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Feedback Compensation of the In-Domain Attenuation of Inputs in Diffusion Processes

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***Abstract*— This paper addresses the problem of compensating for the in-domain attenuation of inputs for a class of process control applications. Specific examples include the challenge of maintaining uniform temperatures or through-cure during thick-film radiative drying and curing processes in the face of the effective input's variation with film depth due to the so–called Beer-Lambert effect. These distributed parameter control problems are modeled with parabolic PDEs for the diffusion processes along with an in-domain input with a spatial attenuation function. The approach presented in this paper involves transforming the original model to an equivalent boundary input problem to which existing output feedback backstepping boundary control design methods can be applied. The resulting compensation scheme includes a mechanism for tuning the closed-loop performance. The performance of this scheme is compared with a controller designed via modal approximation of the PDE.**

Keywords: distributed parameter control, in-domain actuation, boundary control, compensation of Beer-Lambert effect, curing process, stereolithography

I. INTRODUCTION

The choice of control design methods for distributed parameter systems modeled by partial differential equations (PDEs) often starts with categorizing whether the control inputs are applied at the boundary (boundary control) or distributed throughout or at a few locations in the spatial domain (in-domain control). These classes of problems have been studied extensively, and many PDE control approaches have been proposed and demonstrated [1-4].

A particular class of PDE control problems that has received limited attention is one where a single actuator, often physically located outside the domain, has a distributed in-domain input to the physical process under consideration. Examples can be found in various industrial applications that use radiant energy sources. These processes, include letter pressing, production of holograms, microelectronics and integrated circuits, curing of dental fillings and rapid prototyping processes such as stereolithography [5]. Additional to this list are ultraviolet (UV) curing and infrared (IR) drying processes in automotive and aerospace paint/coating applications [6-7]. In all of these processes, the single actuating input is often a lamp, laser or LED energy source physically located outside the target process, and transferring energy to the target process by radiation as an in-domain input. However, this transfer is subject to the attenuation of the input intensity and thereby the reduction of its effectiveness farther in the domain of the target. The so-called Beer-Lambert law is often used to explain this attenuation of intensity with depth.

Practical challenges related to this Beer-Lambert effect (also referred to as photo-absorption) are worth highlighting. The main one is a restriction it imposes on the depth of effective processing achievable using open-loop methods. In UV curing of thick-film coatings, the Beer-Lambert effect could lead to poor performance of coatings cured with open-loop methods. Typically, over-dosing, i.e., more energy than necessary, is applied to counteract the effect of depth attenuation. In stereolithography curing applications, the depth of cure limitation necessitates layered or multi-pass approaches (Figure 1). Current solutions involve offline open-loop optimization runs to determine a critical depth of cure for each curing pass [8]. If one can find compensation algorithms for the Beer-Lambert effect, it may be possible to change the stepped curing approach and speed up production (with less passes needed), improve product quality and reduce processing energy needs.

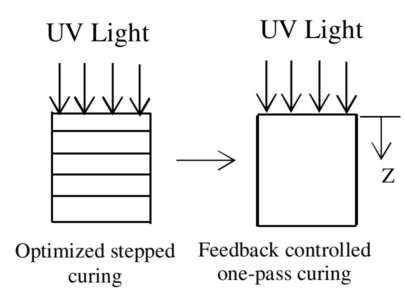

Figure1. Potential Benefit of Compensating for the Beer-Lambert Effect

While there appears to be no prior work on direct feedback compensation algorithms for the Beer-Lambert effect from control or actuation point of view, there have been some practical solutions proposed for the related observation/measurement problem. One is found in the field of bioengineering, where a depth compensation algorithm (DCA) is applied to compensate the decay of light propagation in a tissue so as to accurately localize absorbers in deep tissue by using depth sensitive Diffuse Optical Tomography (DOT) [9]. DCA is based on inversion to create a balancing weight matrix to compensate measurement sensitivity with depth. In combustion engine research [10], an iterative compensation algorithm is developed to compensate laser attenuation in optically dense fuel sprays (fuel image obtained by planner laser imaging). In this case, the

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compensation is done by adding up the lost light along the path of the laser image iteratively pixel by pixel. In both of the above works, compensation is achieved by off-line post processing of measurements.

In this paper, we formulate the problem of in-domain control, focusing on compensating Beer-Lambert effects in diffusion-dominated processes using output feedback. The key approach is that of transforming the body input problem into a standard boundary control problem for which solutions readily exist ([2], [11-12]). In our particular transformation, we imbed free parameters in the desired stable target systems that allows us to tune the performance of the resulting compensator. We assume that boundary measurements are available and construct PDE observers to estimate spatial state distributions. The distributed nature of the system is kept and the control design is performed based on the infinite-dimensional model without resorting to model reduction.

The remainder of the paper is organized as follows. Section II narrows down the problem statement starting with a generalized UV curing process model. Section III and IV detail the controller and observer designs, respectively. Section V provides demonstrative simulation results including comparisons with a modal approximation based control design. Section VI gives the conclusions of the paper.

II. Problem Statement

We consider UV curing as an example process for which there is an in-domain attenuation of input. The curing of a liquid polymer (resin) through exposure to UV light involves the attenuation of the thermal radiation through the resin in accordance with Beer-Lambert's law. The curing process modeled is given by the coupled PDEs given by [13]:

$$\begin{cases} \rho c_p \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T(x,t)}{\partial z}\right) - \Delta H_{pm} \frac{\partial M(x,t)}{\partial t} & in\ \Omega_T, PDE \\ \qquad +I_t \beta_t e^{-\beta_t x} & \\ \frac{\partial M(x,t)}{\partial t} = K_p[M(x,t)].\left[\frac{\emptyset \varepsilon[S(x,t)] I_o e^{-\varepsilon[S(x,t)]x}}{K_t}\right]^{\frac{1}{2}} & in\ \Omega_T, PDE \\ \frac{\partial S(x,t)}{\partial t} = -\ \emptyset \varepsilon[S(x,t)] I_o e^{-\varepsilon[S(z,t)]x} & in\ \Omega_T, PDE \\ k \frac{\partial T(0,t)}{\partial x} = h(T(0,t) - T_\infty) & on\ \Gamma_1, BC1 \\ T(l,t) = T_\infty & on\ \Gamma_2, BC2 \\ T(x,0) = T_0(x) & in\ \Omega,\ IC \end{cases} \tag{1}$$

where, ρ is density, c_p is heat capacity, k is thermal conductivity, ΔH is heat of polymerization, I_t is incident heat flux intensity, β_t is extinction coefficient of heat flux. I_0 is the incident UV light intensity; ε is the molar absorptivity; $T(x,t)$ is temperature; $S(x,t)$ and $M(x,t)$ are the photo-initiator and monomer concentration, respectively; $\emptyset$ is the quantum yield of initiation; K_p is propagation rate constant; h is convective heat transfer coefficient; K_t is termination rate constant; $\Omega_T \in [0,l] \times [0,\infty)$, $\Omega \in [0,l]$, $\Gamma_1 \in \{0\} \times (0,\infty)$, $\Gamma_2 \in \{l\} \times (0,\infty)$ and, T_∞ is constant ambient temperature. The Beer-Lambert effect is described by the exponential attenuation function multiplying the radiant input intensities in both the concentration and temperature equations.

For the following analysis, we reduce the problem and focus on the thermal diffusion part subjected to the attenuation of the input according to Beer-Lambert's law, where the heat input is multiplied by a spatial attenuation function. We neglect the monomer and photo-initiator consumption rates and associated nonlinearities. The simplified model is given by:

$$\begin{cases} \rho c \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T(x,t)}{\partial x}\right) + I_t \beta_t e^{-\beta_t x} & in\ \Omega_T, PDE \\ k \frac{\partial T(0,t)}{\partial x} = h(T(0,t) - T_\infty) & on\ \Gamma_1, BC1 \\ \frac{\partial T(l,t)}{\partial x} = 0 & on\ \Gamma_2, BC2 \\ T(x,0) = T_0(x) & in\ \Omega, IC \end{cases} \tag{2}$$

Introducing non-dimensional spatial variable, time, and temperature, respectively, as:

$$z = \frac{x}{l} \tag{3}$$

$$t' = \frac{k}{\rho c_p l^2} t \tag{4}$$

$$w(z,t') = \frac{T_\infty - T(z,t')}{T_\infty} \tag{5}$$

and, subsequently - omitting the superscript in t', we obtain the non-dimensionalized form:

$$\begin{cases} w_t(z,t) = w_{zz}(z,t) + \gamma e^{-\sigma z} u(t) & in\ \Omega_T, PDE \\ w_z(0,t) = -q w(0,t) & on\ \Gamma_1, BC1 \\ w_z(1,t) = 0 & on\ \Gamma_2, BC2 \\ w(z,0) = w_0(z) & in\ \Omega,\ IC \end{cases} \tag{6}$$

where, $\gamma = -\frac{\beta_t l^2}{k T_\infty}$, $\sigma = l\beta_t$, $q = \frac{hl}{k}$, $u(t) = I_t$. $u(t)$ is the control signal representing the incident heat flux intensity (reaching the surface). The notation w_z represents $\frac{\partial w(z,t)}{\partial z}$. Note that this model specifically considers the exponential input decrement over the spatial domain. However, the approach we present below can be used with a general spatial function representing the attenuation.

The control problem is to devise the input $u(t)$ such that some regulation or tracking of the spatially distributed temperature state is achieved despite the spatial input attenuation. We assume that the surface temperature is measured at the $z = 0$ end. The structure of the feedback control scheme sought is summarized in Figure 2.

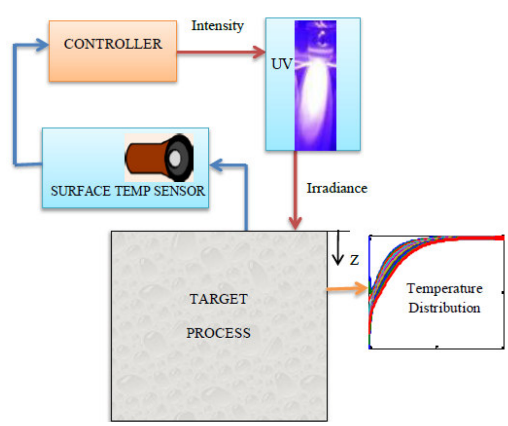

Figure2. Closed Loop Process Control System

III. CONTROLLER DESIGN

Existing approaches for in-domain input control problems like (6) include a) model reduction to a finite number of ODEs to which standard linear control design methods apply [1] or, b) use of mobile actuators [14]. However, the former is subject to approximation errors, while the latter cannot be applied to the depth problem where moving of the actuator across depth is impossible. In this paper, we outline a three-step process that directly deals with PDE control design for the system given in (6), without model reduction and with a spatially fixed actuator applying irradiance on top surface. In this section, we design a state feedback controller assuming availability of the distributed state. This assumption will be lifted when we discuss observer design in the next section.

A. Transforming In-domain Input to Boundary Input

In this step, we transform the spatially distributed in-domain input problem (6) to an equivalent mixed boundary input problem (7):

$$\begin{cases} w_t(z,t) = w_{zz}(z,t) & in\ \Omega_T, PDE \\ w_z(0,t) = -qw(0,t) & on\ \Gamma_1, BC1 \\ w_z(1,t) = h(t) & on\ \Gamma_2, BC2 \\ w(z,0) = w_0(z) & in\ \Omega,\ \ IC \end{cases} \tag{7}$$

The idea is to assume that the original PDE (6) without a boundary input term is equivalent to a hypothetical PDE (7) with the boundary input term $\mathrm{h(t)}$. This is inspired by the transformation sought in [4] where a PDE with localized in-domain inputs is converted to a standard Cauchy boundary value problem and the feedback control law is sought via energy multiplier methods.

The next step is developing a relationship that transforms the effect of control signal appearing in (6) to a boundary input, $\mathrm{h(t)}$ as in (7). This can be done by equating the weak formulations of both equations (6) and (7). To perform the weak solution analysis, we consider a special Sobolev space:

$$H^1_{\Gamma_2}(\Omega) = v(z) \in H^1(\Omega): v(\Gamma_2) \neq 0 \tag{8}$$

where: $H^1(\Omega) = v(z) \in L_2(\Omega) | v_z(z) \in L_2(\Omega)$ is the Hilbert space with the inner product and the norm defined in (9) & (10), respectively:

$$H^1(\Omega) := \langle v, w \rangle_1 = \int_\Omega\ v(z).w(z)dz \tag{9}$$

$$H^1(\Omega) := \|v\|_1 = \left(\int_\Omega\ (v^2 + v_z^2)dz\right)^{1/2} \tag{10}$$

This choice of Sobolov space helps us to maintain information at the second boundary, Γ_2. We multiply the PDE (7) with an arbitrary test function $v(z) \in H^1_{\Gamma_2}(\Omega)$ which is smooth (i.e., continuously differentiable $(C^1(\bar{\Omega}) \cap C(\Gamma))$. Then, performing integration by parts over the domain Ω, the weak form of PDE (7) is

$$\int_\Omega\ w_t(z,t)v(z)dz + \int_\Omega\ w_z(z,t).v_z(z)dz = \int_{\Gamma_2}\ w_z(1,t)v(z)d\gamma + qw(0,t)\int_{\Gamma_1}\ v(z)d\gamma, \forall w,v \in H^1_{\Gamma_2}(\Omega) \tag{11}$$

Applying Lax-Milgram theorem with $V = H^1_{\Gamma_2}(\Omega)$, one can prove the existence and uniqueness of the weak solution for this asymmetric boundary value problem following examples provided in [15].

Similarly, the weak form of the PDE (6) is:

$$\int_\Omega\ w_t(z,t)v(z)dz + \int_\Omega\ w_z(z,t).v_z(z)dz = \int_\Omega\ \gamma u(t)e^{-\sigma z}v(z)dz + qw(0,t)\int_{\Gamma_1}\ v(z)d\gamma, \forall w,v \in H^1(\Omega) \tag{12}$$

Similar reason hold for the existence and uniqueness of the weak solution of (12). For both systems (11) & (12) where w is smooth (i.e., $w \in C^2(\Omega) \cap C(\Gamma)$), it is possible to prove that the unique weak solution can be exact solution with some regularity checks provided in [16-17].

By subtracting (12) from (11) the relationship between the boundary input in (7) and in-domain input in (6) is given by:

$$\mathrm{h(t)} = \frac{\int_\Omega\ \gamma \mathrm{u(t)e^{-\sigma z}v(z)dz}}{\int_{\Gamma_2} \mathrm{v(z)d\gamma}} \tag{13}$$

The condition $\int_{\Gamma_2} \mathrm{v(z)d\gamma} \neq 0$ and the equality constraint (13) are sufficient conditions for (11) & (12) to have an identical weak solution. Note that the solvability of $\mathrm{h(t)}$ from (13) depends strongly on the choice of test function.

Given this transformation, we can use existing boundary control design methods to develop a control law for the boundary input problem (7) and then transform the results back to the original in-domain input problem (6), provided a proper selection is made for the test function $v(z)$. We return to this issue in subsection *C* below.

B. Feedback Control Design Using Backstepping

In the second step, the feedback control law is derived for the boundary control problem (7). In this work, we choose to apply backstepping techniques, detailed in the book [2]. The controller design starts by first defining a stable target system. The choice of the stable target system is not unique. For the current application, the standard heat equation with the boundary conditions in (14) was found suitable:

$$\begin{cases} x_t(z,t) = x_{zz}(z,t) & in\ \Omega_T, PDE \\ x_z(0,t) = 0 & on\ \Gamma_1, BC1 \\ x_z(1,t) = -Kx(1,t) & on\ \Gamma_2, BC2 \\ x(z,0) = x_0(z) & in\ \Omega,\ \ IC \end{cases} \tag{14}$$

It can be shown that this target system is exponentially stable for parameter $\mathrm{K} \geq \frac{1}{2}$ (see Appendix). And as will be illustrated later, this parameter provides a means for tuning the controller.

We define a state transformation along with boundary feedback to bring the unstable PDE (7) to the desired stable form of (14). The suitable state transformation is:

$$x(z) = w(z) - \int_0^z k(z,y)w(y)dy \tag{15}$$

Following the approach detailed in [2], by analytically solving the relevant PDEs for the control gain kernel, one can find the control gain kernel to be:

$$k(z,y) = -qe^{q(z-y)} \tag{16}$$

Then, the Neumann boundary controller that transforms (7) to (14) is determined by first taking the spatial derivative of (15) and then substituting the corresponding boundary condition at $\mathrm{z} = 1$. The controller expression is:

$$h(t) = -(q+K)\left[w(1,t) + q\int_0^1 e^{q(1-y)} w(y,t)dy\right] \quad (17)$$

The backstepping design guarantees stability of the closed-loop consisting of (PDE (7) with controls (17) via the selected invertible transformation (15) and stable target system (14). Next, this stabilizing controller designed for PDE (7) is to be applied back to the original PDE (6) using the transformation map (13) to obtain the original control signal $u(t)$.

C. Back Transformation of Boundary Control to In-Domain Input Signal

In the back transformation, a proper test function $v(z)$ needs to be selected when transforming the boundary control back to the original in-domain control using (13). For the given model, we selected the test function of the form (18) as one that satisfies the fundamental static solution for a unit in-domain input applied to the PDE (6).

$$v(z) = -\frac{z^2}{2} + z - \frac{1}{q} \quad (18)$$

Using (13, 16 ,17 & 18) the original control signal $u(t)$ becomes:

$$u(t) = -\lambda(q+K)\left[w(1,t) + q\int_0^1 e^{q(1-y)} w(y,t)dy\right] \quad (19)$$

where, k, λ, and γ are all constant parameters, and obey:

$$\lambda = \frac{\int_{\Gamma_2} v(z)d\gamma}{\int_{\Omega} \gamma e^{-\sigma z} v(z)dz} \quad (20)$$

IV. Observer Design

In the previous section, we designed a feedback control law that assumes the availability of measurements of the distributed state. In practice, only boundary measurements are available. We therefore need to design an observer to estimate the distributed state based on the available boundary measurements. A number of DPS observer design methods are summarized in [18], including adaptive observer, Lyapunov-based observer and backstepping observer. In this paper, for consistency with the control design, we adopt the backstepping technique to design an observer for the particular class of problems addressed in this paper. We note that the closed loop stability of the combination of separately designed observer and state feedback boundary controller for the transformed boundary actuated PDE (7) has been proved in [2] for the case where identical target PDEs are selected for both the controller and observer design problems. In this work, we found that choosing a different target PDE for the observer problem is more expedient for obtaining the observer gain in closed form. Furthermore, in addition to the steps outlined in [2], we introduce additional transformations for addressing the original in-domain input problem.

Proceeding as above, we transform the in-domain input term to boundary input and design a boundary PDE observer. The construction of transformed observer model with boundary state measurement $w(0,t)$ takes the form:

$$\begin{cases} \hat{w}_t(z,t) = \hat{w}_{zz}(z,t) + p_1(z)[w(0,t) - \hat{w}(0,t)] & in\ \Omega_T, PDE \\ \hat{w}_z(0,t) = -qw(0,t) + p_{10}[w(0,t) - \hat{w}(0,t)] & on\ \Gamma_1, BC1 \\ \hat{w}_z(1,t) = h(t) & on\ \Gamma_2, BC2 \\ \hat{w}(z,0) = \hat{w}_0(z) & in\ \Omega,\ \ IC \end{cases} \quad (21)$$

Defining the error variable as $\tilde{w} = w - \hat{w}$ and subtracting (21) from (7), the error system becomes:

$$\begin{cases} \tilde{w}_t(z,t) = \tilde{w}_{zz}(z,t) - p_1(z)\tilde{w}(0,t) & in\ \Omega_T, PDE \\ \tilde{w}_z(0,t) = -p_{10}\tilde{w}(0,t) & on\ \Gamma_1, BC1 \\ \tilde{w}_z(1,t) = 0 & on\ \Gamma_2, BC2 \\ \tilde{w}(z,0) = \tilde{w}_0(z) & in\ \Omega,\ \ IC \end{cases} \quad (22)$$

The observer gain $p_1(z)$ & p_{10} can be determined by transforming the error system to a stable target system using state transformation along with boundary feedback. The state transformation selected is:

$$\tilde{w}(z) = \tilde{x}(z) - \int_0^z p(z,y)\tilde{x}(y)dy \quad (23)$$

It can be shown that the following traget system is exponentially stable for c>0 (see Appendix):

$$\begin{cases} \tilde{x}_t(z,t) = \tilde{x}_{zz}(z,t) - c\tilde{x}(z,t) & in\ \Omega_T, PDE \\ \tilde{x}_z(0,t) = 0 & on\ \Gamma_1, BC1 \\ \tilde{x}_z(1,t) = 0 & on\ \Gamma_2, BC2 \\ \tilde{x}(z,0) = \tilde{x}_0(z) & in\ \Omega, IC \end{cases} \quad (24)$$

The free parameter c can be used to set the desired observer convergence speed. Applying the approach detailed in [2], we obtain the following PDE and conditions for the observer gain kernel $p(z,y)$ and the observer gains $p_1(z)$ & p_{10}.

The gain kernel PDEs are summarized as follows:

$$p_{zz}(z,y) - p_{yy}(z,y) = -cp(z,y) \quad (25)$$

$$p_z(1,y) = 0 \quad (26)$$

$$p(z,z) = \frac{c}{2}(z-1) \quad (27)$$

The observer gains are:

$$p_1(z) = p_y(z,0) \quad (28)$$

$$p_{10} = p(0,0) \quad (29)$$

Solving (27-29), the observer gain kernel becomes:

$$p(x,y) = -c(1-y)\frac{I_1\left(\sqrt{c(2-z-y)(z-y)}\right)}{\sqrt{c(2-z-y)(z-y)}} \quad (30)$$

The final step is to transform the observer (21) designed for the boundary-actuated problem (7) to that of the original in-domain input PDE (6). To this end, we apply the transformation (13) with the selected test function (18). The final form of the observer is given in (31) below.

V. Final Structure of Proposed Compensator

The structure of the proposed output feedback Beer-Lambert effect compensator is summarized in the schematic block diagram of Figure 3. The compensator consists of the separately designed observer and feedback controller summarized below.

Transforming the observer back to the original in-domain input PDE (6), the final form of the observer takes the form:

$$\begin{cases} \hat{w}_t(z,t) = \hat{w}_{zz}(z,t) + p_1(z)\big(w(0,t) - \hat{w}(0,t)\big) \ in\ \Omega_T, PDE \\ \qquad +\gamma e^{-\sigma z} u(t) \\ \hat{w}_z(0,t) = -qw(0,t) + p_{10}[w(0,t) - \hat{w}(0,t)] \ \ on\ \Gamma_1, BC1 \\ \hat{w}_z(1,t) = 0 \qquad on\ \Gamma_2, BC2 \\ \hat{w}(z,0) = \hat{w}_0(z) \qquad in\ \Omega,\ \ IC \end{cases} \quad (31)$$

where,

$$p_1(z) = \frac{cI_1\left(\sqrt{cz(2-z)}\right)}{\sqrt{cz(2-z)}} + \frac{cI_2\left(\sqrt{cz(2-z)}\right)}{z(2-z)} \quad (32)$$

$$p_{10} = -\frac{c}{2} \quad (33)$$

Finally, substituting the estimated state in the feedback form of (19), the controller takes the form:

$$u(t) = -\lambda(q+K)\left[\hat{w}(1,t) + q\int_0^1 e^{q(1-y)}\hat{w}(y,t)dy\right] \quad (34)$$

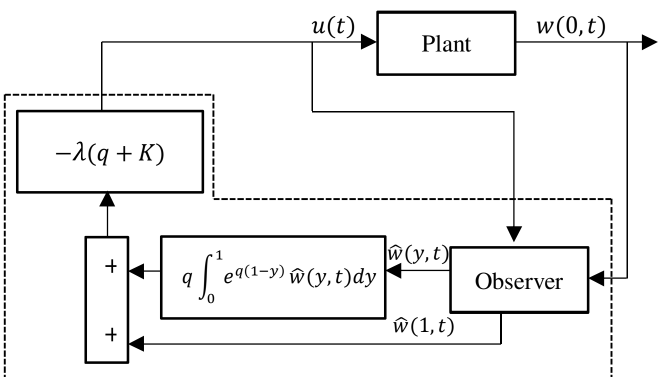

Figure 3. Final Form of the Beer-Lambert Effect Compensator

VI. SIMULATION RESULTS

In this section, we present simulation results to illustrate the performance of the proposed compensator. For comparison purposes, we also consider a common control design method of reducing the Distributed Parameter System (DPS) of PDE (6) in to a Lumped-Parameter System (LPS) using modal analysis (the approach is classical and can be referred from ([1] & [19]). Using a linear state space model generated from the modal analysis, an output feedback controller is designed, including integrator augmentation in order to minimize the steady state error. The design steps are also classical and are detailed in [20]. We applied pole placement on a reduced model that retains the first four modes. Only the unstable first mode is moved to the stable region and the rest of the stable modes are kept in their original locations. A full order LPS observer is also designed with same boundary measurement as the proposed DPS compensator. The observer poles are selected to be an order of magnitude faster than the controller poles.

For the simulations, we consider a film depth of unit length and the non-dimensional PDE (6) as the plant with parameters: $\sigma = 3.8,\ q = 0.166,\ \lambda = 3.84,\ \gamma = 1.$ To illustrate the effectiveness of the proposed compensator, an unstable open-loop plant dynamics is considered by taking these parameters. Since the evolution of temperature with the proposed and lumped parameter compensator are very similar, only the response of proposed compensator is shown in Figure 4. In both cases, the closed-loop system is tuned to stabilize to zero (non-dimensionalized) temperature in about 4 non-dimensional time units. However, as Figure 5 shows, for this comparable performance, the lumped parameter compensator requires a bit higher peak and total control energy input compared to the proposed compensator. While combined control and observation spillover effects that arise from using the reduced model could be potentially destabilizing in general cases, here the overall dissipative nature of the diffusion process prevails [21]. If observation spillover could be ignored (which doesn't hold for the current sensor set up), it was shown in [21] that the effects of control spillover alone are predictable and can be accounted for during tuning of the lumped parameter controller (essentially by including enough modes in the reduced model). The proposed DPS compensator doesn't involve any model reduction and therefore doesn't suffer from control/observation spillover issues.

Another attractive feature of the proposed compensator structure is the ability to further tune its performance using the parameter K that arises from the choice of stable target system. For example, the system is stabilized in less than 2 times units by increasing the value of K from 1 to 1.82. The control effort for both cases is shown in the same plot in Figure 5.

Finally, the performance of the observer is illustrated in Figure 6 where we plotted the evolution of the spatial 2-norm of the observation error on a faster time scale with observer parameter $c = 10$.

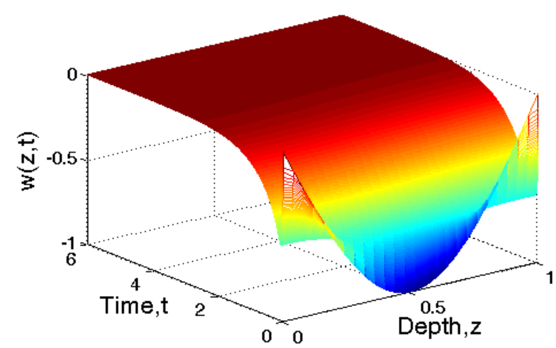

Figure 4: Temperature Evolution with the Proposed Compensator (K=1)

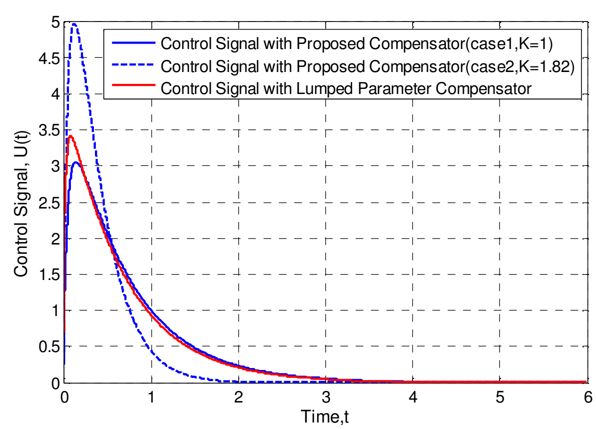

Figure 5: Control Signals

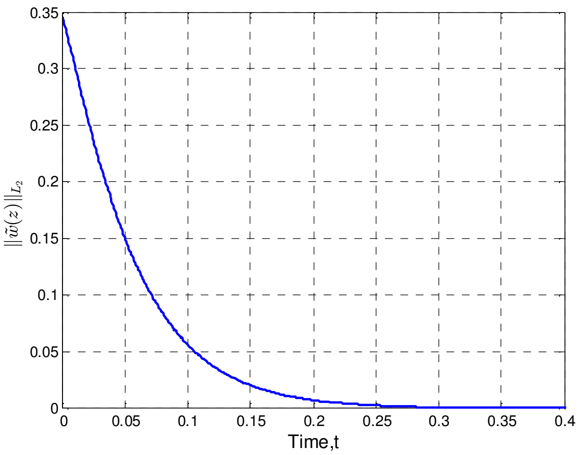

Figure 6: Exponential Convergence of the Observer Error

VII. Conclusions

This paper presented one approach to extend the distributed parameter control design method of backstepping to a class of process control problems involving in-domain attenuation of process inputs. Considering the so-called the Beer-Lambert effect that arises in radiative curing and drying processes as an example, the paper outlined distinct steps that lead to an effective compensation of the in-domain attenuation of input. The first step was to develop a transformation map to change the original in-domain-input PDE to an equivalent boundary control problem. This was done by drawing on the weak formulation of the relevant PDEs and selecting a suitable test function that satisfies necessary regularity conditions. On the transformed system, backstepping boundary control and observer design methods were applied to extract the controller and observer that subsequently form the proposed compensator. This compensator retains the infinite dimensional nature of the original problem and as such does not suffer possible compromises from control/observation spillover effects present in lumped parameter compensator designs. In addition, the proposed compensator includes a simple tuning mechanism embedded in the choice of controller and observer stable target systems.

Appendix

The sufficient condition $K \geq \frac{1}{2}$ for exponential stability of target system(14) is estabilished as follows. Choosing a Lyapunov function candidate: V$= \frac{1}{2}\int_0^1 x^2(z,t)dz$, taking the derivative, integrating by parts, using PDE and BCs in (14):

$$\dot{V} = -\left[Kx^2(1,t) + \int_0^1 x_z^2(z,t)dz\right] \quad \text{(A1)}$$

Using the Poincare inequality [2]

$$\int_0^1 x^2(z,t)dz \leq 2\,x^2(1,t) + 4\int_0^1 x_z^2(z,t)dz \quad \text{(A2)}$$

in (A1), we arrive at:

$$\dot{V} \leq -(K - \tfrac{1}{2})x^2(1,t) - \tfrac{1}{4}\int_0^1 x^2(z,t)dz \quad \text{(A3)}$$

For $K \geq \frac{1}{2}$, this reduces to the following, which proves exponential stability.

$$\dot{V} \leq -\tfrac{1}{4}\int_0^1 x^2(z,t)dz \leq -\tfrac{1}{2}V$$

The condition c>0 for target system (24) is proven similarly.

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