
Geometric Path-finding Algorithm in Cluttered 2D Environments

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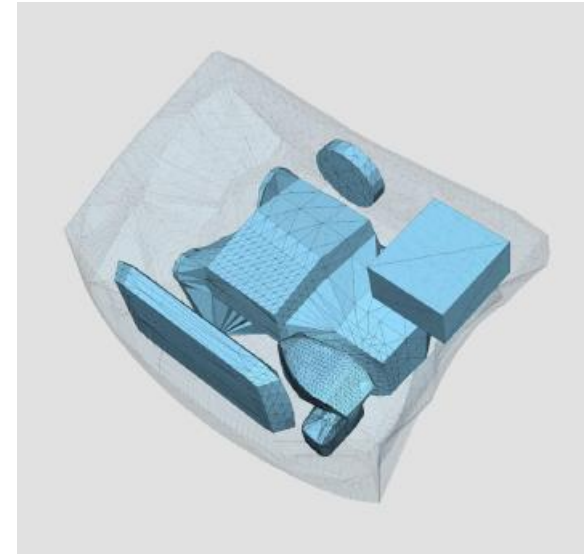
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Outline

- Motivation
- Research Objectives
- Literature Review
- 2D Routing Problem
- Research Approach
- Conclusions
- References

Background

- Packaging Optimization: packaging of components in vehicle under-hood to achieve an optimum center of gravity, accessibility, survivability, dynamic behavior, etc.

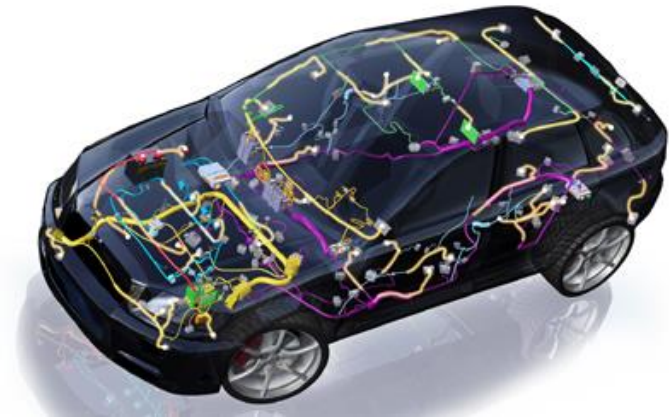


- Connecting the components in an optimal way using cables, wires, and harnesses + placing breakouts



Motivations

- Cable harnesses, the third heaviest and costliest component in a car (Matheus,2015)
- Their layout is currently performed in CAD systems by human designers
- Current process lacks automation and the final solution often times is NOT optimal
- Routing is important in assembly planning, robot motion planning and geographic information



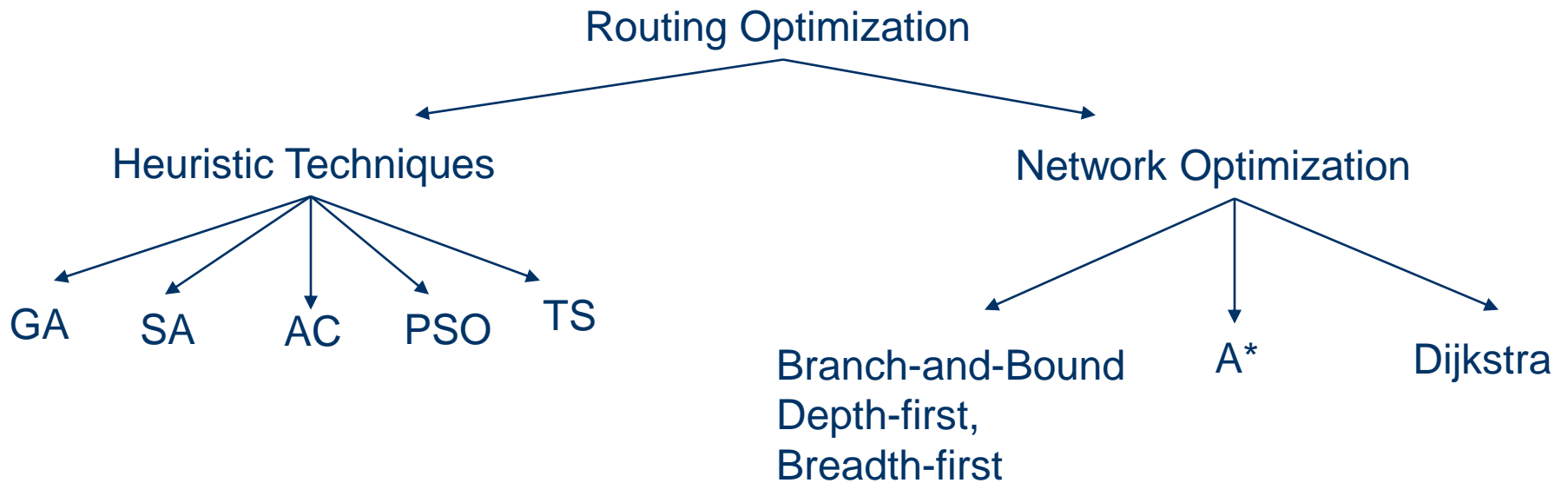
Research Objectives

- Automate the routing process of wires, hoses, and cables (one dimensional components) in electromechanical systems → ultimate objective
- Avoid interference with other components of the environment
- Minimize the total weight of the harness (length) → goal
- Improve the efficiency of the optimizer through the appropriate choice of a graphical representation for the workspace and the free space

Path Planning Literature Review

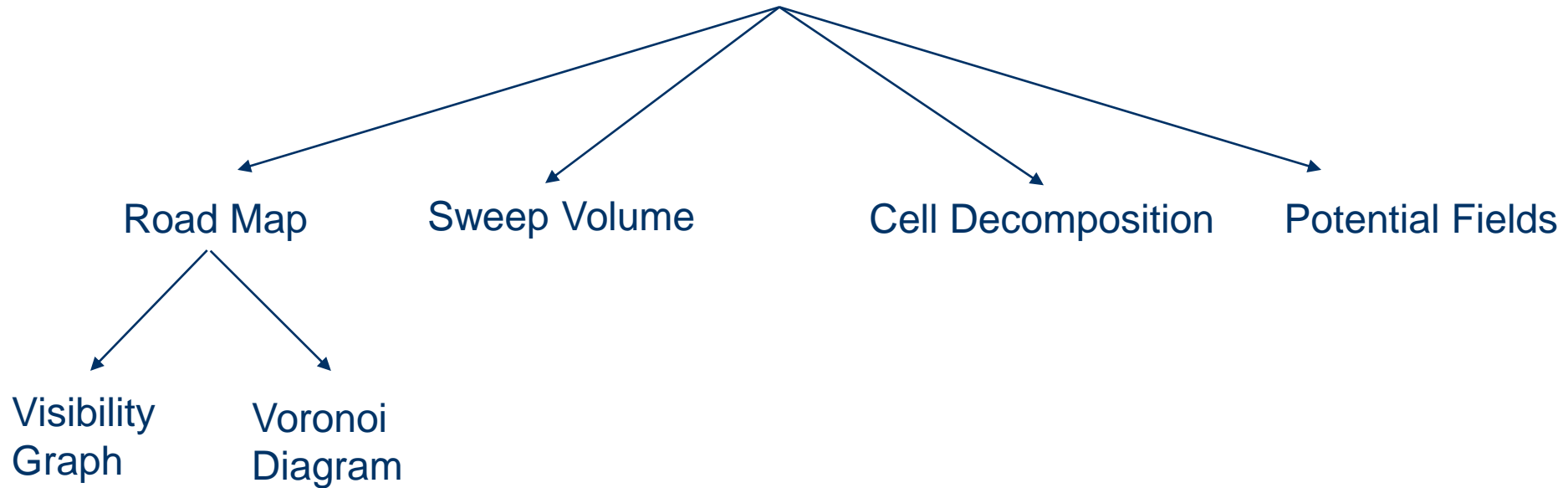
- An automatic pipe router using approximate cell decomposition and A* search algorithm is described in [1]
- 3D pipe routing problem is solved in [3], using convex hulls of barriers and visibility graphs to find candidate segments
- Chen and Sandurkar [4] solve 3D pipe routing using tessellations of obstacles and Genetic Algorithms
- Conru [6] uses GA to find near-optimal solutions to the 3D cable harness routing problem with collision avoidance constraints using cell decomposition
- Automotive wire routing and sizing for weight minimization is addressed in [7] using the Minimal Steiner Tree algorithm

Summary



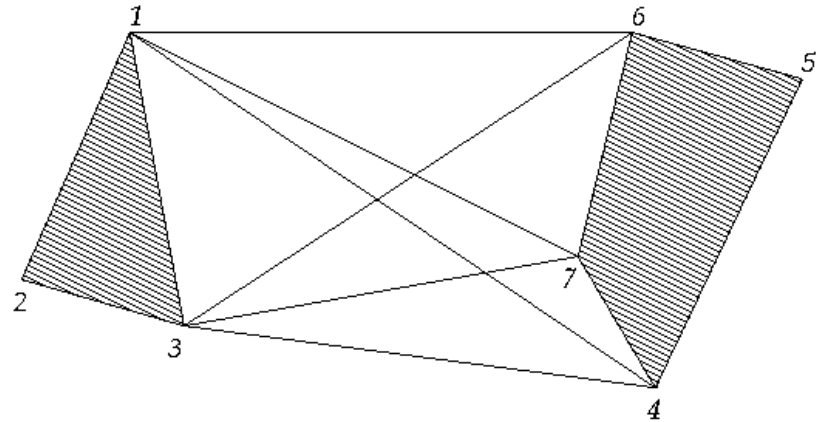
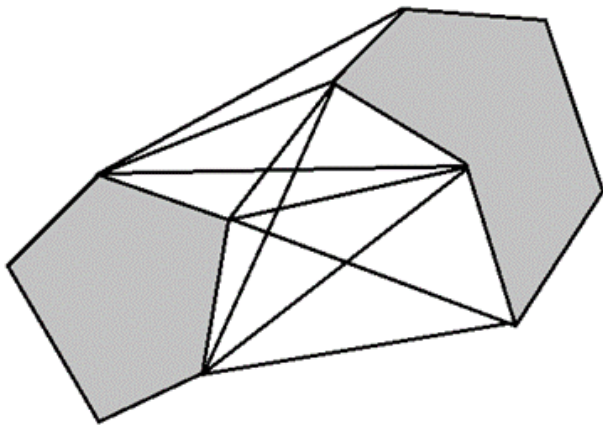
Summary

Collision Avoidance + Free Space Graph Generation



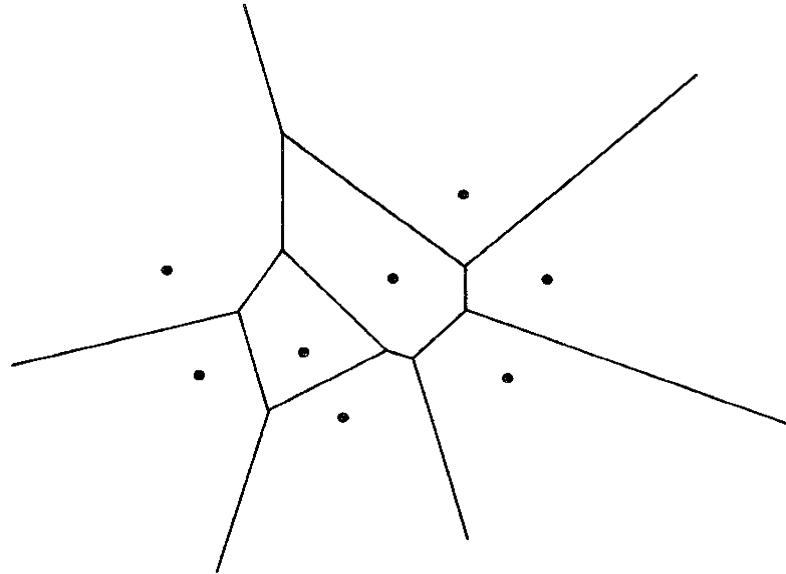
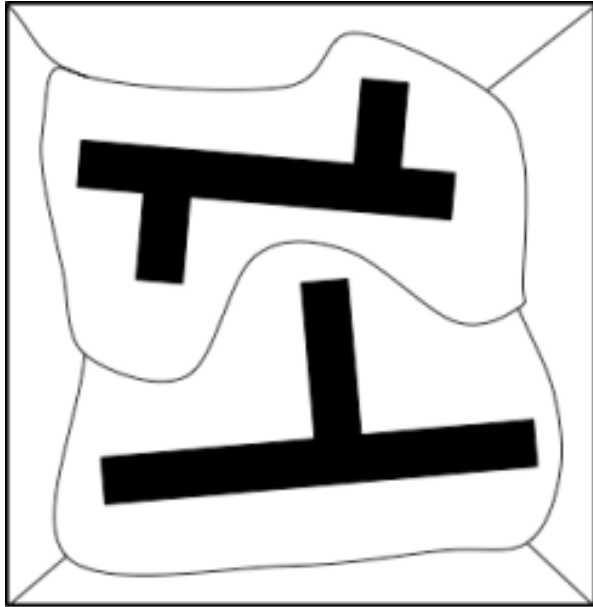
Visibility Graph

- A way to generate the collision free graph
- A Visibility graph: a finite set of nodes and edges. The nodes can “see” one another in the sense that the common edge does not meet the interior of any obstacles



Voronoi Diagram

- Voronoi diagram of n sites partitions the workspace into n convex regions such that any point on an edge is equidistant from exactly two sites, hence generating max-clearance path

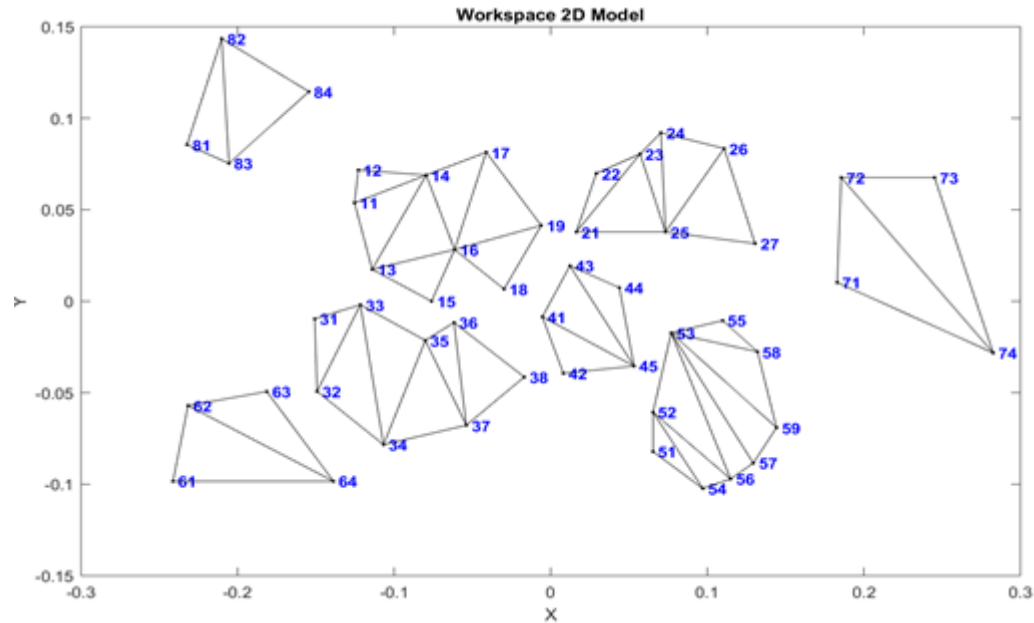


Issues with the previous work

- Visibility graph generates the graph of the visible nodes from a vertex through extensive search of the entire workspace
- Voronoi does not necessarily result in the shortest path
- Roadmaps that work well with 2D routing problems are not fast enough since they explore the entire workspace → memory and time issues
- Non convex shapes are not well-addressed using the previous techniques.

Geometric representation

- Using tessellations, STL data of the workspace
- Efficient handling of convex as well as non convex shapes

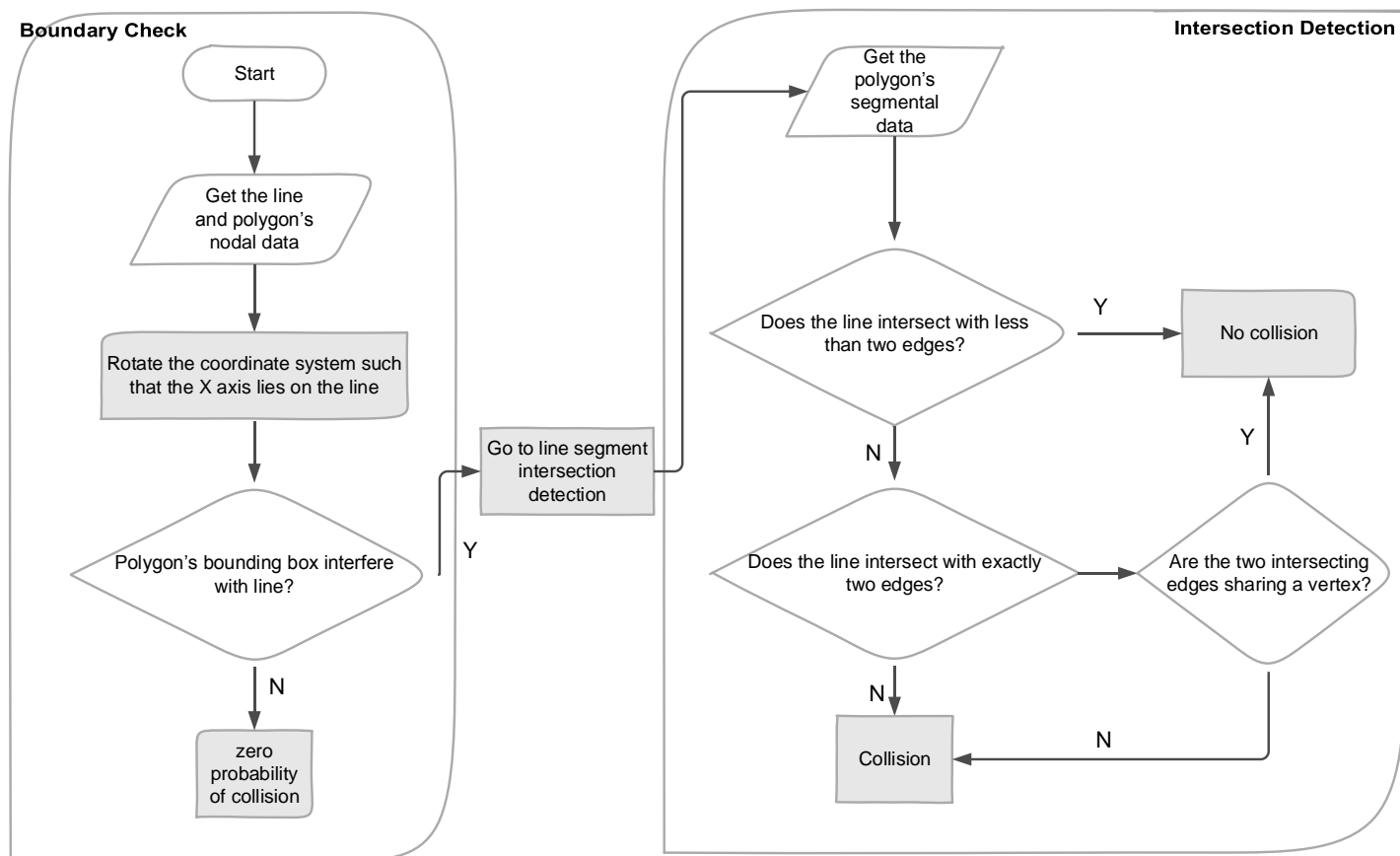


Node numbering:

- first digit=object number
- Other digits=vertex number

Intersection Detection

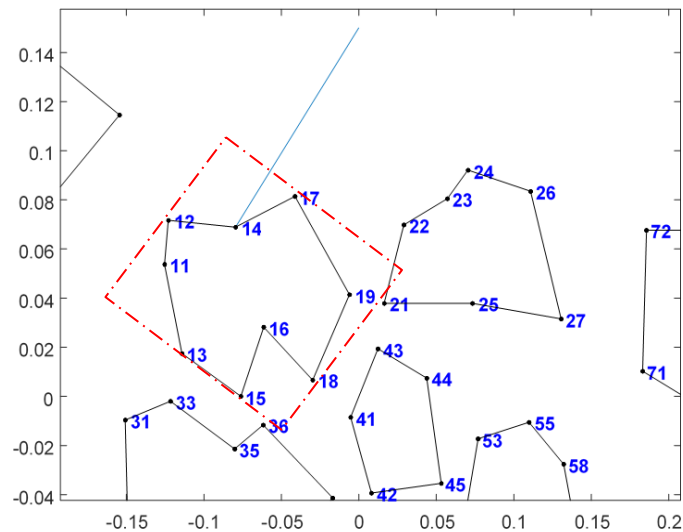
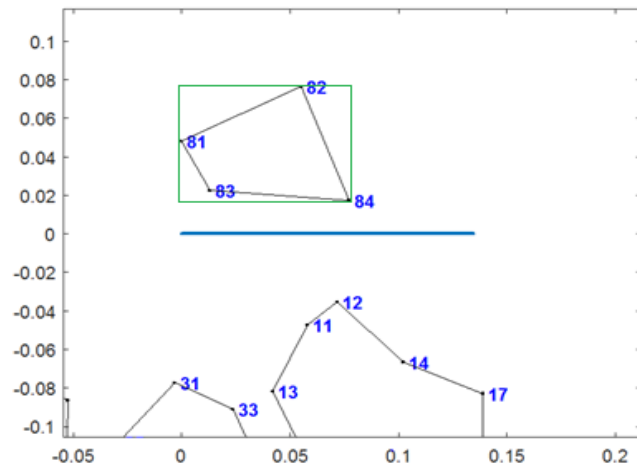
- Bi-level Intersection detector



Intersection Detection

- Bi-level intersection detector:
 1. Filtering out the out-of-bound obstacles
 2. Checking the intersection between line segments for in-bound obstacles

Out-of-bound Example



Line segment intersection detection

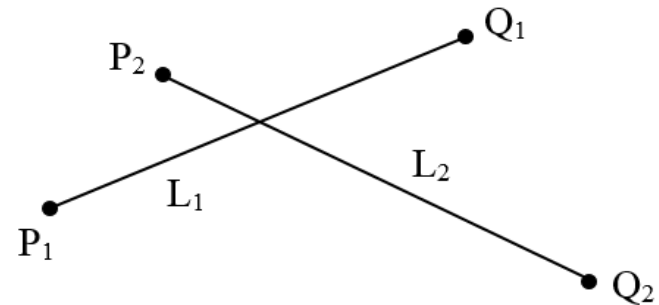
- Determining intersection point of the two line segments

$$L_2 = (1 - \mu)P_2 + \mu Q_2$$

$$L_1 = (1 - \lambda)P_1 + \lambda Q_1$$

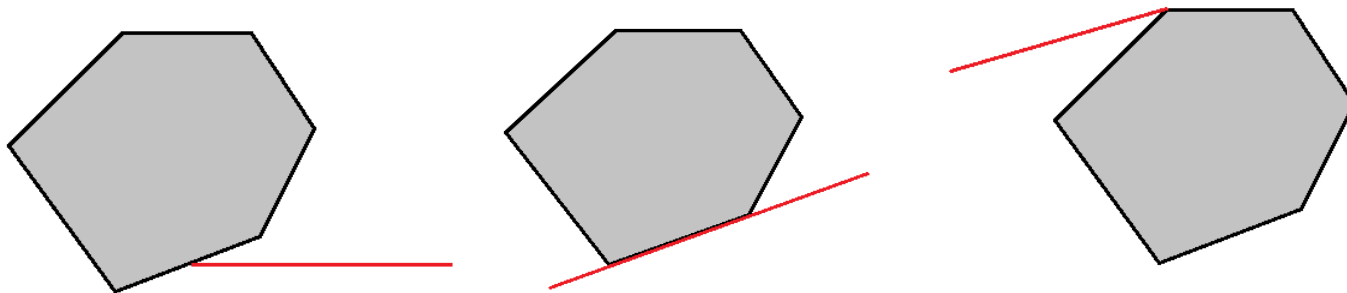
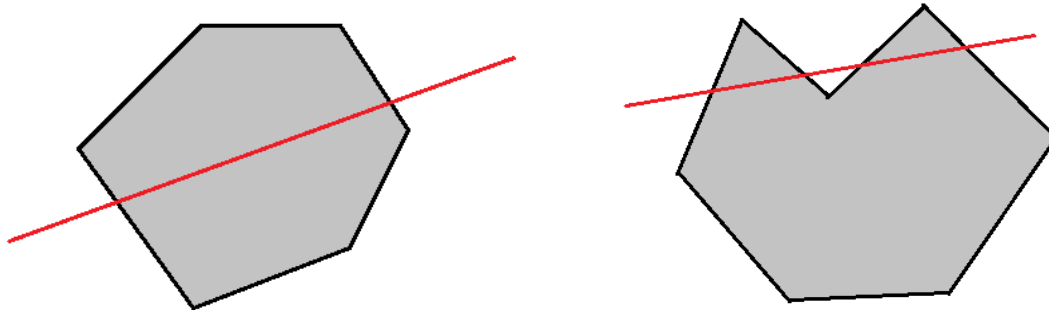
$$(1 - \lambda)X_{P_1} + \lambda X_{Q_1} = (1 - \mu)X_{P_2} + \mu X_{Q_2}$$

$$(1 - \lambda)Y_{P_1} + \lambda Y_{Q_1} = (1 - \mu)Y_{P_2} + \mu Y_{Q_2}$$



- Intersected if $0 \leq \lambda, \mu \leq 1$

Intersection Examples



Finding the free space graph

W is the workspace; $W \subseteq \mathbb{R}^2$

there are n polygonal obstacles in the workspace: P_1, \dots, P_n

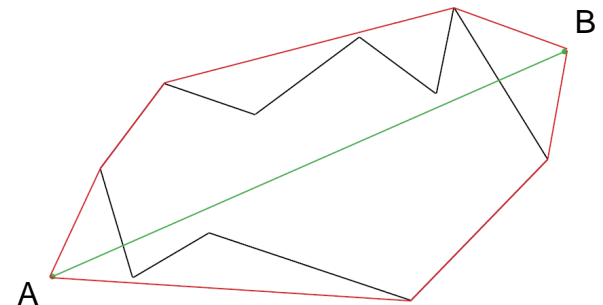
Known: geometry and location of all obstacles+ start and end points

Assume: obstacles are stationary and disjoint

$$C_{free} = W \setminus \bigcup_{i=1}^n P_i$$

Find the graph G such that:

$$G = \{V, E\} \subseteq C_{free}$$



Graph Construction

- Assume there is one polygonal obstacle P in the workspace W : $P \subseteq W \subseteq \mathbb{R}^2$ and the start and end points are denoted by $A, B \in \mathbb{R}^2$, and if:

$$\partial \text{Conv}(A, B) \cap P \subseteq \text{Int}(P) \cup \partial(P)$$

Where:

$\text{Conv}(r_1, \dots, r_n)$: convex hull

$\text{Int}(P)$: interior of set P

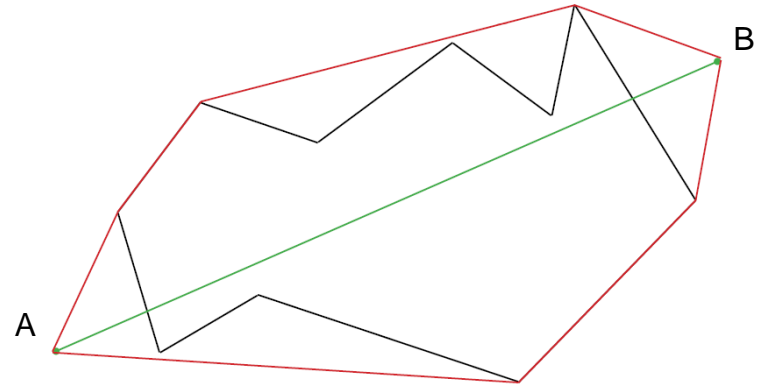
$\partial(P)$: boundary of set P

- There is an intersection between line segment AB and the polygon P . otherwise:

$$\text{Conv}(A, B) \cap P \subseteq \partial(P)$$

Define $\text{Conv}(A, B, P)$ such that:

$$\partial \text{Conv}(A, B, P) \cap P \subseteq \partial(P)$$



Graph Definition

- $G = \{V, E\}$ Graph of the free space
- V : set of all nodes of the free space graph
$$v_i \in V \Leftrightarrow v_i \in \partial \text{Conv}(A, B, P), \quad \exists v_{i,i+1} \text{ or } v_{i-1,i} \in E$$
- E : set of all segments/edges of the free space graph
$$e_{ij} \in E \Leftrightarrow e_{ij} \subseteq \partial \text{Conv}(A, B, P), \quad e_{ij} \cap P \subseteq \partial(P)$$

Optimization Problem

- Given graph $G=\{V,E\}$, find the shortest path between nodes i,j , $i \neq j$;
- Problem formulation:

$$\min \sum_{(i,j) \in G} C_{ij} X_{ij}$$

Where:

C_{ij} : cost, the L2 norm (Euclidean) of arc e_{ij}

$$X_{ij} = \begin{cases} 1 & \text{if } e_{ij} \text{ is in the path} \\ 0 & \text{otherwise} \end{cases}$$

s.t.

$$\sum_{\{j:(i,j) \in G\}} X_{ij} - \sum_{\{i:(i,j) \in G\}} X_{ji} = \begin{cases} 1 & i = 1 \\ 0 & i \neq 1, m \\ -1 & i = m \end{cases}$$

Dijkstra's Algorithm

- Originated from Dynamic Programming (DP); solving an optimization problem by breaking it into multiple sub-problems
- Starting from s , in a given graph, at each step i , the node with the minimum distance from node among all adjacent nodes is added to the path until it reaches t (explain step by step)

Dijkstra

Dijkstra's Algorithm (Sneidovich,2006)

- Initialization:

$j = 1$; $F(1) = 0$; $F(i) = \infty$, $i \in \{2 \dots, n\}$; $U = C = \{1, \dots, n\}$;

j : current node, $F(j)$: the objective value assigned to node j .

U : set of unvisited nodes

- Iteration:

While ($j \neq n$ and $F(j) < \infty$) Do:

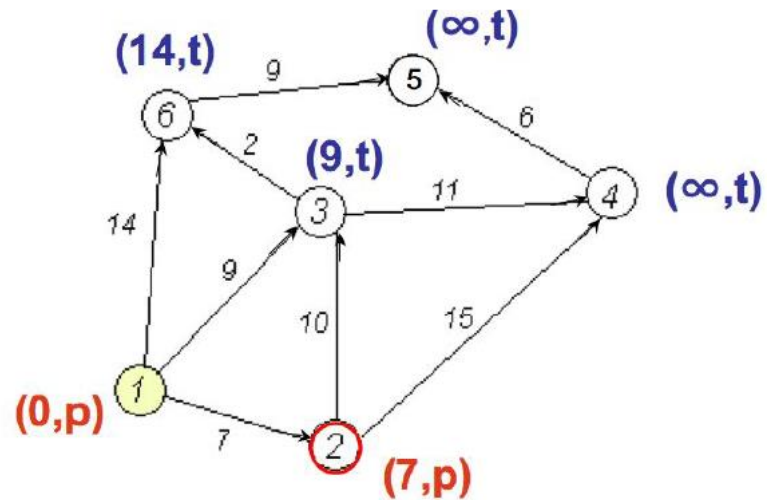
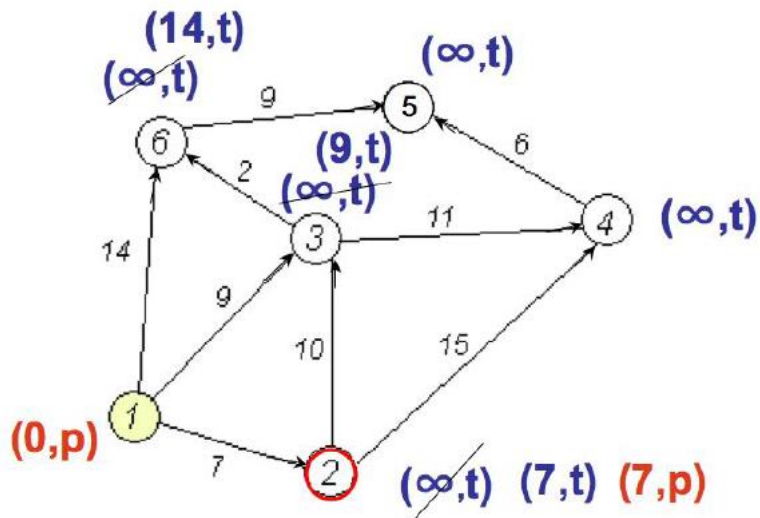
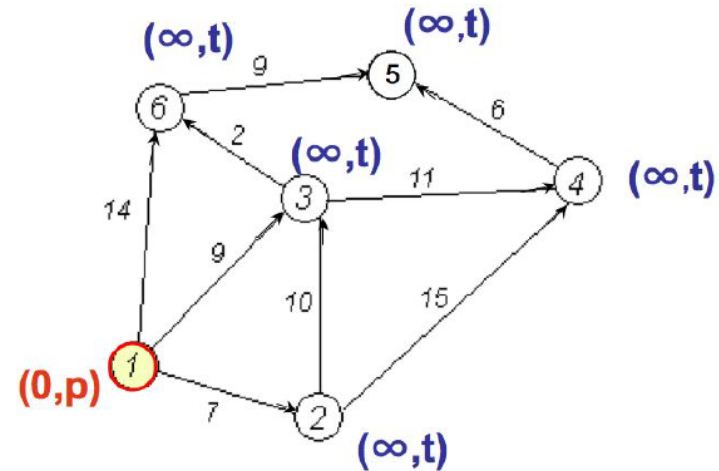
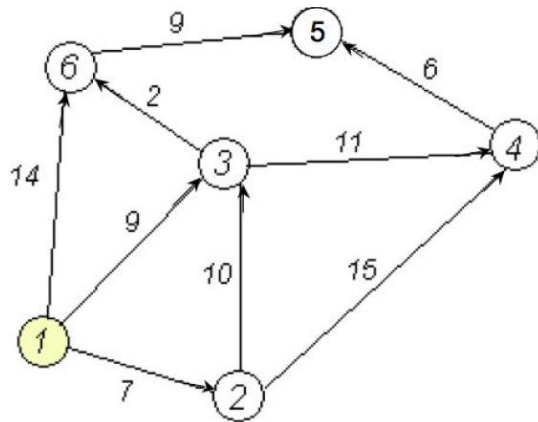
Update U : $U = U \setminus \{j\}$

Update F : $F(i) = \min\{F(i), F(j) + D(j, i)\}$, $i \in A(j) \cap U$

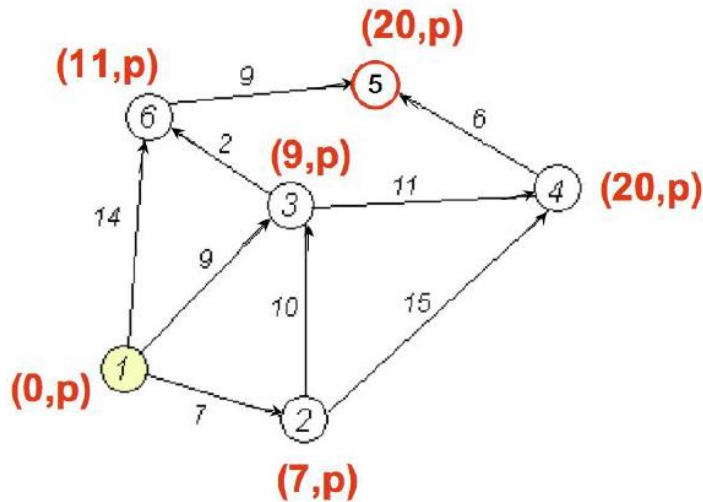
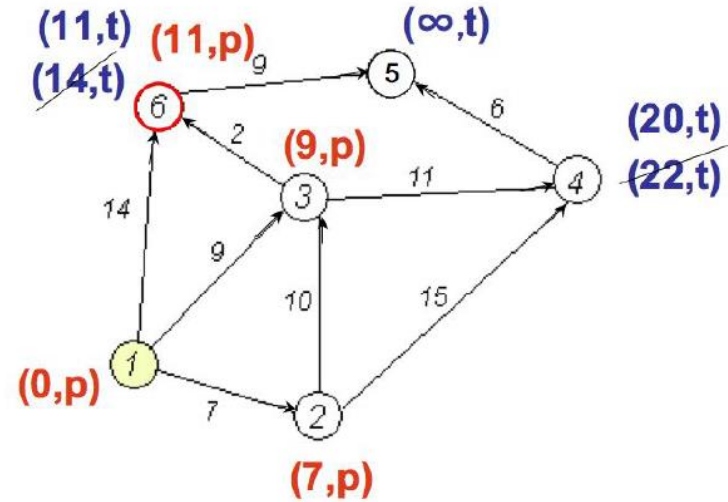
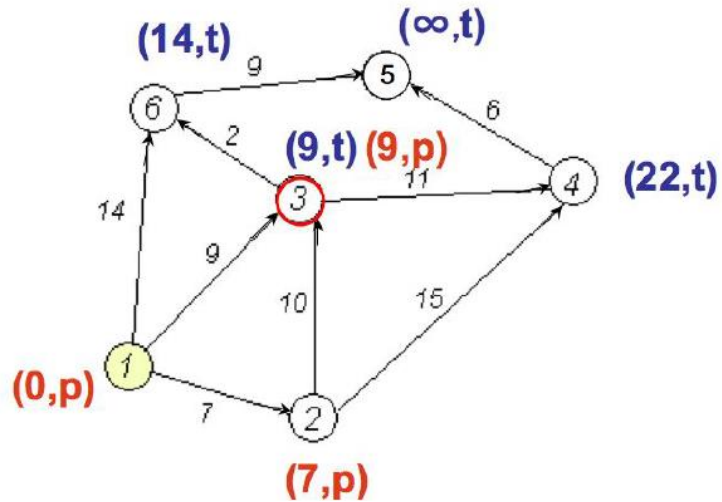
Update j : $j = \operatorname{argmin}\{F(i) : i \in U\}$

Where; $A(j)$ denotes the set of node j 's immediate successors

Dijkstra Example



Example Cont'd

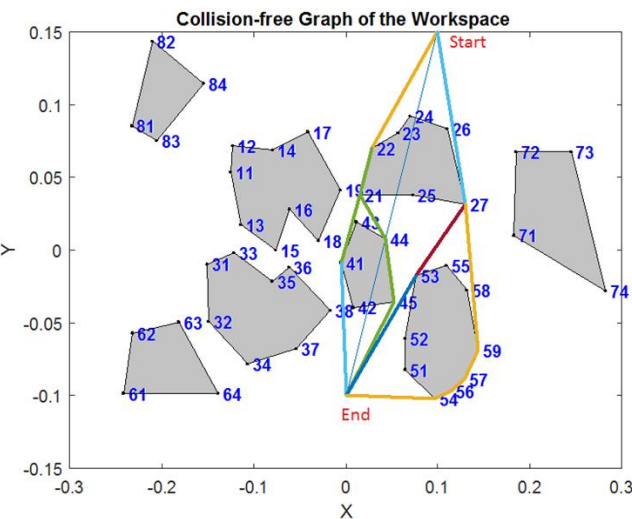
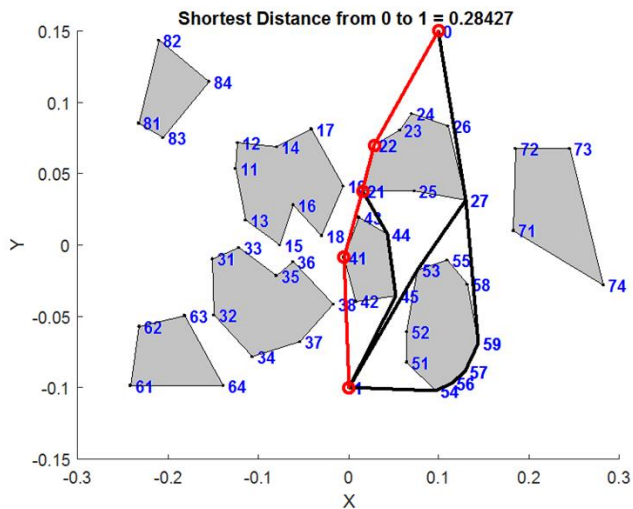
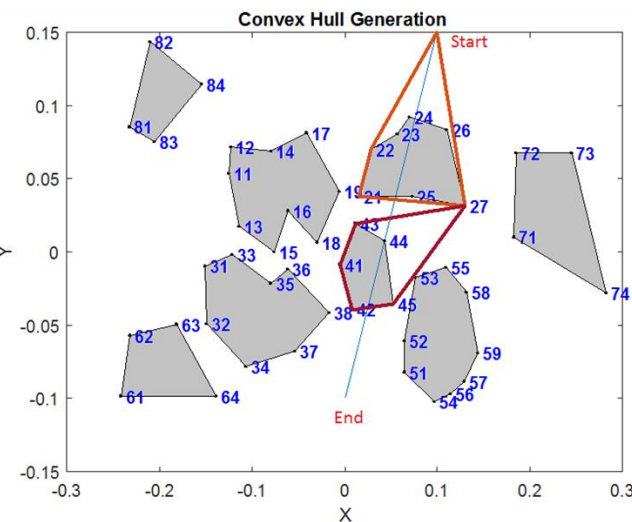
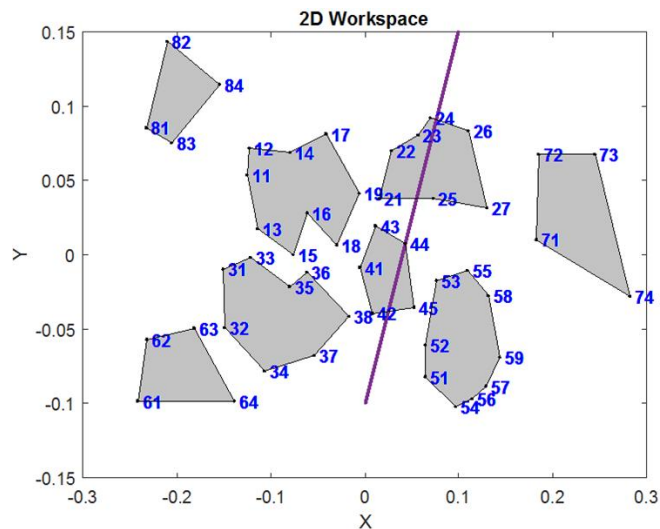


Shortest Path = {1,3,6,5}
Path Length = 20

A* Search Algorithm

- Similar to Dijkstra in finding the shortest path based on DP
- But, it also has a heuristic term which helps favoring vertices that are close to the goal
- Cost: $f(n) = g(n) + h(n)$
- Definition of the heuristic term is challenging.

Results



Results

- The algorithm is tested on different cases with varying number of objects, start and end points and various shaped objects(convex and non convex)
- The algorithm depends on the number of colliding obstacles while being independent of the total number of obstacles
- The algorithm is independent of the shape of the objects

Conclusions

- A two-level collision detection algorithm is developed that checks for intersections
- Instead of generating the entire visibility graph of the workspace, we find a portion of it using convex hulls of the intersecting objects
- A network optimization algorithm, Dijkstra, is implemented to find the global optimal solution of the SP problem (if one exists)

Future Work

- A sensitivity analysis will be deployed to analyze the changes of the path with respect to small changes in the workspace configuration
- The algorithm will be modified as needed and implemented on a 3D environment
- An optimizer will be developed to optimize the path of harnesses for automation of harness assembly process in manufacturing lines

References

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Thank You

Questions ?