (3), respectively. Note that we have temperature measurement directly from the system, which we shall use to predict the most probable future state. However, for the equivalent capacitor voltage  $V_c$ , we do not have direct

measurement. We use the following equation to estimate  $V_c$  from the measurement of terminal voltage  $V$ .

$$V_{c,estimated} = E_0 - IR - V \quad (14)$$

The schematic of the diagnosis scheme is shown in Figure 6.

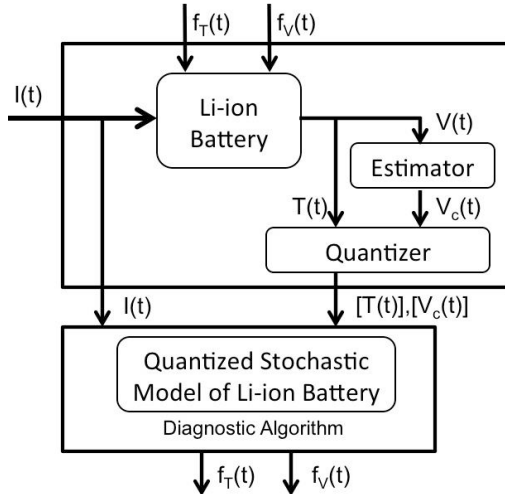


Figure 6: Diagnostic scheme for Li-ion battery fault detection

#### 4. SIMULATION RESULTS

To validate the effectiveness of the proposed diagnostic scheme, a commercial Li-ion battery is used from which we extracted the parameters using experimental data of voltage, current and temperature. This particular Li-ion battery cell has the following characteristics: LiFeP<sub>4</sub>-Graphite chemistry, 3.3 V, 2.3 Ah, maximum discharge current 70 A. To extract the battery model parameters, an optimization problem was solved minimizing the difference between experimental and model-simulated data. The identified parameters are  $R=0.206 \Omega$ ,  $R_0=0.008 \Omega$ ,  $C_0=12000 \text{ F}$ ,  $\alpha_1=2.939$ , and  $\alpha_2=0.01939$ .

After extracting the battery model parameters, simulation studies were conducted in which faults were injected. Two different fault cases have been used to verify the diagnostic approach. First, an additive fault has been injected in the thermal model at  $t = 200 \text{ sec}$  ( $f_T=0.0001$ ). The residual response to the fault is shown in Figure 7. Note that, the residual is zero under non-faulty conditions ( $t = 0$  to  $200 \text{ sec}$ ) and becomes nonzero after the fault occurrence at  $t = 200 \text{ sec}$ . Next, an additive fault has been injected into the electrical model at  $t = 200 \text{ sec}$  ( $f_V=0.001$ ). The residual response for the fault is given in Figure 8. As expected, the residual goes up from zero to a nonzero value after the fault occurrence. Note that, there is a delay in this residual response. This is due to the particular choice of quantization step we used. Note that, in this proposed approach the main tuning variable is the quantization step. The particular selection of the quantization step dictates the minimum detectable fault size.

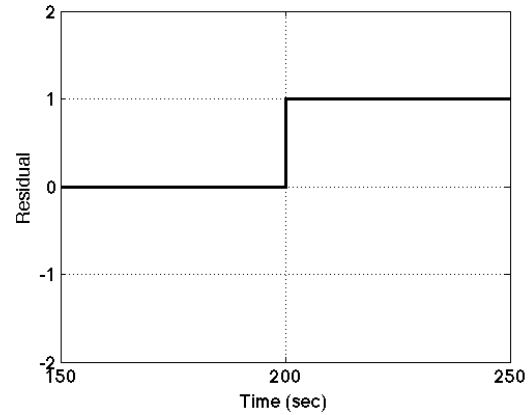


Figure 7: Residual for temperature fault ( $f_T$ )

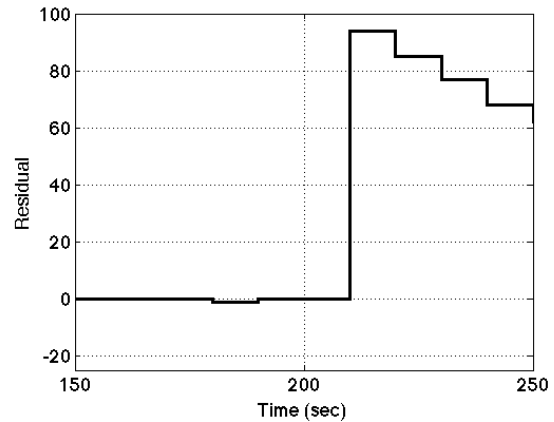


Figure 8: Residual for voltage fault ( $f_V$ )

#### 5. CONCLUSIONS

In this paper, a fault detection scheme based on a quantized stochastic modeling approach has been applied for Li-ion batteries. This scheme used a quantized stochastic model derived from the equivalent circuit model to predict the most probable future states/outputs from the measured inputs and quantized outputs. Fault detection is achieved by generating transition probability matrices and comparing the values of the expected event and the actual event. The effectiveness of the approach is illustrated via simulation studies using experimentally identified parameters of a commercial Li-ion battery.

Although the simulation studies have shown promising results, there are some aspects, which can be considered as future work of this study. First, no modeling uncertainties have been considered in this diagnostic scheme. To make the scheme more effective in real-time scenarios, modeling uncertainties should be taken into account. Next, the approach will be experimentally validated via real-time computations to verify the trade-offs identified between quantization steps and the minimum detectable fault sizes.

## ACKNOWLEDGEMENT

This research is supported by the US Department of Energy GATE program under grant number DE-EE0005571.

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