Two Level Structural Optimization:

Designing Varying Mesostructures To Form A Macro Gradient

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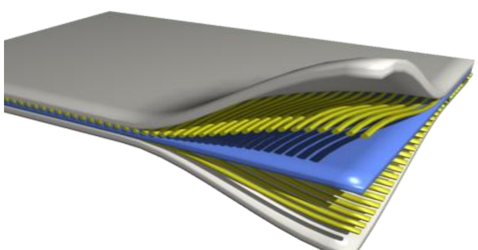
Overview

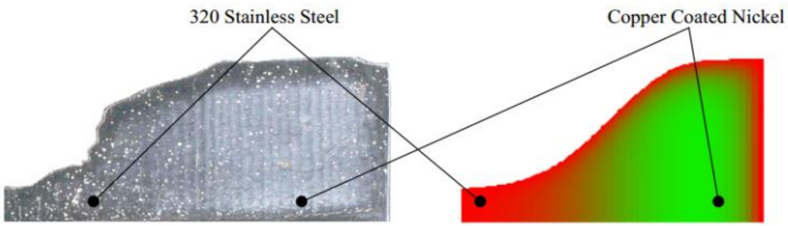
- Why do we care about material gradients?
- Two level optimization
  - Current methods
  - Current problems
- Our Solution
  - Macro Problem
  - Meso Problem
  - Comparison of Meso Optimization techniques

Functional Gradient Material

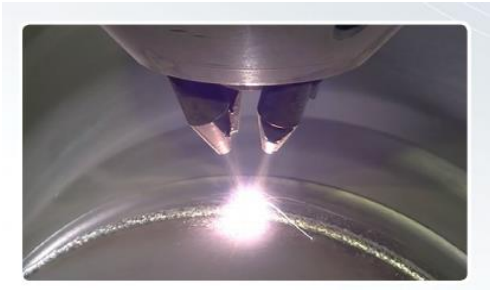
- Two or more materials within a single object.
- Smooth transition between materials.

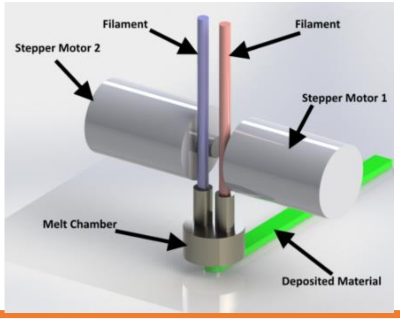
Composite vs. Gradient material





Point by Point Additive manufacturing allows fabrication of gradient designs





Why do we care about material gradients?

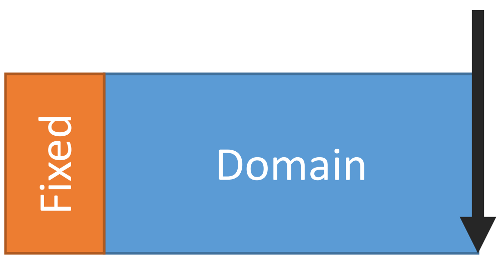
- Using a wide range of material properties within a single design results in ‘more’ optimal solutions.

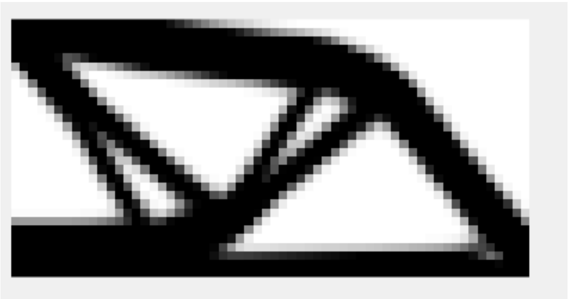
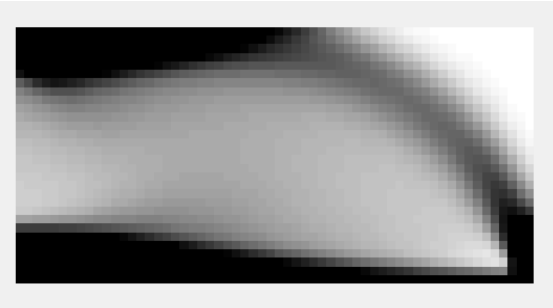
Maximize Stiffness = minimize strain energy

$$\min U^T K U$$

s.t.

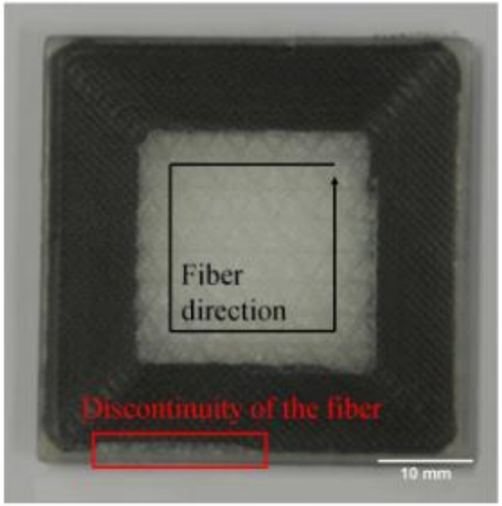
50% of domain is filled



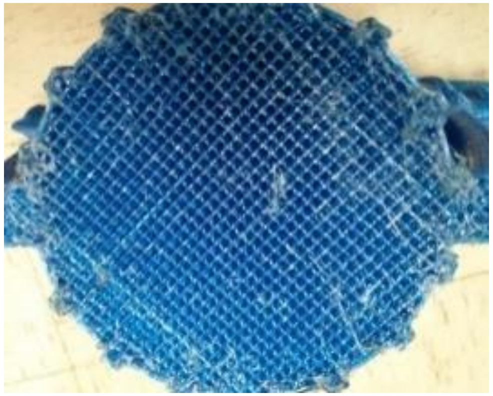
$U^T K U$ Objective	Design
81.3	
67.0	

How do we achieve target properties?

Vary fiber density and direction	**Vary volume composition of two or more materials**	**Vary meso structures**

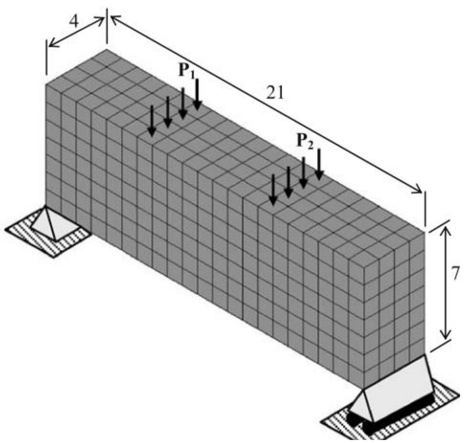


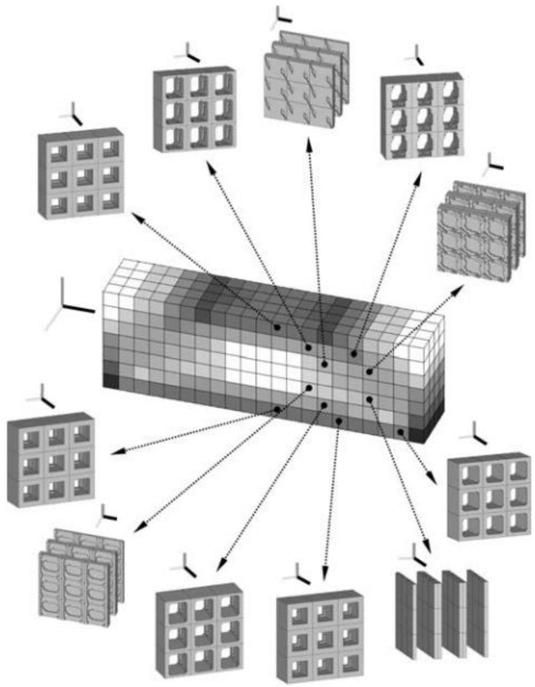
[1]

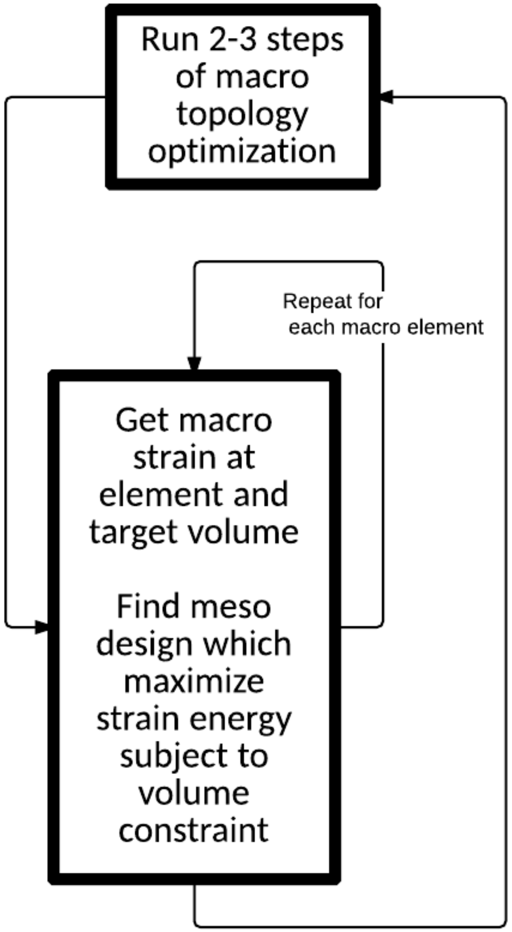


Two Level Optimization (Current Methods)

- Find optimal topology
- Find optimal gradient mesostructure



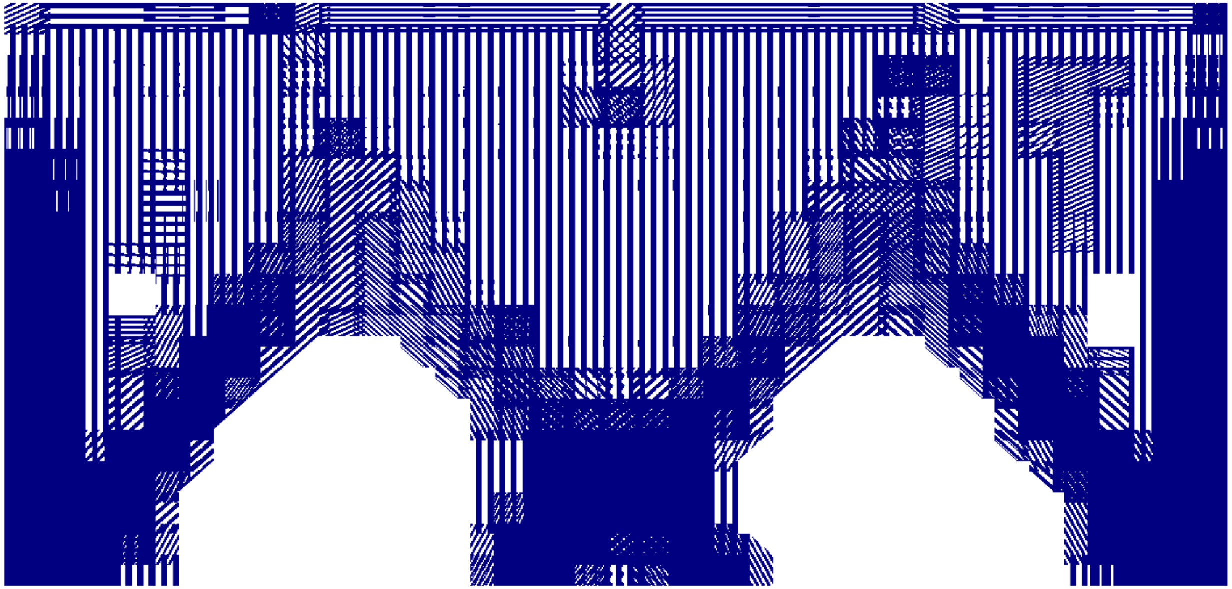




[2]

Problems

- Coordination of macro and meso levels is minimal
- Excessive computational power required
  - 4*21*7=588 (meso problems) each iteration.
- The meso-level problem never converges

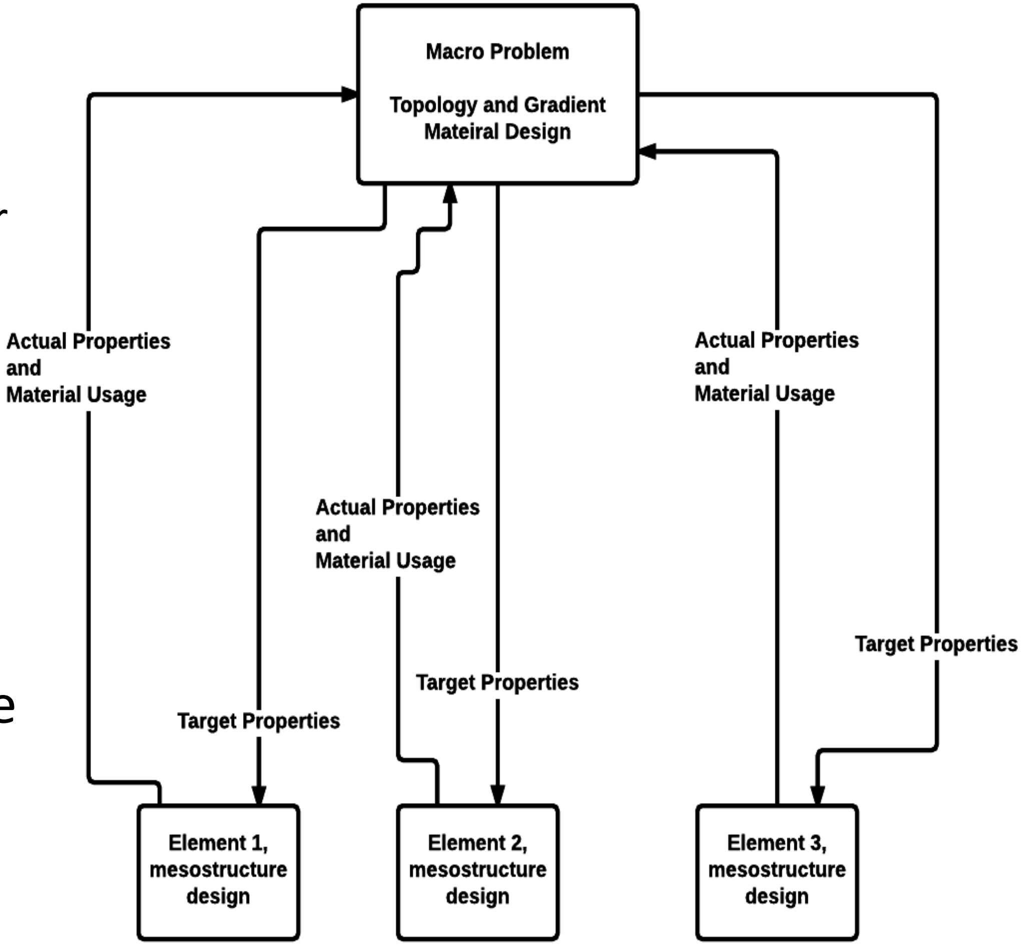


Solution

1. Macro problem designs
   - The overall shape/topology of the structure
   - Selects the target properties for the sub-problems

2. Meso-level subproblems
   - Find a meso design with the homogenized target properties

- Use ATC method of system optimization to coordinate the systems.



Macro Problem

1. Macro problem designs
   - The overall shape/topology of the structure
   - Selects the target properties for the sub-problems

- Design Variables (4 per location)
  - ρ Topology (artificial SIMP density)
  - $E_{xx}$and E_{yy} components of an orthogonal material
  - θ rotation of the orthogonal material.

$$D_{orth} = \begin{bmatrix} \frac{E_{xx}}{1-v^2} & \frac{E_{12}v}{1-v^2} & 0 \\ \frac{E_{12}v}{1-v^2} & \frac{E_{yy}}{1-v^2} & 0 \\ 0 & 0 & \frac{0.5(1-v))E_{12}}{1-v^2} \end{bmatrix}$$

$$E_{12} = \frac{E_{xx} + E_{yy}}{2}$$

$$c = \cos(\theta)$$

$$s = \sin(\theta)$$

$$T(\theta) = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$D_{eff} = \rho^n T(\theta)^{-1} D_{orth} R\, T(\theta) R^{-1}$$

Macro Problem

- Maximize stiffness

$$\min U^T K U$$

- Subject To:
  - Constitutive equations

$$F = KU$$

  - Space occupied by any type of material is limited to 60% of domain

$$\int_{\Omega} \rho d\Omega - V_{total} = 0$$

  - Minimum average elastic modulus at each point, and Maximum upper limit.

$$E_{xx} \leq E_{base}$$
$$E_{yy} \leq E_{base}$$
$$E_{xx} + E_{yy} > E_{min}$$

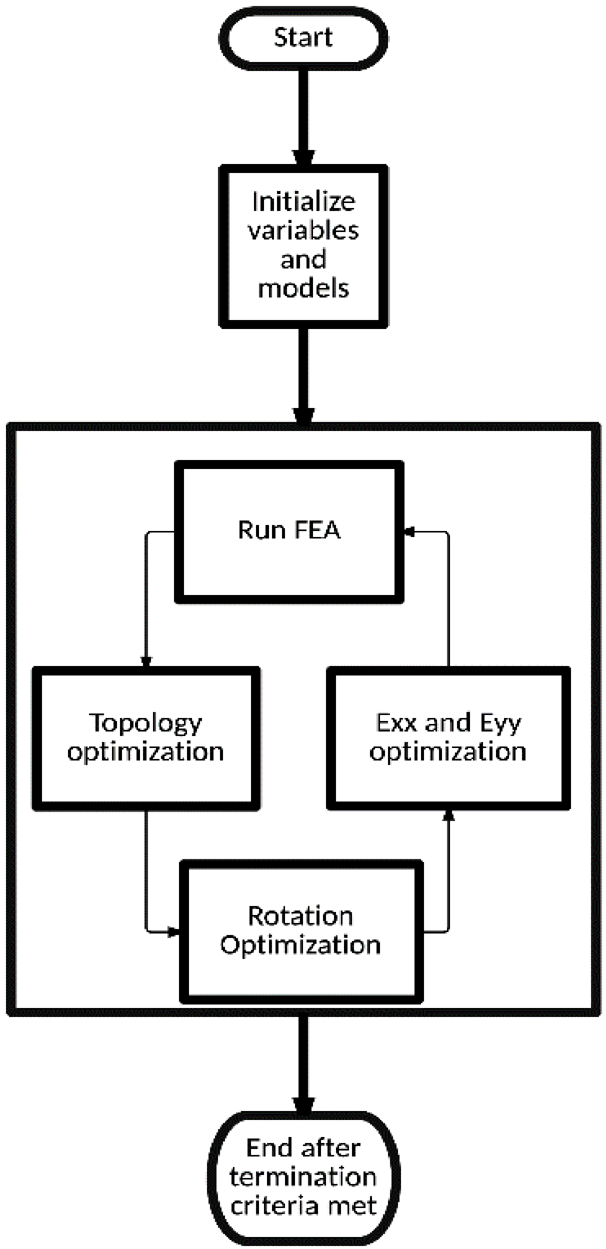
  - Average Elastic Modulus Target in the regions with material

$$\sum_{e=1}^{N} \frac{(E_{xx,e} + E_{yy,e})\rho_e}{2\rho_e} - E_{target} = 0$$

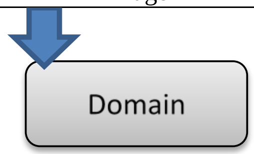
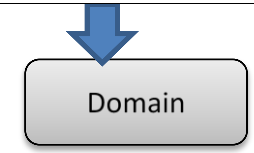
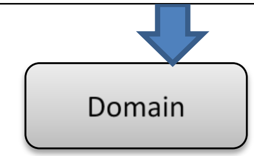
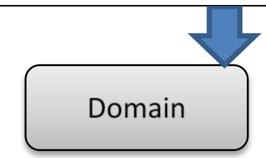
  - Domain of θ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Optimization

- Sequential optimization of each variable type
- Optimal Criteria method for ρ, E_{xx}, and, E_{yy}
- Golden Section algorithm for θ since it is unconstrained



Design Problem

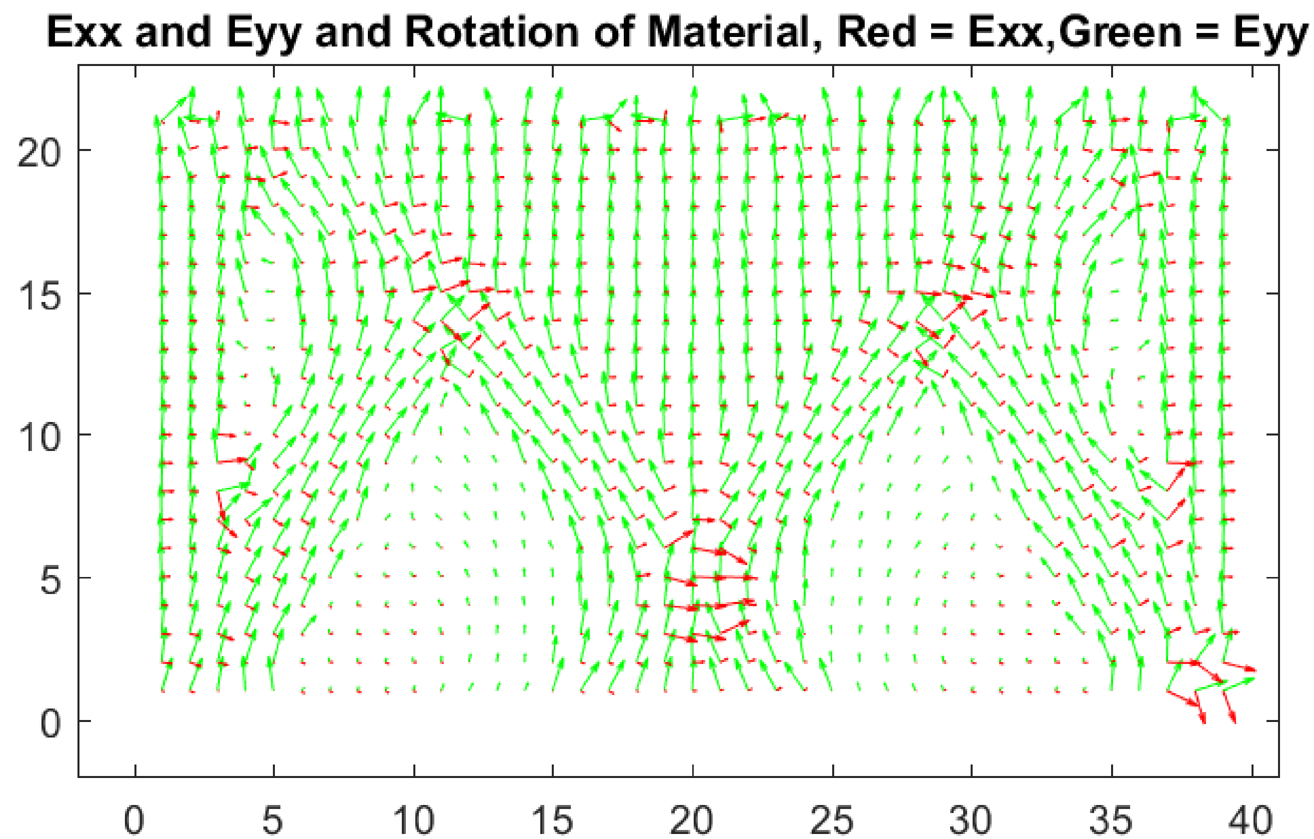
Force	Image
Load top left quarter.	
Load top left-middle quarter.	
Load top right-middle quarter.	
Load top right quarter.	
Load on top layer in the $-x$ direction	
Load on top layer in the $+x$ direction	

- Bridge Design
- 6 loading conditions
  - 500 N
  - Applied evenly
- Maximize stiffness
- 39x21 grid (39*21*4=3276 vars)
- $E_{base} = 100000\ Pa$
- $E_{min} = 25000\ Pa$
- $E_{target} = 62500\ Pa$

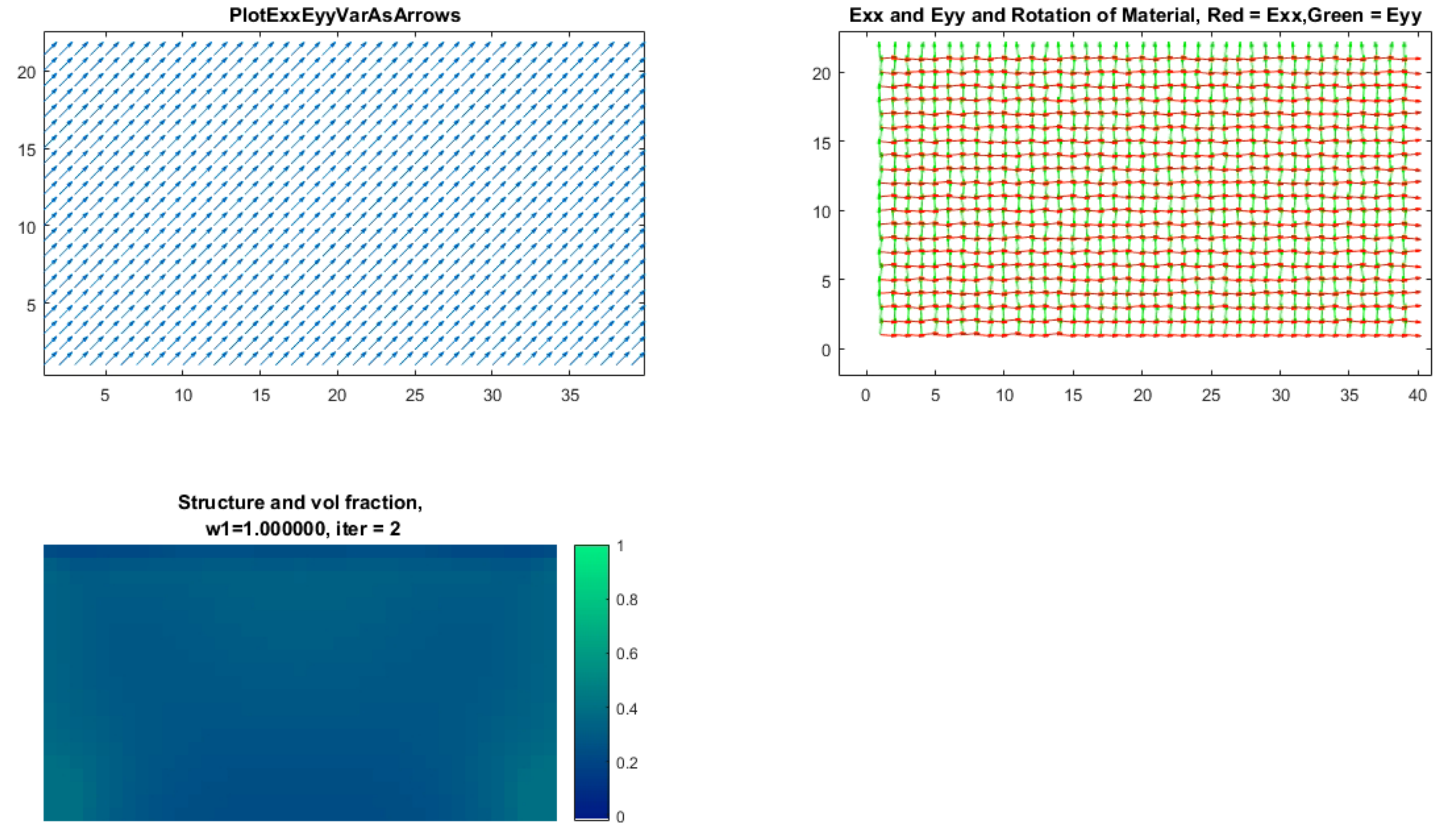
Results

$$v_1 = [E_{xx}\cos(\theta), E_{xx}\sin(\theta)]$$ (red)

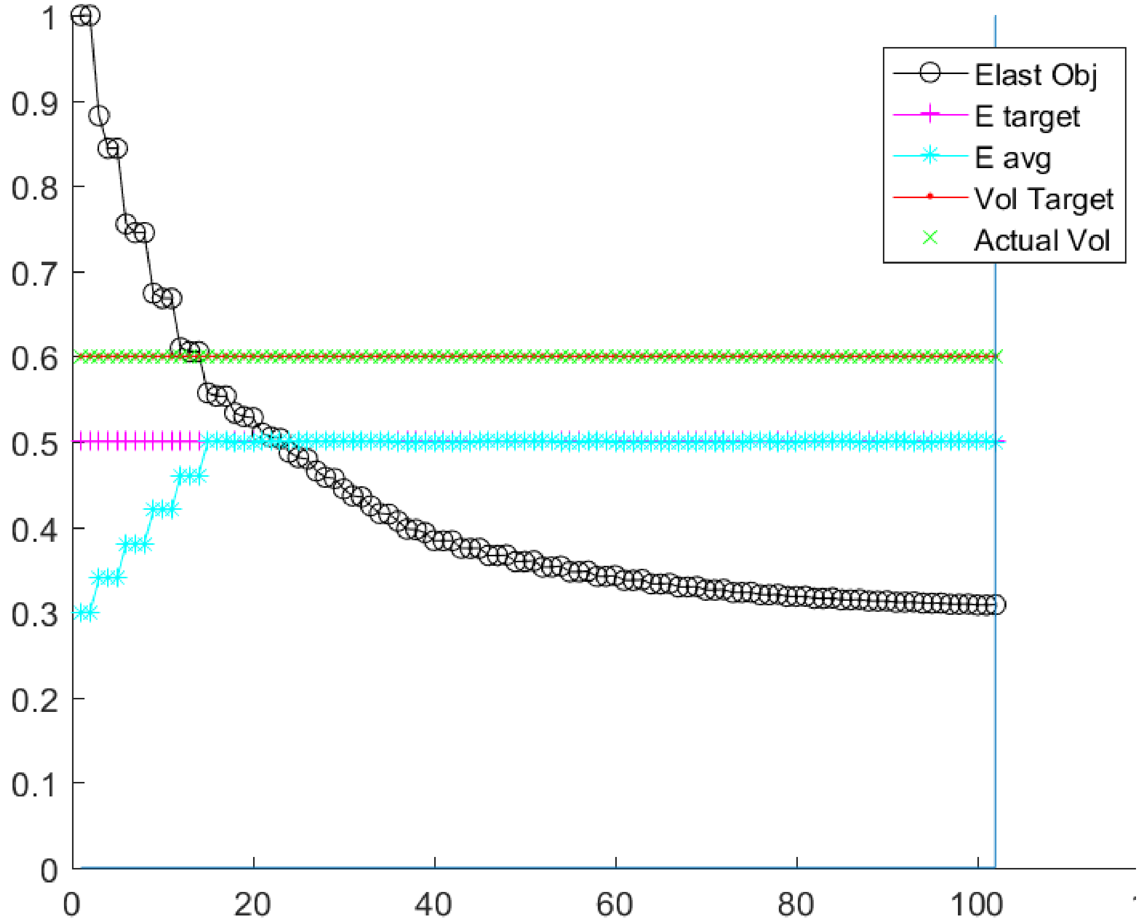
$$v_2 = \left[E_{yy}\cos\left(\theta + \frac{\pi}{2}\right), E_{yy}\sin\left(\theta + \frac{\pi}{2}\right)\right]$$ (green)



Results


PlotExxEyyVarAsArrows
Exx and Eyy and Rotation of Material, Red = Exx,Green = Eyy
Structure and vol fraction,
w1=1.000000, iter = 2

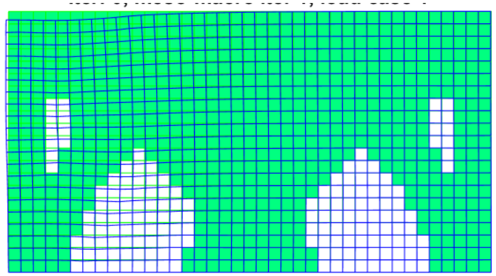
Tracking Optimization Parameters

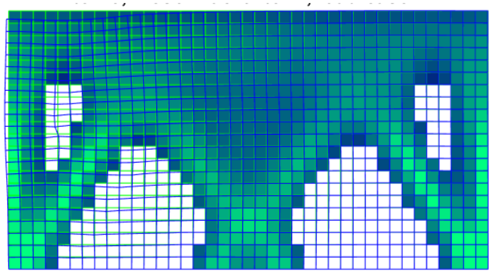


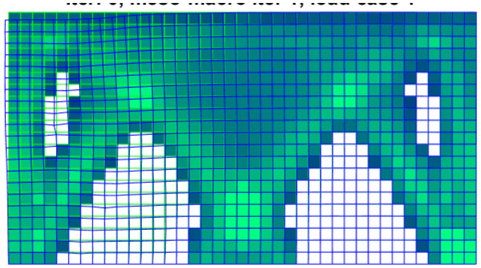
Objective is decreasing

Constraints met

Compare







	Final Objective value	Average Elastic modulus of occupied regions

29% improvement from just topology optimization!

Meso Design

- At each macro element/region, find a meso structure that has the same homogenized properties as the corresponding macro element
- Minimize material usage

$$D_{eff,target} = \rho^n T(\theta)^{-1} D_{orth} R\, T(\theta) R^{-1}$$

- 60% $*$ 39*21 = 491 meso design problems.

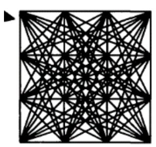
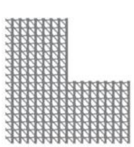
Comparison of Meso Design Methods

- Target a specific D constitutive matrix
- Volume usage
  - Minimize when an objective
  - Target when constraint
- Most of the literature is limited to single property optimization

$$\max E_{11} + E_{12} + E_{21} + E_{22}$$
$$\max E_{33}$$

s.t.

$$\frac{1}{V}\int_{\Omega} \rho_e - \rho_{target} = 0$$

	Maximize Strain Energy.[2]	Target material properties with ground structure [3]	BESO inverse topology optimization [4]
Objective (Standardized)	Maximize the strain energy $\max \epsilon^T D_{sub}\epsilon$ ϵ is the macro strain, which is constant. D_{sub} is the homogenized matrix sub-system matrix	Minimize a cost function W $\min \sum_{e=1}^{NE} \gamma_e \rho_e$ $\gamma_e = l_e \left(\frac{l_{pref}}{l_e}\right)^{\eta} \mu_e$ η is not 1. μ is 1 for interior bars, and >1 for exterior bars. ρ is cross section area of the beams	Minimize compliance of the macro structure. Fixed topology. Only changing meso structure's topology. $\min J$ $J = \frac{1}{2}FU$ On the meso level, maximize the strain energy where the displacements/strains are fixed. $\max \epsilon^T D_{sub}\epsilon$
Constraints (Standardized)	$F = KU$ $\frac{1}{V}\int_{\Omega} \rho_e - \rho_{target} = 0$	$F = KU$ $\sum_{e=1}^{NE}(D_{sub,e} - D_{system}) = 0$ D_{system} has 4 the 4 target matrix terms	$F = KU$ $\frac{1}{V}\int_{\Omega} \rho_e - \rho_{target} = 0$
Lagrangian (λ is the multiplier)	$\mathcal{L} = -\epsilon^T D_{sub}\epsilon + \lambda\left(\frac{1}{V}\int_{\Omega} \rho_e - \rho_{target}\right)$	$\mathcal{L} = \sum_{e=1}^{NE} \gamma_e \rho_e + \sum_{i=1}^{4} \lambda_i \left(\sum_{e=1}^{NE} D_{sub,e} - D_{system}\right)$	Not shown. Since the evolutionary technique is used.
Sensitivity for an element density ρ	$\frac{\partial \mathcal{L}}{\partial \rho_e} = -\epsilon_{macro}^T \frac{\partial D_{sub}}{\partial \rho} \epsilon_{macro} + \lambda_1$ SIMP is used. $\rho^{\zeta} D_0 = D_{eq}$ ζ is the SIMP power function exponent	$\frac{\partial \mathcal{L}}{\partial \rho_e} = \rho_e - \sum_{i=1}^{4} \lambda_i \frac{\partial D_{sub,e,i}}{\partial \rho_e}$ $\frac{\partial D_{sub,e,i}}{\partial \rho_e} = \frac{D_{sub,e,i}}{\rho_e}$ 	$\frac{d\mathcal{L}}{d\rho_e} = -\frac{\zeta \rho_e^{\zeta-1}}{2\lvert Y\rvert}\sum_{i=1}^{M} \boldsymbol{U}_i^T \left\{\int_{\Omega} \boldsymbol{B}^T \left[\int_{Y_i} (I-bu)^T D_0 (I-bu) dY\right] \boldsymbol{B} dV\right\} \boldsymbol{U}_i$ $\frac{d\mathcal{L}}{d\rho_e} = \sum_{i=1}^{M} \boldsymbol{\epsilon}^T \frac{\partial D_{sub,e}}{\partial \rho_e} \boldsymbol{\epsilon}$ **B** is the macro derivative of the shape function for FEA **U** is the macro element displacement $\boldsymbol{\epsilon}$ is the effective macro strain $\boldsymbol{\epsilon} = \boldsymbol{BU}$
Optimization method	Optimal Criteria	Optimal Criteria, but the Lagrangian multipliers are updated using Newton-Raphson procedure	Evolutionary.
Results	Works. Pushes toward a 0 or 1 design. Not targeting specific properties Constraints are fully met.	Works well. Constraints met. They say 'all kinds of anisotropic and orthotropic materials …. Can be constructed with the proposed algorithm.'	Meets target volume.
		Ground structure has a 4x4 grid of nodes. The bars can go through each other, so not necessarily a physically realizable design.	 

Methods, we have tried

	Directly Target Material Properties with continuous structure	New Method. Find Pseudo Strain
Objective (Standardized)	Minimize material usage $$\min(\rho^2)$$	Minimize material usage $$\min(\rho)$$ Solve an alternate problem of maximizing stain energy of using a pseudo strain. $$\max e^T D_{sub} e$$ e is the pseudo strain. Solve for the pseudo strain $$\max e^T D_{sys} e$$ s.t. $e(1) + e(2) + e(3) = 1$
Constraints (Standardized)	$$F = KU$$ $$\left(D_{system} - D_{sub,e}\right)^2 = 0$$ D_{system} has 4 the 4 target matrix terms	For the alternate problem $$\frac{1}{V}\int_{\Omega} \rho_e - \rho_{target} = 0$$ ρ_{target} is adjusted until $D_{sys} = D_{sub}$
Lagrangian (λ is the multiplier)	$\mathcal{L} = \sum_{e=1}^{NE} \rho^2 + \sum_{i=1}^{4} \left(\lambda_i\left(D_{system} - D_{sub,e}\right) +\right.$	$$\mathcal{L} = -e^T D_{sub} e + \lambda\left(\frac{1}{V}\int_{\Omega} \rho_e - \rho_{target}\right)$$
Sensitivity for an element density ρ	$\frac{\partial\mathcal{L}}{\partial\rho_e} = 2\rho + \sum_{i=1}^{4}\left(\lambda_i\left(-\frac{\partial D_{sub,e}}{\partial\eta}\right) + p\left(D_{system} -\right.\right.$	$$\frac{\partial\mathcal{L}}{\partial\rho_e} = -e^T \frac{\partial D_{sub}}{\partial\rho} e^T + \lambda_1$$
Optimization method	Augmented Lagrangian multiplier	Optimal Criterial
Results	Constraint is partially met. Design is not pushed toward 0 or 1 like expected.	The pseudo strain, e, does not seem correct. Otherwise, it will probably work.

What next?

- Still working on the meso-design problem. Target specific properties and minimize volume
- Coordinate the maco and meso levels until the problem converges

Sources

- [1] Klift, F. Van Der, Koga, Y., Todoroki, A., Ueda, M., and Hirano, Y., 2016, “3D Printing of Continuous Carbon Fibre Reinforced Thermo-Plastic (CFRTP) Tensile Test Specimens,” (January), pp. 18–27.
- [2] Coelho, P. G., Fernandes, P. R., Guedes, J. M., and Rodrigues, H. C., 2008, “A hierarchical model for concurrent material and topology optimisation of three-dimensional structures,” Struct. Multidiscip. Optim., **35**(2), pp. 107–115.
- [3] SIGMUND, O., 1995, “Tailoring Materials With Prescribed Elastic Properties,” Mech. Mater., **20**, pp. 351–368.
- [4] Zuo, Z. H., Huang, X., Rong, J. H., and Xie, Y. M., 2013, “Multi-scale design of composite materials and structures for maximum natural frequencies,” Mater. Des., **51**, pp. 1023–1034.

Questions?

Homogenization

- Given a single repeating unit cell, find the equivalent properties of the matrix of the cells.
- Chris Czech’s work shows that 8x8 grid of repeating unit cells are needed.

- The effective properties are found by applying 3 unit strains.

$$\epsilon^1 = (1,0,0)$$
$$\epsilon^2 = (0,1,0)$$
$$\epsilon^3 = (0,0,1)$$

Homogenization

- The equivalent force of the strains is calculated at each element.

$$f^{i} = \sum_{e} \int_{V_e} B_e \, C_{eq} \epsilon^{i} dV_e$$

The stiffness of each element is calculated then combined

$$K = \sum_{e=1}^{N} \int_{V_e} B_e^T C_{eq} B_e dV_e$$

The 3 problems are solved for the displacements

$$KX^{i} = f^{i}$$

Homogenization

- Converting the displacements to strain
- Subtract from the unit strains
- Find the equivalent macro (homogenized) properties of the unit cell

$$C^H = \frac{1}{V}\sum_{e=1}^{R}\int_{V_e}(I - B_e X_e)^T C_{eq}\,(I - B_e X_e)\,dV_e$$

Limitations of Macro Optimization

1. Performance constraints instead of volume constraints would be better. (max displacement, max strain energy)
2. Mesh refinement at regions or rapidly changing stress
3. Make the Poisson's ratio a design variable
4. Some combinations of E_{xx} and E_{yy} are not physically realizable.

   If $E_{xx} = E_0$, ie it is max strength, so a totally solid meso structure, then E_{yy} must also be E_0

5. Targeting an average meso density would be better
   - The function relationship between $\rho(E_{xx}, E_{yy})$ is not known.
   - Until it is known, we cannot target a density