Proceedings of the 2007 American Control Conference
Marriott Marquis Hotel at Times Square
New York City, USA, July 11-13, 2007

Robustness to Friction Estimation for Nonlinear Position Control of an Electrohydraulic Actuator

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***Abstract*—A near input-output (IO) linearizing position tracking controller is a nonlinear controller based on a nonlinear model of an electrohydraulic actuator. It uses feedback of piston friction force. In this paper, a friction model is identified from offline experiments and is subsequently approximated with differentiable functions. The effects of altering the friction estimate and even of ignoring it altogether, are investigated. For the test system considered, the near IO linearizing controller is moderately robust to this uncertainty in friction.**

Keywords: electrohydraulic actuator, IO linearization, position control, friction estimation, robustness

I. Introduction

Electrohydraulic actuators are among the most ubiquitous actuation systems for a variety of positioning and force generation applications. However, electrohydraulic actuators exhibit significant nonlinearities in their dynamics. To obtain satisfactory performance in the presence of these nonlinearities, elaborate nonlinear controllers are often necessary. The approaches suggested in the literature include variants of linear state feedback [1], adaptive control [1-5], variable structure control [6] and Lyapunov-based controller designs [2, 7-9]. Each approach has its own strengths and limitations as outlined in the respective listed references.

In this paper, a partial feedback (input-output) linearizing controller is considered [10, 11]. Feedback linearization involves the transformation of a nonlinear system to a linear one via nonlinear state feedback and input transformation. The linear system can then be handled using results from linear control theory.

Perhaps the earliest report on the application of feedback linearization to electrohydraulic actuators was that of Axelson and Kumar [12] in 1988. They presented the derivation of the control laws emphasizing the nonlinearity of valve flow only. Hahn, et al [13] derived a more detailed controller for the position tracking case and presented limited results from simulations with an inertia load. Vossoughi and Donath [14] presented an analysis and derivation of feedback linearizing controllers for velocity tracking in a robotic application. Prior papers by the author [15-17] include further references and successful applications in simulations and some experiments, as well as a suggested tuning procedure for input-output (IO) linearizing controllers in pressure/force and position tracking applications.

The focus of the present paper is to study the effect of friction estimation on the performance of the near IO linearizing position controller. Section II, describes the system model adopted for this study. Section III outlines the expressions for the near IO linearizing controller together with some necessary assumptions for its implementation. These assumptions, which are often tacitly ignored in other work, make it explicit that the adopted form is only a ‘near’ IO linearizing controller, and thereby, that an exact IO linearizing controller is not feasible. Section IV describes the friction identification experiments and the models adopted for controller implementation. Section V presents the discussion on the robustness of the controller to changing friction estimates and Section VI presents the conclusions.

II. System Model

Physical models of electrohydraulic actuators are quite widely available in the literature [1, 18-21]. The model used here applies to a four-way servovalve close-coupled with a rectlinear actuator as shown in Fig. 1. q_t and q_b, are flow rates from the top chamber and to the bottom chamber of the cylinder, respectively. q_i represents internal leakage flow and $q_{e,t}$ and $q_{e,b}$ are external leakage. A_t and A_b represent the effective piston areas, and V_t and V_b designate the volumes of oil in the top and bottom chambers, respectively, corresponding to the center position (x_p=0) of the piston.

Manuscript received September 22, 2006.

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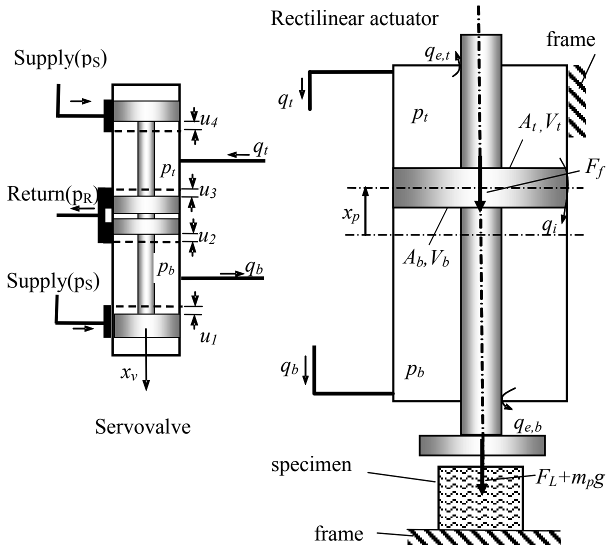

Fig. 1. Schematic of a rectilinear actuator and servovalve

Considering flow continuity and the state equation with the effective oil bulk modulus, β_e, for the cylinder chambers, and introducing the load (differential) pressure, p_L, [19], it can be shown that the load pressure dynamics are given by (see, for example, [15]) :

$$\dot{P}_L = f_{p_L}(x_p,\dot{x}_p,p_L) + g_{p_L}(x_p,p_L,\mathrm{sgn}(i_v))\,i_v \tag{1}$$

where the load pressure, p_L, is:

$$p_L = p_b - p_t \tag{2}$$

and

$$f_{p_L}(x_p,\dot{x}_p,p_L) = -\beta_e \dot{x}_p\left(\frac{A_b}{V_b + A_b x_p} + \frac{A_t}{V_t - A_t x_p}\right) - \beta_e C_L p_L\left(\frac{1}{V_b + A_b x_p} + \frac{1}{V_t - A_t x_p}\right) \tag{3}$$

$$g_{p_L}(x_p,p_L,\mathrm{sgn}(i_v)) = \beta_e C_v\sqrt{\left(\frac{p_S - p_R}{2}\right)} \times \sqrt{1 - \frac{p_L}{p_S - p_R}\mathrm{sgn}(i_v)} \times \left(\frac{1}{V_b + A_b x_p} + \frac{1}{V_t - A_t x_p}\right) \tag{4}$$

Here, the external leakages, $q_{b,e}$ and $q_{t,e}$, are neglected. Note that the first term on the right in Eq (3) shows the explicit dependence of the pressure dynamics on the piston velocity. The second term has its roots in the cross chamber leakage, which is assumed to be laminar with leakage coefficient, C_L. The expression for the coefficient of the current input i_v, lumped into g_{pL}, arises from turbulent flows through the sharp-edged control orifices of a spool valve to and from the two sides of the cylinder chambers. The valve is assumed to be matched and symmetrical ($u_1=u_2=u_3=u_4=0$) with valve coefficient C_v. Also the valve spool dynamics are assumed to be fast enough to be neglected for the purpose of controller derivation.

The state equations governing piston motion are derived considering the loading model for the actuator. For the test system, the actuator cylinder is rigidly mounted on a load frame, which is considered as the inertial frame. For a symmetric actuator ($A_b=A_t=A_p$), the upward force on the actuator piston due to the oil pressure in the two cylinder chambers is given by:

$$F_p = A_p p_L \tag{5}$$

The friction force on the piston in the cylinder is denoted by F_f, and the external loadings, including specimen stiffness and damping forces, are lumped together in F_L. In Fig. 1, F_L is considered tensile positive. The equations of motion are derived by applying Newton's Second Law:

$$\dot{x}_p = v_p \tag{6}$$

$$\dot{v}_p = \frac{1}{m_p}[A_p p_L - F_L - F_f - m_p g] \tag{7}$$

Equations (1), (6) and (7), constitute the state space model for the servovalve and loaded actuator subsystem under consideration. These equations also contain the major modeled nonlinearities in the system, which are the variable hydraulic capacitance and the turbulent flow rate versus pressure drop relations. Nonlinearity is also introduced in Eq (7) by the nonlinear friction force, which includes Coulomb, static, and viscous components [22], as will be detailed in Section IV.

III. Near-IO Linearizing Position Tracking Controller

The first and second derivatives of the output position, x_p, as given by Eqs (6) and (7) do not contain the control input, i_v , However, further differentiation of (7) gives:

$$\ddot{x}_p = f_p(x_p,\dot{x}_p,p_L,\dot{F}_f,\dot{F}_L) + g_p(x_p,p_L,\mathrm{sgn}(i_v))i_v \tag{8}$$

where the nonlinear functions, f_p and g_p, are respectively:

$$f_p(x_p,\dot{x}_p,p_L,\dot{F}_f,\dot{F}_L) = \frac{1}{m_p}[f_F(x_p,\dot{x}_p,p_L) - F_f - F_L] \tag{9}$$

$$g_p(x_p,p_L,\mathrm{sgn}(i_v)) = \frac{1}{m_p}g_F(x_p,p_L,\mathrm{sgn}(i_v)) \tag{10}$$

The form of Eq (8) leads to a piecewise IO linearization suggesting the control law:

$$i_v = \frac{1}{g_p(x_p, p_L, \mathrm{sgn}(i_v))}(v - f_p(x_p, \dot{x}_p, p_L, \dot{F}_f, \dot{F}_L)) \tag{11}$$

The closed loop position dynamics reduce to:

$$\dddot{x}_p = v \tag{12}$$

It leads to an exponentially convergent tracking when the new input v is chosen as:

$$v = \dddot{x}_d - k_3(\ddot{x}_p - \ddot{x}_d) - k_2(\dot{x}_p - \dot{x}_d) - k_1(x_p - x_d) \tag{13}$$

where x_d is the desired position profile. The dynamics of the closed loop position tracking error, $e=x_p-x_d$, reduce to:

$$\dddot{e} + k_3\ddot{e} + k_2\dot{e} + k_1 e = 0 \tag{14}$$

The control law is rewritten as:

$$i_v = \frac{1}{g_p(x_p, p_L, \mathrm{sgn}(i_v))}(\dddot{x}_d - k_3\ddot{e} - k_2\dot{e} - k_1 e - f_p(x_p, \dot{x}_p, p_L, \dot{F}_f, \dot{F}_L)) \tag{15}$$

The three gains k_1, k_2, and k_3 can be chosen to place the poles of the closed loop tracking error dynamics (14) strictly in the left half s-plane. This could be done by using direct pole placement or posing the problem as a linear optimal control (such as LQR) problem. Yet another approach that exploits equivalence to a cascade form was revealed in [15].

It is important to note that (15) cannot be solved "as is", since it contains the control variable, i_v, on both sides of an equation involving the *sgn* function. A practical solution to this problem becomes evident when considering the digital implementation of this piecewise IO linearizing controller. The sign of the value of i_v at the previous time step can be used to compute the value of i_v at the current time step, if it can be supposed that the control current does not change signs at a rate faster than the sampling rate (This is approached by using a fast sampling rate). However, it is difficult to analytically prove that this approach does not lead to control chatter. This chatter problem has not been previously reported in the literature that discusses feedback linearization for hydraulic drives [13, 14, 16]. Nevertheless, the term near input-output (IO) linearization is adopted here to make the explicit admission that the present controller is not an exact IO linearizing controller.

IV. Friction Estimation Experiment and Modeling

Friction affects the dynamics of the electrohydraulic servovalve as well as the dynamics of the actuator piston. Friction in the servovalve is generally considered to be predominantly of Coulomb type, acting on the spool of the valve, and can in practice be sufficiently eliminated by using dither signals [20]. The particular friction effect of interest in this section is the friction force that appears in the equations of motion of the actuator piston (F_f). The literature offers various empirical models applied to specific hydraulic actuators [1, 9, 23, 24]. In the most general case, friction in the actuator cylinder is considered to be a function of the position and velocity of the piston, the chamber pressures (or the differential pressures when the piston is sticking near zero velocity), the local oil temperature and also running time.

In a previous work [22], open-loop and closed-loop tests were performed to identify the friction force on the actuator piston by assuming it to be a function of velocity. The open-loop tests involved changing the set current input to the servovalve while measuring the steady-state cylinder chamber pressure responses as well as estimates of the steady-state velocity obtained by differentiating piston position responses which were measured with an LVDT.[1] The friction force is then estimated using Eq (7), assuming the acceleration and the external force to be zero in the steady-state. Strong scatter was observed in the friction estimated from these open loop tests.

Improved and more realistic friction force estimates, including hysteresis effects, were obtained by performing friction estimation with closed loop position control tests after warm up periods to stabilize oil temperatures. The tests involved tracking a 2 Hz 35 mm sine wave position command under P-control while measuring acceleration, piston position and chamber pressures. Equation (7) was again used to estimate the friction force without having to assume zero acceleration. The velocity was computed by taking the finite difference derivative of the position response. The acceleration was measured with a accelerometer mounted on the piston rod. Fig. 2 shows the result form one such closed loop test. It shows that the hysteretic behavior of friction is especially strong in the upward (positive velocity) motion. It can also be observed that the friction force is slightly asymmetric with respect to the direction of motion. This asymmetry is thought to be due to asymmetric porting volumes and behaviors in the seals and is expected to vary between actuators.

[1] The actuator approximates a velocity source in the open-loop.

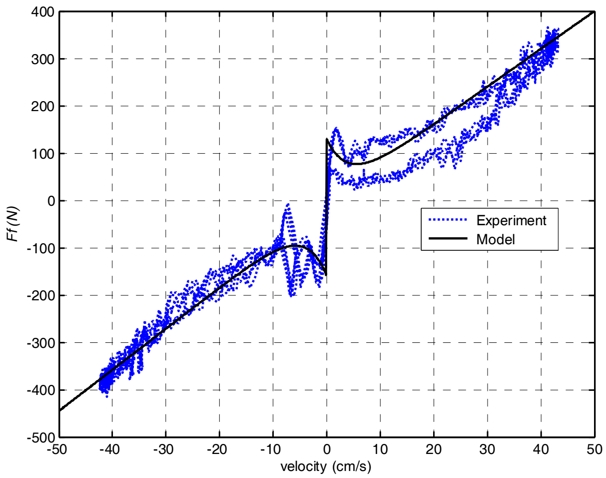

Fig. 2 Estimation of piston friction force (Experiment)

For our purposes, the following common memory-less analytical model of the friction force (without hysteresis) was adopted [1, 24].

$$F_f = F_v^{\pm}\dot{x}_p + \mathrm{sgn}(\dot{x}_p)\left(F_c^{\pm} + \left(F_s^{\pm} - F_c^{\pm}\right)e^{-\left|\frac{\dot{x}_p}{C_S^{\pm}}\right|}\right) \qquad (16)$$

where F_v is the viscous coefficient, F_c is the Coulomb term and F_s is the static term. The friction force is a combination of the so called Stribeck (declining friction at low velocity), Coulomb and Viscous terms. The coefficients were computed by fitting this equation to the experimental data shown in Fig. 2. The observed asymmetry of the experimentally determined friction force with respect to the sign of the velocity can be taken into account by taking different coefficients for the up and down motions (denoted by ± superscripts in Eq (16)).

The expression in Eq (16) has a strong discontinuity and sharp corners near zero velocity. Since the implementation of the nonlinear control law (Eq (15)) for position tracking requires the time derivative of the friction force, it is necessary to make the expression given by Eq (16) smooth with respect to velocity before the derivative can be taken.

The following approximations of the sign function ($\mathrm{sgn}(x)$) and the absolute value function ($|x|$) are taken [1]:

$$\mathrm{sgn}(x) \approx \frac{2}{\pi}\arctan(\gamma x) \qquad (17)$$

$$|x| \approx \frac{2x}{\pi}\arctan(\gamma x) \qquad (18)$$

The parameter γ is used to adjust the degree of smoothening applied to the friction estimation of Eq (16). Figure 3 shows typical results from applying these approximations. Note that the higher the value of the parameter γ the better the approximation, but the sharper the corners at zero velocity (i.e, when the piston motion changes direction). A compromise value of γ=5 was selected for the remainder of the results presented in this work. If the asymmetry with the sign of velocity is to be considered, there still remains some corner at exactly zero velocity, but the severity of the discontinuity is reduced when the approximations are applied.

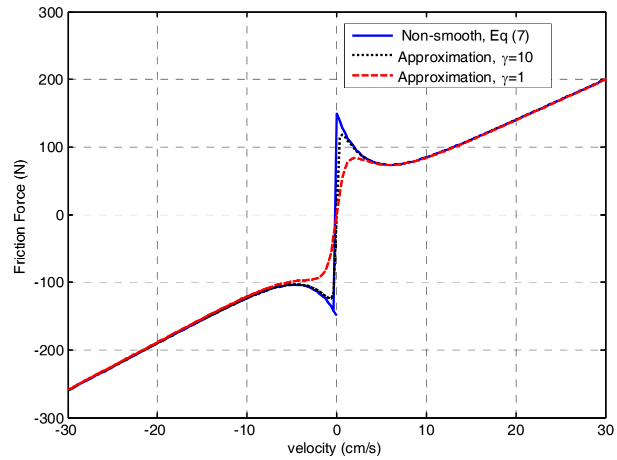

Fig. 3 Smooth approximations of the friction force

V. Robustness To Uncertainty in Friction Estimation

In this section, we look at the robustness of the near IO linearizing position tracking controller to uncertainty in the estimation of friction. Since it is hardly possible to exactly determine and alter the friction in the actual system in a quantifiable manner, we use system simulations to evaluate the robustness of the near IO linearizing controller to representative cases of friction uncertainty. The overall system model, including the accumulators and transmission lines, and the validation experiments as well as the nominal controller parameters were detailed in previous papers [16, 22]. In all cases, the specimen is replaced by an 11 kg mass attached to the piston (i.e, F_L=0).

For the near IO linearizing controller, all three closed-loop poles are placed at s=-300 in the left half s-plane. We consider the performance of the near IO linearizing controller when tracking a 20 mm-2 Hz sine wave reference trajectory where the velocity is limited to ±25 cm/s. This velocity range is where the nonlinearity due to friction is the strongest as shown in the experimental data of Fig. 2.

As a first case, we suppose that the nonlinear and smooth friction estimation with the smoothening parameter γ = 5 is used as the nominal friction estimate in the controller, and we alter the friction in the actuator model to be higher or lower by 100% than the nominally identified model of Fig. 2. In practice, significant variations in friction can happen with changes in temperature, oil viscosity, operating pressure, and also length of running time for the actuator. As a second case, we consider changing the nonlinear friction estimation model used in the controller to a nominal linear viscous case, or to even no friction estimation while we keep the nominal nonlinear estimate of friction in the actuator model. Both of these cases are compared with the case of perfect knowledge of the nonlinear friction in the system by the controller.

Figure 4 shows results for the first case, where the friction estimation used in the controller is mismatched from the actual friction in the actuator by ±100%. It also includes a case where the friction in the actuator model is set to zero (which is a hypothetical case of zero friction).

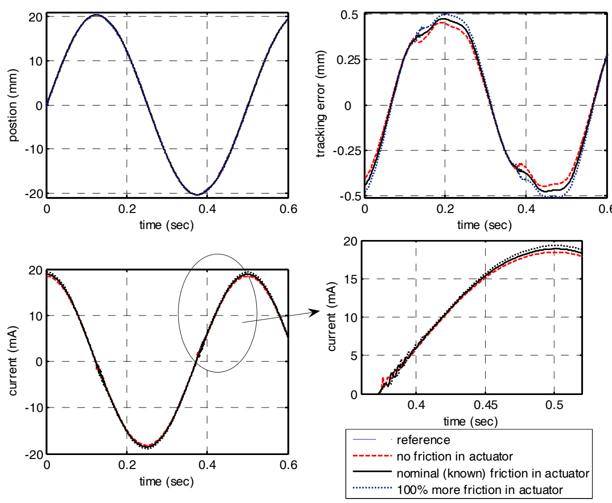

Fig. 4 Effect of uncertainty in the actuator friction on the tracking performance of the Near IO linearizing controller (simulation)

It can be seen from Fig. 4 that the effect of friction uncertainty on the tracking error with the near IO linearizing controller is minimal. When there is more friction in the actuator than estimated by the controller, slightly higher control current is required, and the peak tracking error is also correspondingly higher. When there is less friction in the actuator than estimated by the controller, the peak tracking error is lower. However, the overall effect doesn't appear to be significant for this system. Any significant deterioration in tracking performance appears mainly near zero velocity, when the piston is coming to rest and changing direction of motion. This is the evidence of the effects of the stick-slip on transition of friction from static to kinetic values near zero velocity. For precision positioning applications, this observation may be an important one.

Now that it is determined that uncertainty in the nonlinear friction estimate does not appear to significantly influence the tracking performance of the near IO linearizing controller over the range of motion, as a second case, we look at the effect of using less accurate friction estimation for the controller. Figure 5 shows a comparison of cases of no-friction estimation, nominal nonlinear estimation and (nominal) viscous estimation on the controller model, while only the nominal nonlinear friction model is kept in the actuator model. Here by nominal, it is meant to refer to the experimentally identified friction as shown in Fig. 2 and the respective smooth approximation in Fig. 3 with γ=5.

Figure 5 shows the results for this second case. Again, the significant difference in the tracking error from the use of a viscous (linear) friction or nonlinear friction or no friction estimation in the near IO linearizing controller is mainly near zero velocity, accompanied by the stick-slip phase of the motion, as can be seen in the magnified insert of Fig. 5. It is slightly better to have an estimate of friction in the controller, even if only viscous, than to ignore it altogether.

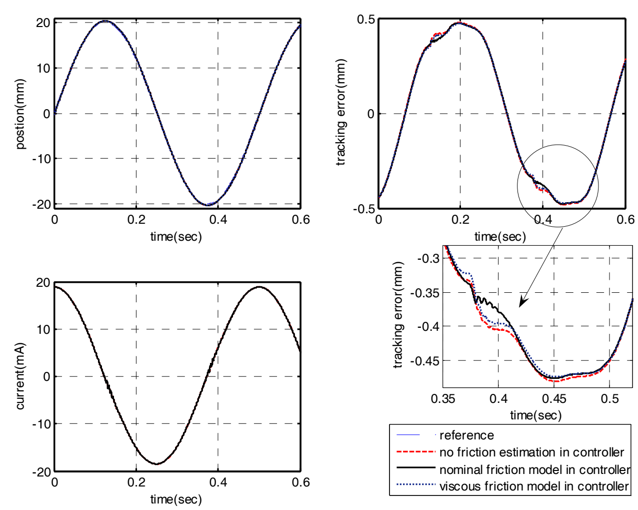

Fig. 5 Effect of changing the friction estimation model in the Near IO linearizing controller (simulation)

VI. Conclusions

This paper presented a study of the significance of piston friction force estimation on the tracking performance of a near IO linearizing controller for an electrohydraulic actuator. A common memory-less nonlinear friction model is experimentally identified and subsequently approximated with smooth functions for use in the controller implementation, which requires the derivative of the friction force.

It was observed from system simulation results that, for the test system considered, the tracking performance of the near IO linearizing controller does not appear to be significantly affected by uncertainty in the friction estimation. A simple viscous estimation may be sufficient for this particular system, depending on whether the observed difference in tracking error is considered critical for a particular application or not.

REFERENCES

1. Mohieddine Jelali, and Kroll, Andreas, *Hydraulic Servo-Systems: Modelling, Identification and Control*. -(Advances in Industrial Control). 2003, London: Springer-Verlag.
2. Andrew Alleyne, and Hedrick, Karl J., *Nonlinear Adaptive Control of Active Suspensions.* IEEE Transactions on Control Systems Technology, 1995. **3**(1).
3. A.R. Plummer, and Vaughan, N.D., *Robust Adaptive Control for Hydraulic Servosystems.* Transactions of ASME, Journal of Dynamic Systems, Measurement and Control, 1996. **118**: p. 237-244.
4. K. Ziaei, and Sepheri, N., *Design of a Nonlinear Adaptive Controller for an Electrohydraulic Actuator.* Journal of Dynamic Systems, Measurement and Control, 2001. **123**: p. 449-456.
5. B. Yao, Bu, F., and Chiu, G.C.T., *Nonlinear Adaptive Robust Control of Electrohydraulic Systems Driven by Double-Rod Actuators.* International Journal of Control, 2001. **74**(8): p. 761-775.
6. Q.H. Nguyen, Ha, Q.P., Rye, D.C, and Durrant-Whyte, H.F. *Force/Position Tracking for Electrohydraulic Systems of a Robotic Excavator*. in *The 39th IEEE Conference on Decision and Control*. 2000. Sydney, Australia.
7. C.-L. Hwang, *Sliding Mode Control Using Time-varying Switching Gain and Boundary Layer for Electrohydraulic Position and Differential Pressure Control.* IEEE Transactions on Control Theory Applications, 1996. **143**(4).
8. Mohammad R. Sirouspour, and Salcudean, S.E. *On the Nonlinear Control of Hydraulic Servo-systems*. in *The IEEE International Conference on Robotics and Automation*. 2000. San Francisco, California.
9. Garret A. Sohl, and Bobrow, James E., *Experiments and Simulations on the Nonlinear Control of a Hydraulic Servosystem.* IEEE Transactions on Control Systems Technology, 1999. **7**(2).
10. Jean-Jacques E. Slotine, and Li, Wieping, *Applied Nonlinear Control*. 1991, Englewood Cliffs, New Jersey: Prentice Hall.
11. Hassan K. Khalil, *Nonlinear Systems*. 3rd ed. 2002, Upper Saddle River, New Jersey: Prentice Hall.
12. Steve Axelson, and Kumar, K. S. P. *Dynamic Feedback Linearization of a Hydraulic Valve-Actuator Combination*. in *The 1988 American Control Conference*. 1988. Atlanta, Georgia.
13. H. Hahn, Piepenbrink, A., and Leimbach, K.-D. *Input/Output Linearization Control of an Electro Servo-Hydraulic Actuator*. in *The 3rd IEEE Conference on Control Applications*. 1994. Glasgow, UK.
14. Gholamreza and Donath Vossoughi, Max, *Dynamic Feedback Linearization for Electrohydraulically Actuated Control Systems.* Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control, 1995. **117**: p. 468-477.
15. Beshahwired Ayalew and Bohdan T. Kulakowski. *Cascade Tuning for Nonlinear Position Control of an Electrohydraulic Actuator*. in *Proceedings of the American Control Conference*. 2006. Minneapolis, MN.
16. Beshahwired Ayalew, Bhondan T. Kulakowski, and Kathryn W. Jablokow. *A Near Input-Output Linearizing Force Tracking Controller for an Electrohydraulic Actuator*. in *Proceedings of ASME IMECE:International Mechanical Engineering Congress and Exposition*. 2005. Orlando, FL.
17. Beshahwired Ayalew, and Kulakowski, Bohdan T. *Nonlinear Decentralized Control of Electrohydraulic Actuators in Road Simulation*. in *Proceedings of IMECE 2005: International Mechanical Engineering Congress and Exposition*. 2005. Orlando, FL: ASME.
18. Peter Dransfield, *Hydraulic Control Systems: Design and Analysis of their Dynamics*. 1981: Springer-Verlag.
19. Herbert E. Merritt, *Hydraulic Control Systems*. 1967, New York, New York: John Wiley and Sons.
20. Gerard Van Schothorst, *Modelling of Long-Stroke Hydraulic Servo-Systems for Flight Simulator Motion Control and System Design*. 1997, Delft University of Technology: Delft, The Netherlands,
21. Andreas Kugi, *Non-linear Control Based on Physical Models*. 2001, London, UK: Springer.
22. Beshahwired Ayalew, and Kulakowski, Bohdan T., *Modeling Supply and Return Line Dynamics for an Electrohydraulic Actuation System.* ISA Transactions, 2005. **44**(3).
23. Andrew Alleyne, and Liu, Rui, *A Simplified Approach to Force Control for Electrohydraulic Systems.* Control Engineering Practice, 2000. **8**: p. 1347-1356.
24. B. Manhartsgruber. *Singular Perturbation Analysis of an Electrohydraulic Servo-drive with Discontinuous Reduced Dynamics*. in *The 1999 ASME Design Engineering Technical Conference*. 1999. Las Vegas, Nevada.