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CASCADED ROBUST NONLINEAR POSITION TRACKING CONTROL OF AN ELECTROHYDRAULIC ACTUATOR: SLIDING MODE AND FEED FORWARD

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ABSTRACT

An approach to position tracking control based on a cascade of a nonlinear force tracking controller derived from a near input-output linearization framework and a simple feedback plus feed forward position controller is presented. The method exploits the cascade structure to employ a sliding mode pressure force tracking controller as inner-loop and the position tracking controller as an outer-loop. Furthermore, it is highlighted that Lyapunov backstepping analysis can be used to drive performance bounds and reveal trade-offs between the size of uncertainty and measurement errors and the tracking accuracy. The performance of the proposed cascaded robust controller is demonstrated with experiments and simulations on a test system that doesn't necessarily satisfy all of the assumptions made for controller derivation. In particular, a typical comparison of the robust and nominal cascade controllers shows the robust version can recover the performance of the nominal near IO linearizing controller. In addition, model simulation results are included to show the performance of the controller in the presence of some combinations of perturbations or difficult to estimate parameters such as valve coefficient, supply pressure, piston friction, and inclusion of servovalve spool dynamics.

Key words: electrohydraulic actuators, sliding mode control, cascade position control, feedback linearization

INTRODUCTION

Electrohydraulic actuators constitute important positioning and force generation elements in a variety of industrial applications. However, they exhibit significant nonlinearities in their dynamics that may necessitate the use of improved control techniques to achieve accurate positioning and force tracking objectives.

Various such techniques have been investigated in the literature including optimal linear state feedback [1], adaptive control [1-5], variable structure control [6, 7] and Lyapunov-based controller designs [2, 4, 8-10]. Each approach has its own strengths and limitations, which are outlined in the respective references. The approach presented in this paper combines the features of feedback linearization [11-14], sliding mode force tracking control [6-8, 15, 16] and application of cascade control to electrohydraulics [17-20].

The cascade control of hydraulic actuator piston position employing classical and linear state feedback was described in [17-19]. The central idea of the method lies in treating the actuator as a force generator with an inner-loop force (or differential pressure) tracking controller, and a feedback plus feed forward outer-loop position controller that computes the desired force profile for the inner-loop. The inner-loop force controller generally comprises of a high-gain force (or differential pressure) feedback term in addition to positive velocity feedback. The latter is intended to cancel the natural velocity coupling of the piston motion with the pressure dynamics. This expectation is reasonable if flow-pressure and variable compliance nonlinearities could be ignored, as can easily be seen in the expression of the pressure dynamics (or its local linearization about operating points). Then, with the piston motion decoupled from the pressure dynamics by velocity feedback, the outer-loop control would provide compensation for external loads and friction, and enable tracking of the desired piston motion (position, velocity and acceleration). The outer-loop solves a standard motion control problem and may even include adaptive algorithms to compensate for load and parameter changes [18].

The same basic idea of cascading was extended to the realm of nonlinear control in the work of Heintze and Van der Welden [17], who compared an inner-loop controller based on dynamic inversion with a cascade controller which includes nonlinearity compensation in the original constant gain cascade form of Sepehri, et al [18]. Starting with a Lyapunov-like analysis, Sohl and Bobrow [10] also presented a cascade position tracking controller, with a nonlinear pressure force controller as an inner-loop. The proposals in [17] and [10] are similar in structure to the discussion of the nominal cascade control presented below. Eryilmaz and Wilson [21] arrive at a slightly different cascade control structure from a singular perturbation point of view.

A common approach for sliding mode position tracking controller design involves defining a sliding manifold to represent well-behaved position tracking error dynamics or some weighted sum of position and force tracking errors and their integrals [6-8, 16]. In this paper, we use the cascade interpretation and implement a sliding mode force tracking controller as an inner-loop to a feedback plus feed forward position controller to achieve a robust version of the cascade controller.

The rest of the paper is organized as follows. We first describe a simplified mathematical model of the electrohydraulic actuator that is used for controller derivation. We then discuss the nominal cascade form that neglects uncertainty followed by the robust cascade form that considers parametric and measurement uncertainty. This is followed by a discussion of some experimental and simulation results. Finally, the conclusions of the paper are presented. Certain mathematical details are postponed to the Appendix.

NOMENCLATURE

A_b, A_t	piston areas for the bottom and top chambers, respectively
A_p	piston area for a symmetric actuator
C_L	leakage coefficient used in controller
C_v	valve coefficient used in controller
e_F	pressure force tracking error
f_F	nonlinear feedback term given by Eq (9)
F_f	friction force on piston
F_L	load force or specimen reaction on piston
F_p	fluid pressure force on piston
$F_{p,d}$	desired or reference pressure force trajectory
f_{pL}	nonlinear feedback term given by Eq (3)
g_F	nonlinear feedback term given by Eq (10)
g_{pL}	nonlinear feedback term given by Eq (4)
i_v	servovalve current
K	gain in sliding mode controller
k_o	constant gain of force error dynamics, Eq (12)
m_p	lumped mass of piston, fixture and oil mass
p_b ,p_t	pressure in the bottom and top cylinder chambers, respectively
p_L	load or differential pressure (p_L=p_b-p_t)
p_R	return pressure at servovalve
p_S	supply pressure at servovalve
q_b, q_t	flow to the bottom and from the top cylinder chambers, respectively
$q_{e,b}$, $q_{e,t}$	external leakage from the bottom and top chambers, respectively
q_i	internal leakage in cylinder
S	sliding manifold variable
u_1, u_2, u_3, u_4	valve underlap/overlap lengths
V_b, V_t	bottom and top cylinder chamber volumes, respectively
v_p	piston velocity
x_p	piston position
x_v	servovalve spool displacement
β_e	effective bulk modulus
Φ	thickness of boundary layer
δf_F, δg_F	bounds on the uncertain functions f_F, g_F, Eq (19)

MODEL OF SYSTEM

Physical models of electrohydraulic actuators are quite widely available in the literature [19, 22-24]. The model used here applies to a four-way servovalve close-coupled with a piston actuator as shown in Fig. 1. q_t and q_b, are flow rates from the top chamber and to the bottom chamber of the cylinder, respectively. q_i represents internal leakage flow and $q_{e,t}$ and $q_{e,b}$ are external leakage. A_t and A_b represent the effective piston areas, and V_t and V_b designate the volumes of oil in the top and bottom chambers, respectively, corresponding to the center position (x_p=0) of the piston. These include the volumes of oil in the short pipelines between the close-coupled servovalve and actuator. It is assumed that the supply (p_S) and return (p_R) pressures are constant at the ports of the servovalve.

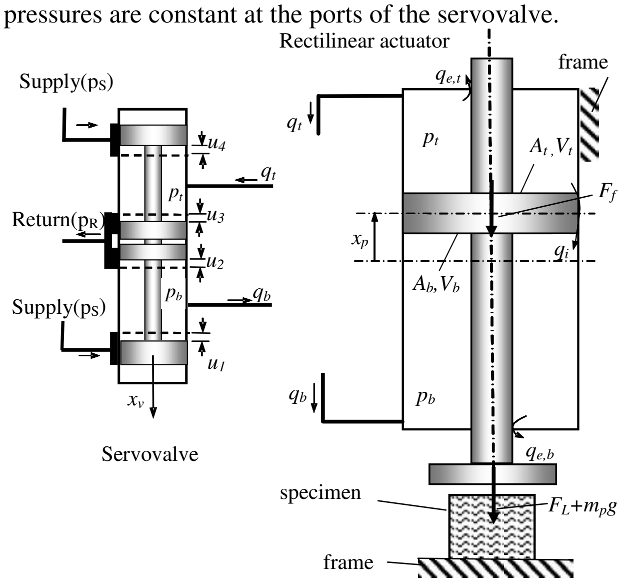

Figure 1 Schematic of rectilinear servovalve and actuator

Considering flow continuity and the state equation with the effective oil bulk modulus, β_e, for the cylinder chambers, and introducing the load (differential) pressure, p_L, [23], it can be shown that the load pressure dynamics are given by (see, for example, [20]) :

$$\dot{P}_L = f_{p_L}(x_p, \dot{x}_p, p_L) + g_{p_L}(x_p, p_L, \mathrm{sgn}(i_v))\, i_v \quad (1)$$

where the load pressure, p_L, is:

$$p_L = p_b - p_t \quad (2)$$

and

$$f_{p_L}(x_p, \dot{x}_p, p_L) = -\beta_e \dot{x}_p \left(\frac{A_b}{V_b + A_b x_p} + \frac{A_t}{V_t - A_t x_p} \right) - \beta_e C_L p_L \left(\frac{1}{V_b + A_b x_p} + \frac{1}{V_t - A_t x_p} \right) \quad (3)$$

$$g_{p_L}(x_p, p_L, \mathrm{sgn}(i_v)) = \beta_e C_v \sqrt{\left(\frac{p_S - p_R}{2} \right)} \times \sqrt{1 - \frac{p_L}{p_S - p_R}\mathrm{sgn}(i_v)} \times \left(\frac{1}{V_b + A_b x_p} + \frac{1}{V_t - A_t x_p} \right) \quad (4)$$

Here, the external leakages, $q_{b,e}$ and $q_{t,e}$, are neglected. Note that the first term on the right in Eq (3) shows the explicit dependence of the pressure dynamics on the piston velocity (motion). The second term has its roots from the cross chamber leakage, which is assumed to be laminar with leakage coefficient, C_L. The expression for the coefficient of the current input i_v, lumped into g_{pL}, arises from turbulent flows through the sharp-edged control orifices of a spool valve to and from the two sides of the cylinder chambers. The valve is assumed to be matched and symmetrical ($u_1=u_2=u_3=u_4=0$) with valve coefficient C_v. Also the valve spool dynamics are assumed to be fast enough to be neglected for the purpose of controller derivation.

The state equations governing piston motion are derived considering the loading model for the actuator. For the test system, the actuator cylinder is rigidly mounted on a load frame. The load frame is used as an inertial frame. For a symmetric actuator ($A_b=A_t=A_p$), the upward force on the actuator piston due to the oil pressure in the two cylinder chambers is given by:

$$F_p = A_p p_L \quad (5)$$

The friction force on the piston in the cylinder is denoted by F_f, and the external loadings, including specimen stiffness and damping forces, are lumped together in F_L. In Fig. 2, F_L is considered tensile positive. The equations of motion are derived by applying Newton's Second Law:

$$\dot{x}_p = v_p \quad (6)$$

$$\dot{v}_p = \frac{1}{m_p}[A_p p_L - F_L - F_f - m_p g] \quad (7)$$

Equations (1), (6) and (7), constitute the state space model for the servovalve and loaded actuator subsystem under consideration. These equations also contain the major modeled nonlinearities in the system, which are the variable hydraulic capacitance and the turbulent flow rate versus pressure drop relations. Nonlinearity is also introduced in Eq (7) by the nonlinear friction force, which includes Coulomb, static, and viscous components [25].

CONTROL LAW DERIVATION

Nominal Cascade Control

First, we consider the case that all model parameters are known and we refer to this case as the nominal case.

The pressure force dynamics are given by:

$$\dot{F}_p = f_F(x_p, \dot{x}_p, p_L) + g_F(x_p, p_L, \mathrm{sgn}(i_v))i_v \quad (8)$$

where,

$$f_F(x_p, \dot{x}_p, p_L) = A_p f_{p_L}(x_p, \dot{x}_p, p_L) \quad (9)$$

$$g_F(x_p, p_L, \mathrm{sgn}(i_v)) = A_p g_{p_L}(x_p, p_L, \mathrm{sgn}(i_v)) \quad (10)$$

A near input-output (IO) linearizing pressure force controller can be derived [1, 26]:

$$i_v = \frac{1}{g_F(x_p, p_L, \mathrm{sgn}(i_v))}(\dot{F}_{p,d} - k_o e_F - f_F(x_p, \dot{x}_p, p_L)) \quad (11)$$

where $F_{p,d}$ is the desired force trajectory which is assumed to be differentiable, and e_F is the force tracking error, $e_F=F_p-F_{p,d}$. Note that the IO linearization achieved with this controller is only a near IO linearization, since some assumptions need to be imposed to solve Eq (11) across the $i_v=0$ discontinuity of the sign function. In particular, during the digital implementation of the controller, it is assumed that the sign of the value of i_v at the previous time step can be used to compute the value of i_v at the current time step. This supposes that the current does not change signs at a rate faster than the sampling rate. However, it is difficult to analytically prove that this approach does not lead to control chatter. This chatter problem has not been previously reported in the literature that discusses feedback linearization for hydraulic drives [13, 14, 27], nor has it been experienced during any of the experiments performed by the authors of this paper.

With the controller given by Eq (11), the force tracking error dynamics are given by:

$$e_F + k_o e_F = 0 \quad (12)$$

where e_F is the force tracking error, $e_F=F_p-F_{p,d}$. Note that the near IO linearization has reduced the nonlinear pressure force dynamics to a first order linear tracking problem. With a proper choice of $k_o>0$, one can obtain a desired degree of exponential force tracking performance, regardless of the nonlinearities in Eq (8), provided the internal dynamics are stable.

Since the original system is of order 3, second order internal dynamics remain, which were rendered "unobservable" during the near IO linearization. It is straightforward to show that the piston velocity and position can be used as state variables to describe the internal dynamics and thereby establish the stability.

The near IO linearizing pressure force controller of Eq (11) cancels the natural feedback of piston velocity on the pressure force dynamics via the first term of the nonlinear function, f_F. This nonlinearity cancellation decouples the dynamics of the piston motion from the hydraulic pressure/force dynamics. The cascade control of piston position exploits this fundamental result.

We start by constructing the desired pressure force profile ($F_{p,d}$) in terms of the desired piston position profile in such a manner that when the pressure force output is driven to the desired force profile, the piston position output approaches the desired position. Define $F_{p,d}$ as:

$$F_{p,d} = m_p \ddot{x}_d - k_v(\dot{x}_p - \dot{x}_d) - k_p(x_p - x_d) + F_L + F_f + m_p g \quad (13)$$

It is assumed here that accurate estimates of the friction force and the load force are available and the piston mass is known. The choice of the gains of k_v and k_p will become evident shortly. To further appreciate the choice of the form of $F_{p,d}$ in (13), recall the equation of motion:

$$m_p \ddot{x}_p = F_p - F_L - F_f - m_p g \quad (14)$$

Combining Eqs (13) and (14), the closed loop dynamics can be expressed in terms of the position error, $e=x_p-x_d$:

$$m_p \ddot{e} + k_v \dot{e} + k_p e = F_p - F_{p,d} = e_F \quad (15)$$

It has already been argued that an exponentially convergent tracking of the pressure force can be obtained using the near IO linearizing controller of Eq (11). Equation (15) shows that the position error dynamics are given by a second-order linear differential equation driven by the pressure force error provided there are no estimation or measurement errors for the load and friction forces. The gains k_v and k_p can be easily chosen to obtain the desired second-order position error dynamics and ensure that the closed loop system is stable.

Figure 2 shows a schematic of the implementation of the nominal cascade control in the absence of uncertainty in friction and load force. Note that the pressure force loop acts as an inner-loop to the feedback plus feed forward outer loop position controller.

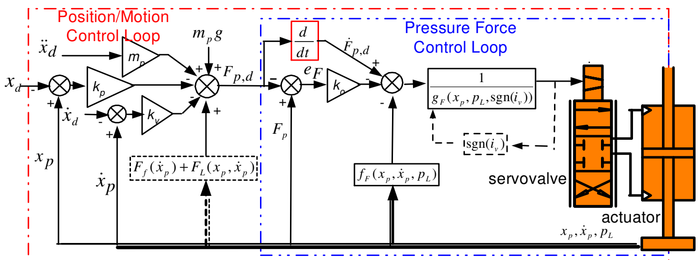

Figure 2 Schematic of the nominal cascade control

Robust Cascade Control

We now extend the nominal cascade controller described above to address issues of uncertainty in model parameters (namely, β_e, C_v, C_L) and measurement and estimation errors (in the load and friction forces, F_L and F_f).

It turns out that the near IO linearizing controller for pressure force tracking can be easily re-considered from a sliding control point of view to formally address the issue of robustness to parametric uncertainty. The reader is referred to [11] for a discussion of sliding mode control and [28, 29] for the derivation and experimental validation of the results briefly outlined here. Dropping the arguments of, and replacing the function f_F and g_F by their estimates $\hat{f}_F$ and $\hat{g}_F$, respectively, the resulting (continuous version) sliding mode controller is:

$$i_v = \frac{1}{\hat{g}_F}(\dot{F}_{p,d} - K sat(S/\Phi) - \hat{f}_F) \tag{16}$$

where Φ is the boundary layer thickness and S is sliding surface variable defined by:

$$S = F_p - F_{p,d} \tag{17}$$

and K is the gain, which should be chosen to satisfy

$$K \geq \delta g_F(\eta + \delta f_F) + |1 - \delta g_F| \left|\dot{F}_{p,d} - \hat{f}_F\right| \tag{18}$$

where δf_F and δg_F are the bounds on the uncertainties(perturbations) in the nonlinear functions f_F and g_F, respectively, and are expressed by:

$$\left|\hat{f}_F - f_F\right| \leq \delta f_F \qquad 0 < (\delta g_F)^{-1} \leq \frac{\hat{g}_F}{g_F} \leq \delta g_F \tag{19}$$

Note that within the boundary layer (S≤Φ), the sliding mode controller given by Eq (16) is identical to the near IO linearizing controller given by Eq (11) in the absence of uncertainty.

Sliding control can also be applied to the piston position tracking case. With piston position as the output, the modeled electrohydraulic system becomes of relative degree 3. A standard sliding mode controller design would start by defining the sliding manifold ($S=0$) to represent well-behaved position tracking error dynamics. In this paper, however, instead of this standard sliding mode approaches to robust position control, which often require higher order derivatives of the position signal, it is desired to take advantage the cascade controller design described above. By keeping the sliding mode pressure force tracking control design as the inner-loop, the robustness of the outer-loop and the overall system is investigated.

To this end, the desired pressure force profile given by Eq (13) is re-defined here considering the uncertainty in the estimation of load and friction forces. Replacing the load and friction forces by their estimates, the desired force profile is computed by:

$$F_{p,d} = m_p\ddot{x}_d - k_v(\dot{x}_p - \dot{x}_d) - k_p(x_p - x_d) + \hat{F}_L + \hat{F}_f + m_p g \tag{20}$$

Using Eq (20) in the equation of motion of the piston, Eq (14), the closed loop position error, $e=x_p-x_d$, satisfies:

$$m_p\ddot{e} + k_v\dot{e} + k_p e = e_F + (\hat{F}_f - F_f) + (\hat{F}_L - \hat{F}_L) \tag{21}$$

where $e_F=F_p- F_{p,d}$ is the pressure force tracking error, and $\hat{F}_f$ and $\hat{F}_L$ are the estimates of the friction and load force respectively. It is assumed that the mass of the piston is known. The friction force is almost always estimated from a model, while the load force can either be measured (with a load cell) or estimated from a relevant model. It should be recalled that the friction force is generally a function of velocity ($\dot{x}_p$), while the load force or specimen reaction is considered a damping and stiffness force, and therefore a function of both piston position (x_p) and velocity ($\dot{x}_p$). Bounds are assumed for the uncertainty in friction and load forces as follows:

$$\left|\hat{F}_f - F_f\right| \leq \delta F_f \qquad and \qquad \left|\hat{F}_L - F_L\right| \leq \delta F_L \tag{22}$$

Unlike the case where there is no uncertainty, Eq (21) shows that this cascade controller cannot guarantee convergence of the position error to zero in the presence of friction and load force uncertainty, even if the force tracking error converges to zero. The position error dynamics are driven by the uncertainty in friction and load force estimation in addition to the force tracking error. The uncertainty enters as a disturbance to the position loop. However, for bounded uncertainty, the position tracking error remains bounded. Figure 3 shows the revised schematic for the cascade controller in the presence of uncertainty in the friction and load forces.

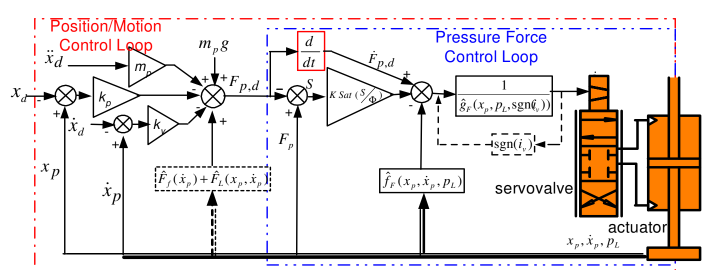

Figure 3 Schematic of the robust cascade control

Even if the force loop has been made robust to the "matched" uncertainty in the functions f_F and g_F , the position loop is still subjected to the effects of uncertainty in friction and load force estimation. The problem of this "unmatched" uncertainty of the force loop can be formally addressed by showing that the choice of the desired force output via Eq (20) has an interpretation from a Backstepping design point of view. One such Lyapunov-based analysis is shown in the Appendix. The approach reveals linear growth bounds that show trade-offs between tracking performance and the size of the unmatched uncertainty as sufficient conditions that render the derivative of the Lyapunov function candidate negative semi-definite. Bounded position and force tracking is achieved provided both the matched and unmatched uncertainty bounds are satisfied.

EXPERIMENTS AND SIMULATIONS

Figure 4 shows the schematic of the electrohydraulic fatigue testing system on which some experiments and system simulations were performed to evaluate the control structures discussed above. The servovalve is a 5 gpm (19 lpm) two-stage servovalve, close-coupled with a 10 kN, 102 mm-stroke symmetric actuator, which is mounted on a load frame. Two pressure transducers are used for sensing the pressures at the output ports of the servovalve. A linear variable displacement transducer (LVDT) is mounted on the actuator piston for position measurement.

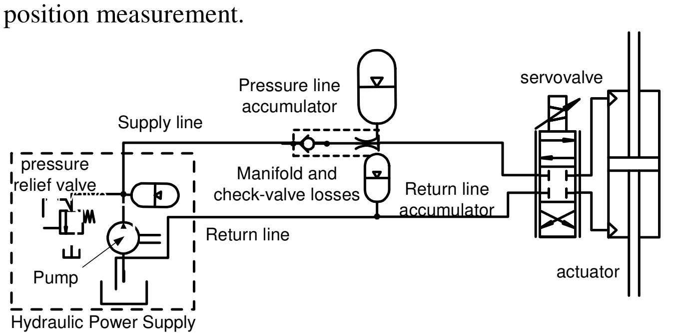

Figure 4 Simplified schematic of the test system

The Hydraulic Power Supply (HPS) unit, including its heat exchanger and drive units, is housed separately and is connected to the service manifold block via 3.048 m-long SAE-100R2 hoses. The manifold block contains an in-line check valve and a filter element on the supply line; it is equipped with a control manifold circuitry to permit selection of high- and low-pressure operating modes, low-pressure level adjustment, slow pressure turn-on and turn-off, and fast pressure unloading. The supply and return accumulators are mounted directly on the manifold, which is in turn connected to the servovalve using 3.048 m-long SAE-100R2 hoses. During normal fatigue testing operations, the manifold circuitry allows flow at full system pressure. A study of the complete system model, including the transmission hoses, the manifold and the accumulators, is published in [25]. It is shown there that in order to effectively exploit the accumulators in eliminating pressure transients at the supply and return ports of the servovalve, the accumulators should be close-coupled to the servovalve (remove the second set of hoses between the accumulators and the servovalve). The present configuration, however, does not provide for this, and therefore, the following simulation and experimental results for the control system are subject to this non-ideal condition, which is neglected during controller derivation. Only cases with F_L=0, i.e, where the actuator is loaded with a known inertia load (m_p=12kg), are considered.

Figure 5 shows an experimental result that compares the robust and nominal cascade controllers. Note that while the design of the nominal cascade controller doesn't take into account parametric and modeling uncertainty, the robust cascade controller does (within bounds). During experimentation, it was observed that, in general, rather large magnitudes of K and/or Φ were necessary to accommodate the unmodeled dynamics of the transmission lines, the servovalve and feedback signal filters in addition to the matched and unmatched uncertainty. The gain, K, and the boundary layer thickness, Φ, were heuristically determined to obtain a chatter free response with the robust cascade controller without saturating the control current. For the data in Fig 5, the following values were set for the nominal controller (and the outer-loop of the robust controller): k_p = 3.0x10^5 kg/s^2, k_v=2000 kg/s, and k_o=502 s^{-1}.; and for the robust controller K=3.0x10^8 kgcm/s^3 and Φ=5000 N. The outer loop gains correspond to a natural frequency of 26 Hz and a damping ratio of 0.6. It can be seen that the performance of the robust cascade controller is comparable to the nominal one for just these settings, while still using lower control current peaks. Note that even higher values of K, with correspondingly higher settings for Φ, could be used to recover the nominal performance with the robust controller.

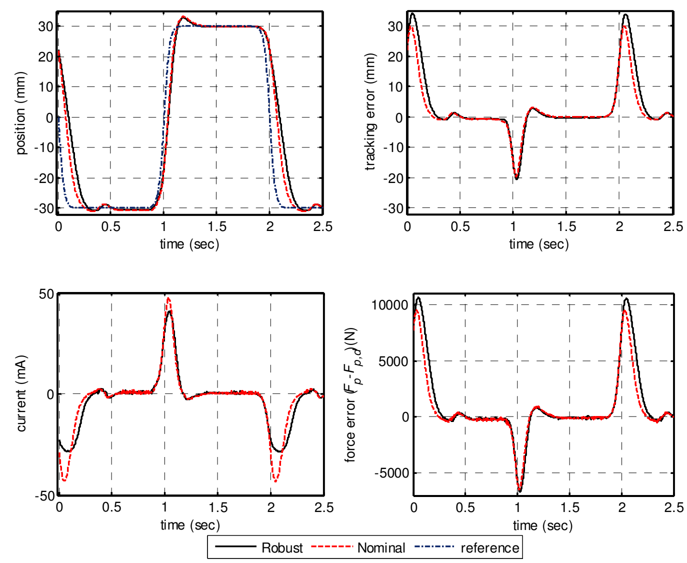

Figure 5 A typical comparison of the robust and nominal cascade controllers (experiment)

We now return to the simulation of the system where we consider the desirable configuration of the system in which the accumulators are close-coupled with the servovalve and the servovalve corner frequency is 240Hz with the damping ratio of 1.1, corresponding to the specifications of the present servovalve with the supply pressure at 21 MPa [30].

As examples, we present two typical perturbations in the model of the electrohydraulic system to show the performance of the robust cascade controller in the presence of uncertainty. For the simulations, the outer-loop position controller is designed to have a natural frequency of 50 Hz and a damping ratio of 1.0, corresponding to k_p=1.1x10^6 kg/s^2 and k_v= 7000 kg/s. Starting values of the sliding control gain K can be estimated from Eq (18) for the force inner-loop, and subsequently tuned, together with the value of Φ, by looking at the tracking error dynamics and the control activity. For the cascade controller considered in this section, the choices are K=3x10^8 kgcm/s^3 and Φ=3000 N.

As a first case, the value of the valve coefficient C_v of the actuator is underestimated in the controller by 10%. That is, the value of C_v in the model of the actuator is increased while a nominal value of C_v is used in the controller. Also, consider at the same time that the friction in the actuator is 100% higher than the estimate used by the controller. Figure 6 shows the simulated tracking performance of the robust cascade controller under these perturbations. For comparison, the case of perfect knowledge (no perturbation) by the robust controller is also shown. It can be seen that the robust controller gives bounded tracking errors, unlike the non-robust cascade (or near IO linearizing) controllers which drive the tracking error to zero. Recall that with the robust cascade controller, boundedness of the tracking error is all that is guaranteed. The boundary layer thickness Φ helps tune this bound on the tracking error. It can also be seen that the robust controller uses slightly lower current peak to give a slightly smaller peak position tracking error. This is generally not the case with the other position controllers discussed so far. As will be shown in the next case, neither is this the universal trend with the robust cascade controller.

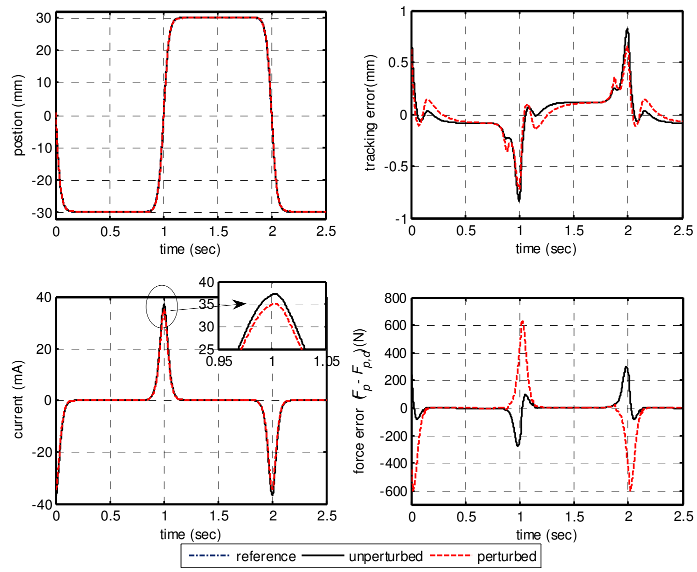

Figure 6 Tracking performance of the robust cascade controller with valve coefficient and friction perturbations (simulations)

As a second and special case that involves the transmission lines, consider the supply line pressure at the pump to drop to a level 20% lower than the nominal value of 21MPa set in the cascade controller. Consider also that the servovalve response is slower with corner frequency of 200 Hz. Figure 7 shows the simulation results for this case. It can be seen that even for this case of the supply pressure uncertainty, the robust cascade controller does a decent job at tracking this particular reference trajectory in the presence of the perturbations. Note also that the current peak and the tracking error are both higher in the perturbed case.

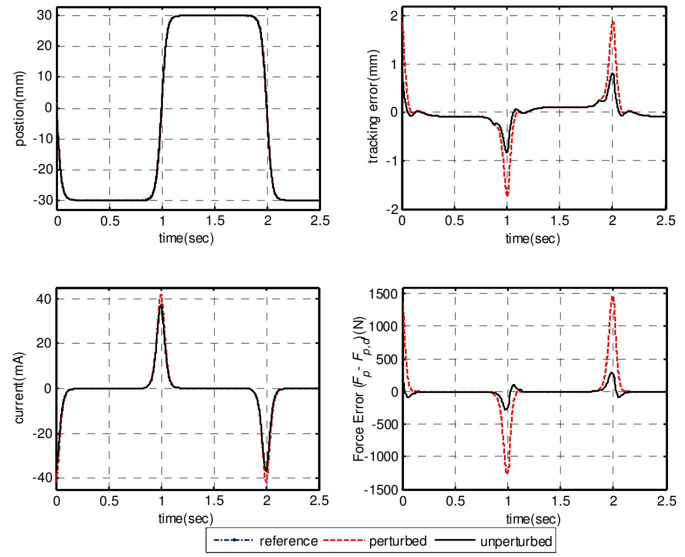

Figure 7 Tracking performance of the robust cascade controller with supply pressure perturbation and slower servovalve dynamics (simulations)

CONCLUSION

Starting from a feedback linearization framework, a nonlinear cascade controller was derived in this paper. The nominal case implements a near IO linearizing pressure force controller as an inner-loop to the feedback plus feed forward outer loop position controller where all model parameters are assumed known. It was shown that the cascade controller exploits the decoupling of the pressure dynamics from the piston motion dynamics by cancellation, during the near IO linearization, of the natural feedback of piston velocity in the pressure force dynamics.

By explicitly considering uncertainty in the model parameters (β_e, C_v, C_L) and measurement and estimation errors in the load and friction forces (F_L and F_f), a robust cascade controller was derived. It implements a sliding mode inner-loop force controller with an outer-loop position controller generating the desired pressure force for the inner-loop. It is discussed that this robust cascade controller can be analyzed from a Lyapunov Backstepping design perspective. It turns out that when the uncertainty magnitudes satisfy certain conditions, the robust cascade controller keeps the position tracking error bounded (and does not necessarily drive it to zero).

An experimental result was shown comparing the robust cascade controller with the nominal cascade controller. And two perturbation cases were simulated to show the tracking performance of the robust cascade controller.

Future work will implement the controllers presented in this paper on an experimental system that is influenced less by transmission line and spool valve dynamics thereby satisfying the assumptions under which the controller derivation was conducted.

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APPENDIX

To show the backstepping interpretation, start by re-writing the system equations including uncertainty in friction and load force as follows:

$$\dot{x}_p = v_p \qquad \text{(A1)}$$

$$\dot{v}_p = \frac{1}{m_p}(Fp - \hat{F}_L + (\hat{F}_L - F_L) - \hat{F}_f + (\hat{F}_f - F_f) - m_p g) \qquad \text{(A2)}$$

$$F_p = f_F + g_F i_v \qquad \text{(A3)}$$

This system is in the so called Strict Feedback Form [12, 31]. Starting with the first two equations, the pressure force, F_p, can be considered as the input and the following Lyapunov function candidate can be taken:

$$V_1 = \frac{1}{2}k_p(x_p - x_d)^2 + \frac{1}{2}m_p(v_p - v_d)^2 \qquad \text{(A4)}$$

Using the choice of the desired force trajectory given before (Eq 20), the derivative of V_1 reduces to:

$$\dot{V}_1 = -k_v(v_p - v_d)^2 + [e_F + (\hat{F}_L - F_L) + (\hat{F}_f - F_f)](v_p - v_d) \qquad \text{(A5)}$$

Using the notation S for the sliding manifold, a Lyapunov function candidate for the whole system can be written as:

$$V = V_1 + \frac{1}{2}S^2 = V_1 + \frac{1}{2}e_F^2 \qquad \text{(A6)}$$

where S satisfies the sliding/reaching condition:

$$\frac{1}{2}\frac{d}{dt}(S^2) \le -\eta|S| \qquad \eta \in \Re^+ \qquad \text{(A7)}$$

Then, the derivative of the function V satisfies:

$$\dot{V} \le -k_v(v_p - v_d)^2 + [e_F + (\hat{F}_L - F_L) + (\hat{F}_f - F_f)](v_p - v_d) - \eta|e_F| \qquad \text{(A8)}$$

When the uncertainty is such that the bounds defined by Eq (22) satisfy the condition:

$$\delta F_L + \delta F_f + |e_F| \le \alpha|v_p - v_d| \qquad \alpha \in \Re^+ \qquad \text{(A9)}$$

the inequality given by Eq (A8) becomes:

$$\dot{V} \le -(k_v - \alpha)(v_p - v_d)^2 - \eta|e_F| \qquad \text{(A10)}$$

For $k_v \ge \alpha$, $\dot{V}$ is rendered negative semi-definite with the control laws given by Eqs (16) and (20) and the uncertainty bounds given by Eqs (18) and (A9). Recall that, when the matched uncertainty bounds satisfy Eq (18), the pressure force tracking error is driven to the boundary layer $|e_F| \le \Phi$ in finite time by the "continuous" sliding mode controller. If, in addition, the unmatched uncertainty can be bounded as in Eq (A9), i.e., to within a compromise linear growth bound that depends on the (acceptable) velocity tracking error, the derivative of the Lyapunov function $\dot{V}$ is negative semi-definite. Bounded position velocity and force tracking is achieved provided both the matched and unmatched uncertainty bounds are satisfied.