Modeling lattice structured materials with micropolar elasticity

Accuracy of the micropolar model

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Outline

1. Background and Motivation
2. Methods
3. Conclusions

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1. **Background and Motivation**
2. Methods
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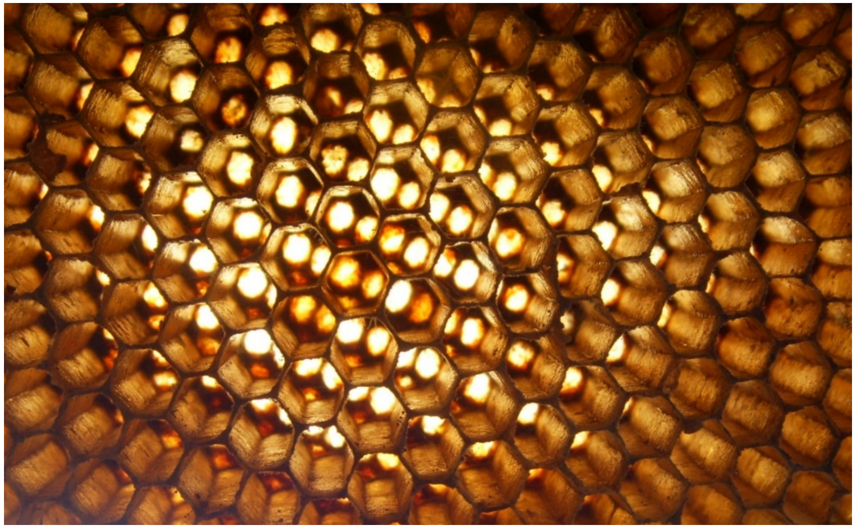
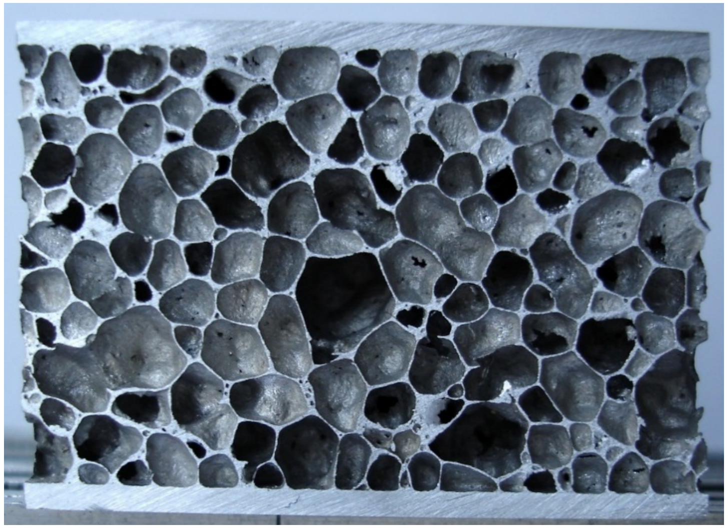
Cellular Materials

- "Cellular materials are made of an interconnected network of solid struts or plates, which form the edges and faces of cells." (Gibson, 1999)
  - Strong and lightweight
  - Or weak and lightweight



Periodic Cellular Material - Honeycomb



Random Cellular Material – Aluminum Foam

Analysis tools

- Alternative Analysis tools
  - Beam lattice model
  - Homogenized elasticity
  - Homogenized micropolar elasticity
- Focus of this work
  - Accuracy of different models
    - as applied to periodic cellular materials with thin walls

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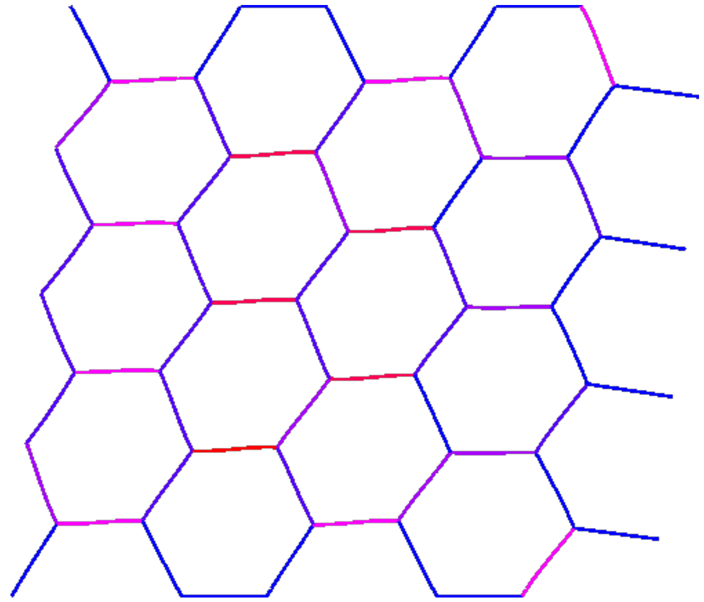
Beam Lattices

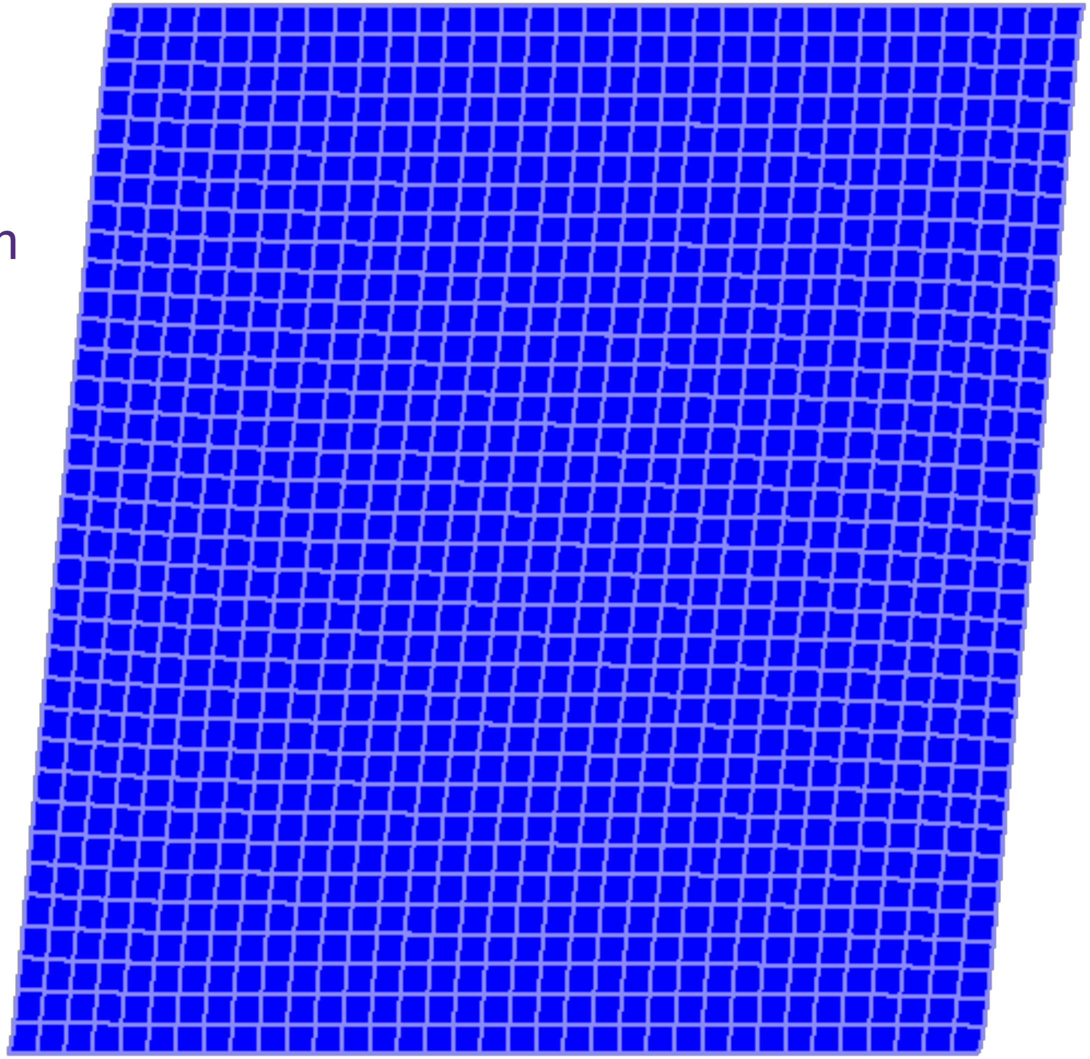


Honeycomb modeled with beam elements

- Beam Lattice Model
- Accepted as correct for purpose of this work

Analysis Methods - Homogenized Elasticity

- Smears discrete lattice behavior over space.
- Young's Modulus and Poisson's ratio calculated from formulas
  - Referred to as *Effective Properties*
- Loses certain details
  - Are these details important?



Honeycomb lattice modeled with a homogenized elasticity model

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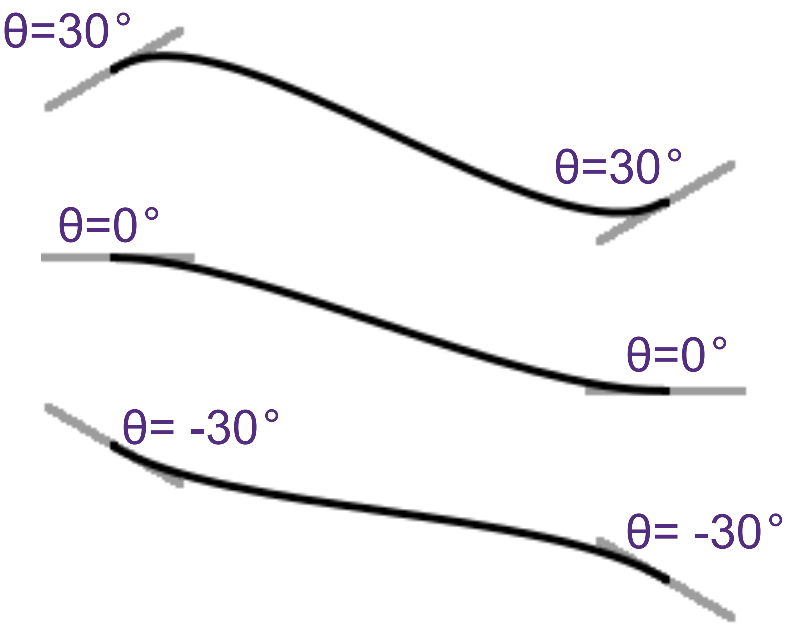
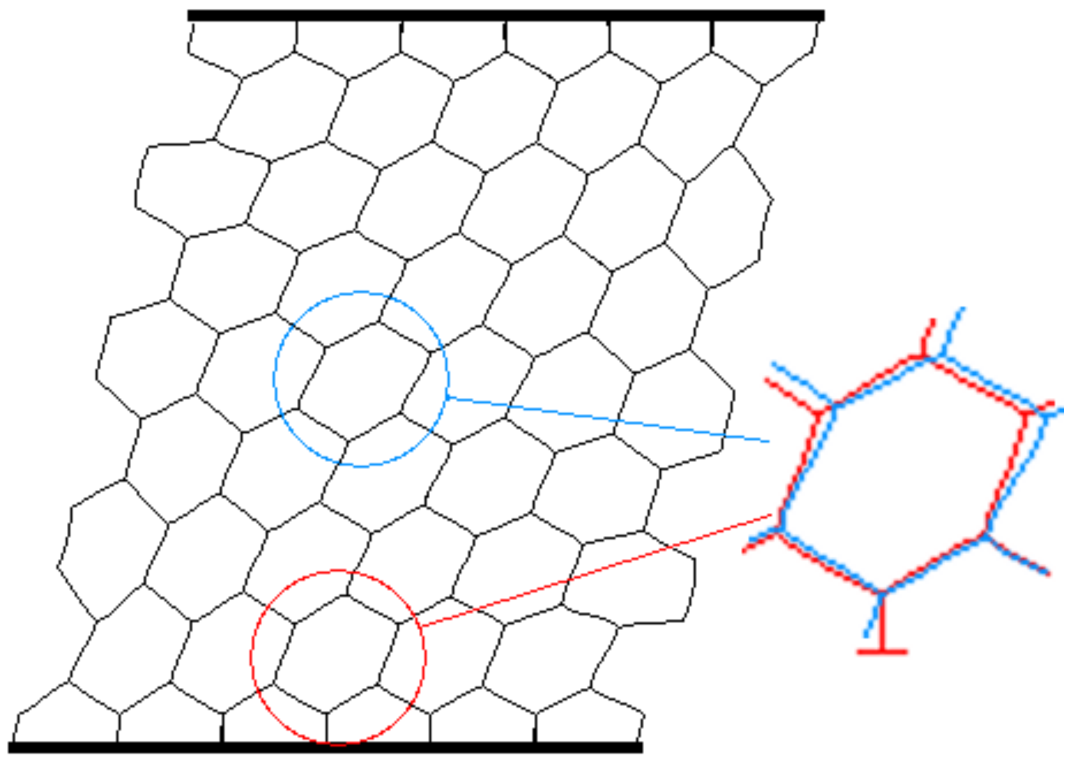
Motivation: Lost Details from Beam Rotation

- Beam theory variables are displacement and beam rotation
  - Beam rotation is lost in transition to homogenized elasticity
  - Fixed boundaries constrain beam rotation (and displacement)
    - Lattice model shows extra stiff behavior near fixed boundary
    - Homogenized model does not show this. (Diebels, 2002)
  - Boundary effects are constant in size

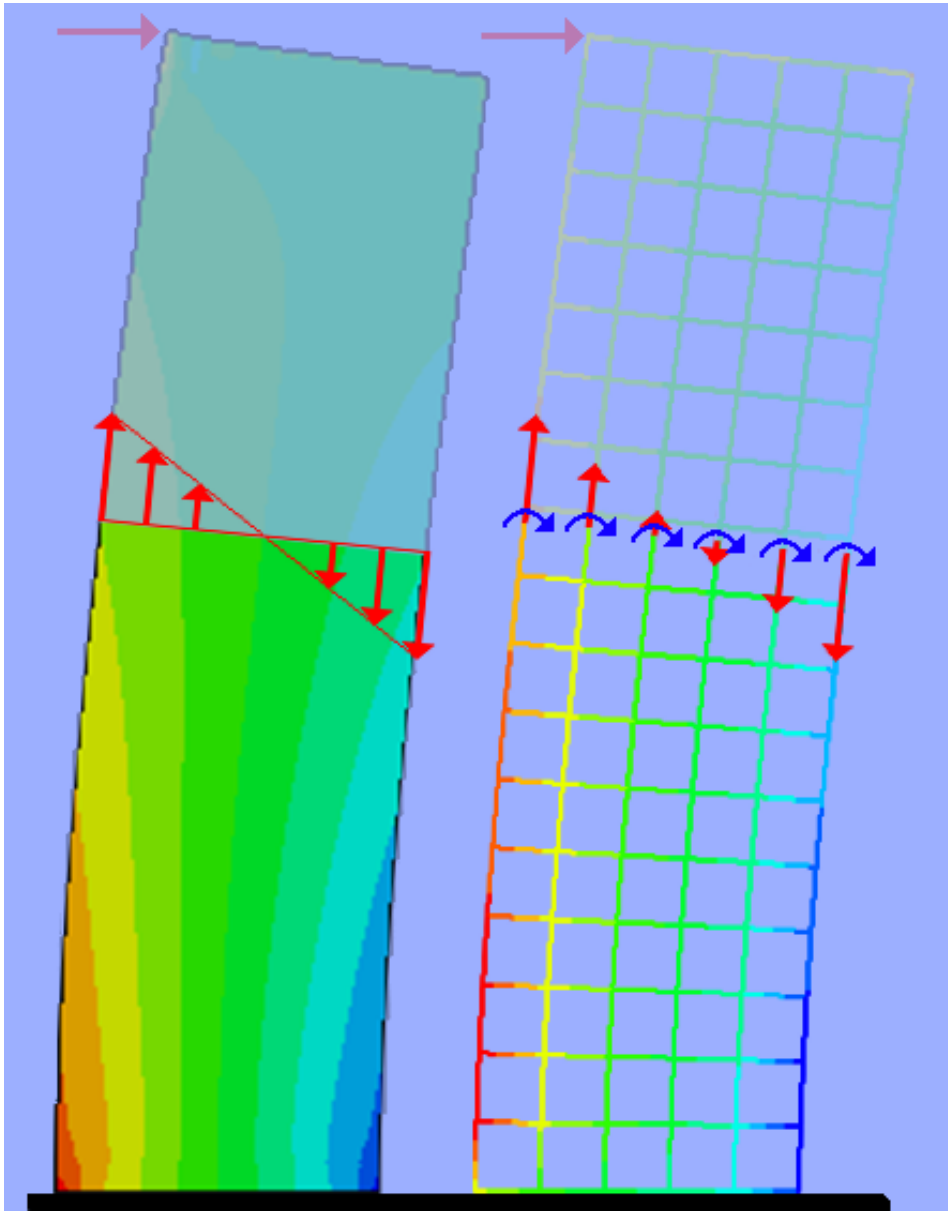


Three beam elements with the same displacement at their ends but different rotations



Motivation: Lost Details from Beam Rotation

- Moments in lattice material from:
  - Distribution of stresses
  - Beam couples
- Homogenized elasticity leaves these out.
- Micropolar elasticity extends classical elasticity to include extra free variable.
  - micropolar rotation, ϕ
- Extra variables and extra equations.



Full Equations

	Classical Elasticity	Micropolar Elasticity
Constitutive Law	$\sigma_{ij} = A_{ijkl}\varepsilon_{kl}$	$\sigma_{ij} = A_{ijkl}\varepsilon_{kl}$ and $m_{ij} = C_{ijkl}k_{kl}$
Strain Definition	$\varepsilon_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$	$\varepsilon_{ij} = u_{j,i} - e_{kij}\phi_k$ and $k_{ij} = \phi_{j,i}$
Equilibrium Equations	$\sigma_{ji,j} = 0$ and $\sigma_{ij} - \sigma_{ji} = 0$	$\sigma_{ji,j} = 0$ and $m_{ji,j} + e_{ijk}\sigma_{jk} = 0$
Strain Energy Density	$w = \frac{\sigma_{ij}\varepsilon_{ij}}{2} = \frac{A_{ijkl}\varepsilon_{ij}\varepsilon_{kl}}{2}$	$w = \frac{A_{ijkl}\varepsilon_{ij}\varepsilon_{kl}}{2} + \frac{C_{ijkl}k_{ij}k_{kl}}{2}$

Equations reference (Eringen, 1999)

Verification from the literature

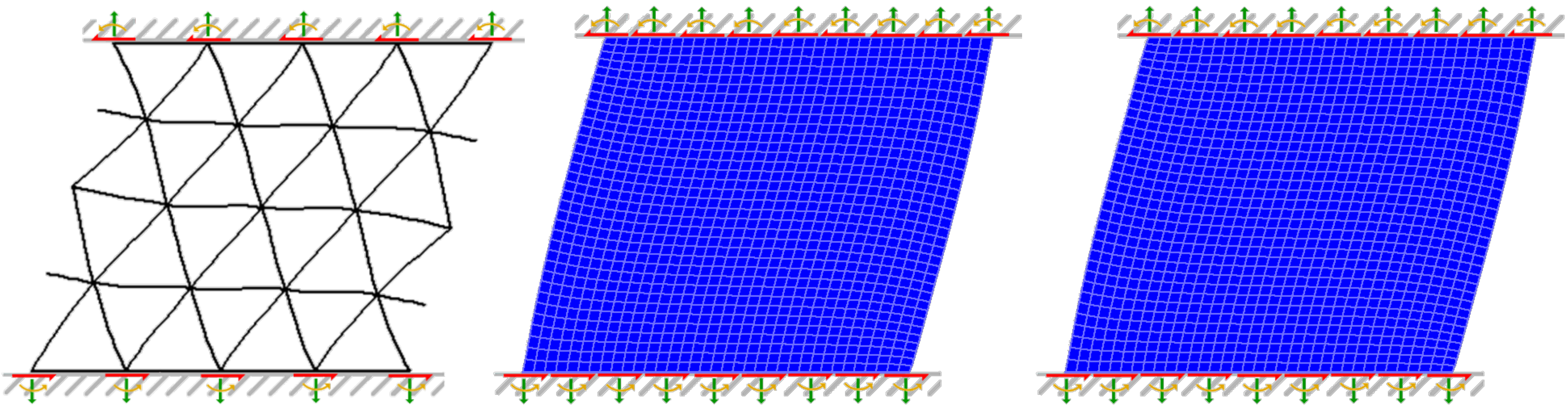
- Research Questions focus on accuracy of continuum models
  - When are continuum models accurate?
  - When is micropolar elasticity more accurate than classical elasticity?

Outline

1. Background and Motivation
2. **Methods**
3. Conclusions

Two equivalent models – Set up models

- Set up a lattice model, equivalent micropolar model, equivalent classical model
  - Solve both using FEA.
    - Three FEA codes I wrote
  - Same overall size,
  - Same boundary conditions,
  - Continuum uses material properties for lattice



Beam Lattice   Classical Elastic   Micropolar elastic

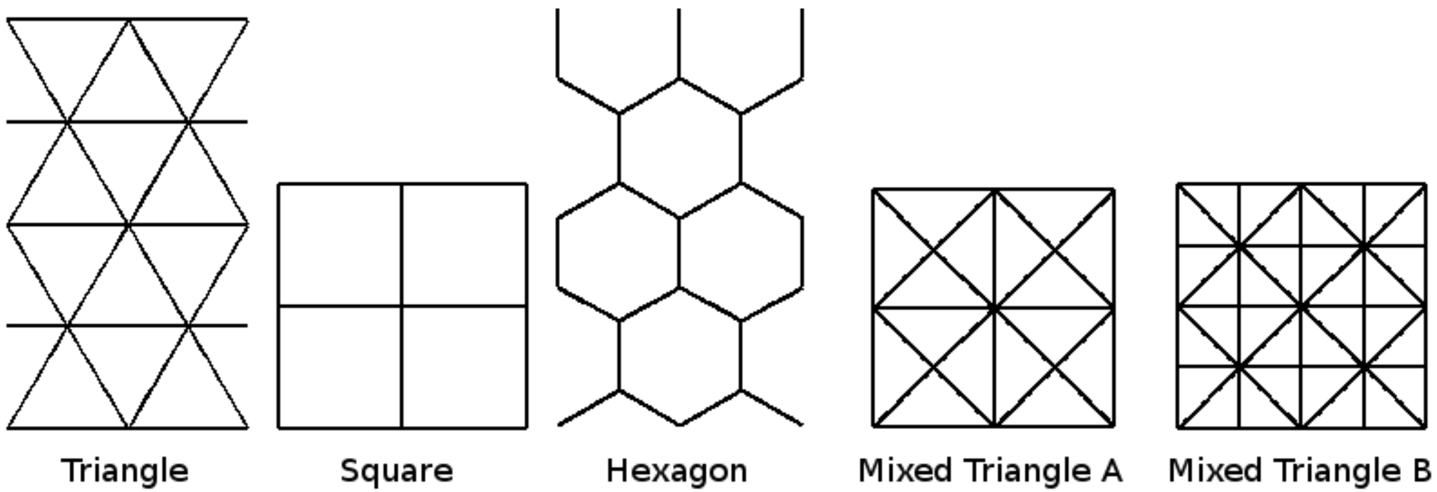
Two equivalent models – Error Formula

$$Error = \frac{L - C}{\sqrt{L\,C}}$$

- L, C are lattice, continuum results
  - For this presentation global strain energy
    - Any pair of comparable results possible

Available lattice geometries

- Certain lattices have formulas for material properties
  - Simulations are limited to these lattice topologies



(Perano, 1983) (Bazant,1972) (Stronge, 1999) (Kumar, 2004) (Kumar, 2004)

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When is micropolar elasticity accurate?

How do unit cell shape and topology, macro-size, and loading conditions affect the accuracy of continuum models when modeling lattice structures?

- My exploratory studies show accuracy influenced by
  1. Lattice type (e.g. Honeycomb, triangle, etc.)
  2. Number of unit cells relative to part dimensions
     - Generally more repeated unit cells means more accurate
  3. Boundary conditions
- All patterns have exceptions

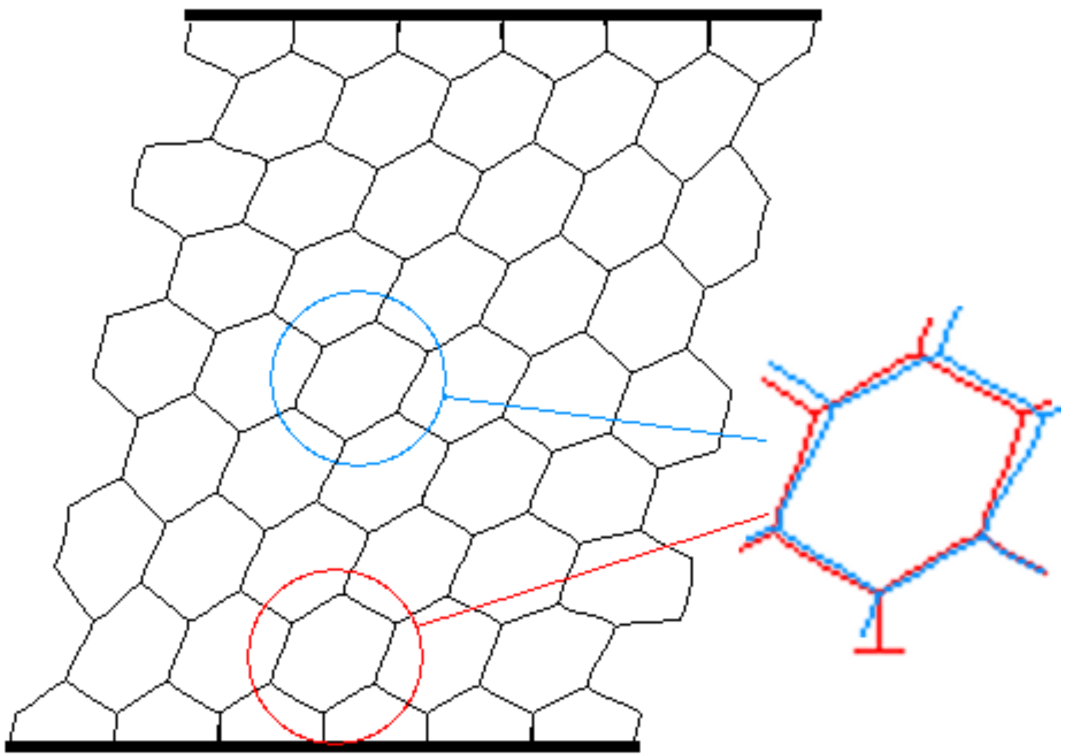
Research Conclusions

- All tested topologies can be accurate at large sizes.
  - Honeycomb is hit and miss.
  - MixedTriAold is not as good as the others.
  - I have hypotheses that explain why
- Certain boundary conditions are not accurate, regardless of size

	square	triangle	hexagon	mixedTriAnew	mixedTriAold	mixedTriBnew	mixedTriBold	diamondNew	diamondOld
stretch11	0.00%	-0.07%	-0.69%	0.10%	2.75%	0.47%	0.48%	0.20%	-0.36%
stretch22	0.00%	0.34%	1.60%	0.10%	2.75%	0.47%	0.48%	0.48%	-0.29%
shear12	-0.70%	-0.35%	-0.38%	-0.19%	1.82%	1.20%	0.03%	-0.11%	-0.55%
shear21	-0.70%	0.24%	1.37%	-0.19%	1.82%	1.20%	0.03%	0.11%	-0.56%
shear21hinge	-0.72%	0.22%	10.83%	-0.21%	1.77%	1.16%	-0.05%	0.10%	-0.58%
bend1free	-0.22%	-0.19%	-0.45%	0.07%	3.06%	0.85%	0.93%	0.37%	-0.43%
bend2free	-0.22%	1.47%	7.86%	0.07%	3.06%	0.85%	0.93%	0.68%	-0.03%
bend1allFree	-0.22%	-0.15%	-0.08%	0.12%	3.24%	0.91%	1.02%	0.45%	-0.29%
bend2allFree	-0.22%	1.63%	10.17%	0.12%	3.24%	0.91%	1.02%	0.76%	0.16%
curve1	0.35%	52.95%	-32.99%	48.84%	90.72%	94.98%	32.69%	94.63%	82.28%
curve2	0.35%	24.56%	161.95%	48.84%	90.72%	94.98%	32.69%	41.73%	33.09%
halfspace	-80.64%	-59.06%	-56.24%	-52.99%	-55.17%	-54.17%	-61.12%	-26.17%	-43.54%

Topologies

- Certain boundary conditions cannot be accurate.
  - Simulations using boundary conditions that activate only local effects are not super accurate
  - For boundary conditions that activate local and global effects, local effects are a small part of total.



Topologies

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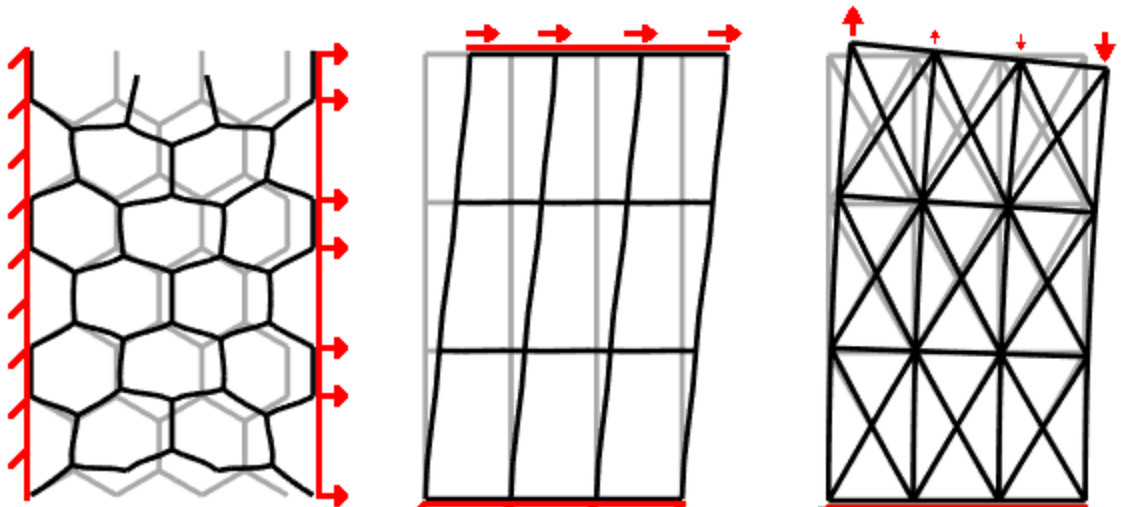
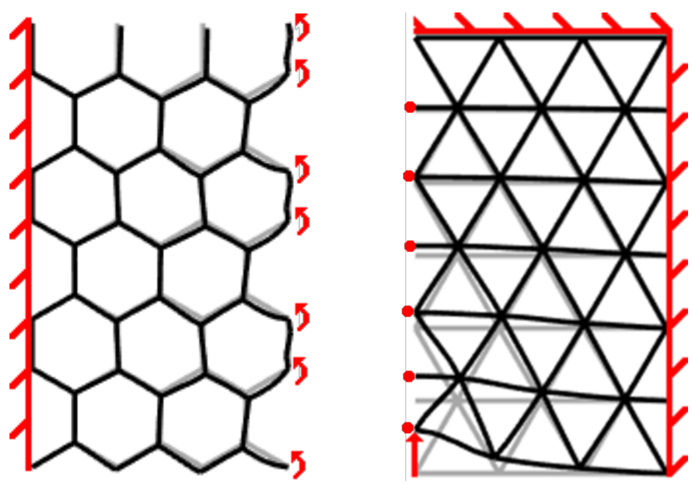
Activating local and global effects

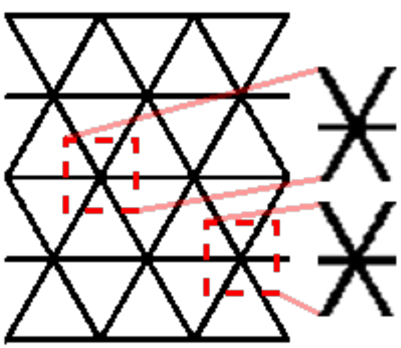


Activating local effects only

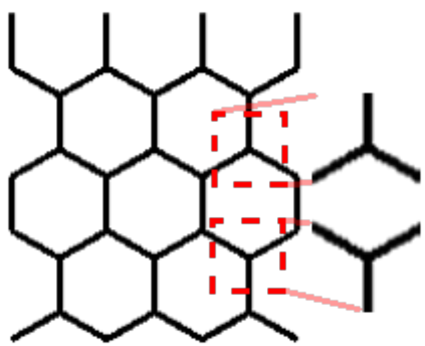


Topologies

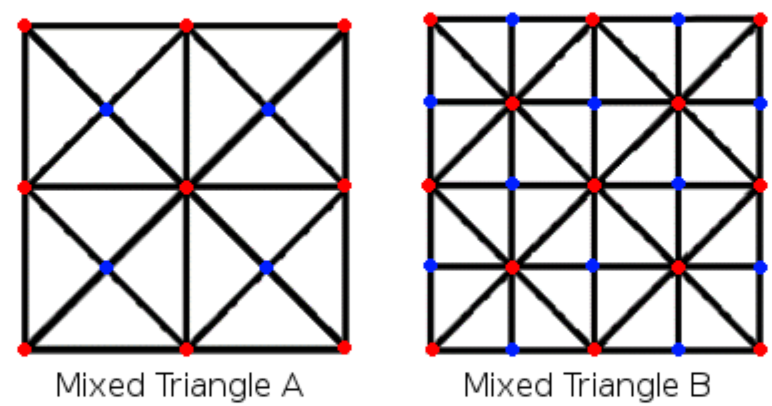
- All tested topologies can be accurate.
- Mixed Triangle topology – Connected vs Disconnected
  - Indistinguishable node points
    - Kumar's method states that it only applies to lattices with one type of node point.
    - He breaks this rule.
    - I reworked his methods "fixing" this.



Indistinguishable



Distinguishable



Red and blue nodes are distinct. If beams pass through each other at blue points, blue points are no longer nodes.

Topologies

- All tested topologies can be accurate.
- Mixed Triangle topology – Connected vs Disconnected
  - Indistinguishable node points
    - Kumar's method states that it only applies to lattices with one type of node point.
    - He breaks this rule.
    - I reworked his methods "fixing" this.
    - My "fix" does not make a clear difference.
      - "Fixed" topologies are labeled new.

	mixedTriAnew	mixedTriAold		mixedTriBnew	mixedTriBold		diamondNew	diamondOld
stretch11	0.10%	2.75%		0.47%	0.48%		0.20%	-0.36%
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curve1	48.84%	90.72%		94.98%	32.69%		94.63%	82.28%
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halfspace	-52.99%	-55.17%		-54.17%	-61.12%		-26.17%	-43.54%

Micropolar effect size

- Micropolar effects are usually small
  - Quantified in terms of global variables
  - Effects bigger on local variables?
- Micropolar effect size
  - $MPE = \frac{W_{MP} - W_{CL}}{\sqrt{W_{MP} W_{CL}}}$
- When the micropolar effect is small, the classical model is about as accurate as the micropolar model for global comparisons.
  - Future work: Look more at local variables
- Micropolar effect is much larger for the boundary conditions that are never particularly accurate.

Questions and References

QUESTIONS?

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Why displacement boundary conditions?

- Lattice structures are relatively flexible
- Loads are applied by connected part assumed relatively rigid.
  - Tweel's boundary condition imposed by ground
  - Test samples boundary condition imposed by rigid steel plate
- Not appropriate for all situations

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Experimental Methods

- Test stiffness of multiple cylindrical samples in bending and torsion
- Calculate micropolar properties from difference between experiment and classical behavior.
  - Imprecise measurements quickly mask micropolar behavior
- (Lakes, 91) describes methods for isotropic
  - My work focuses on transverse isotropic
  - I can follow his logic and adapt his methods

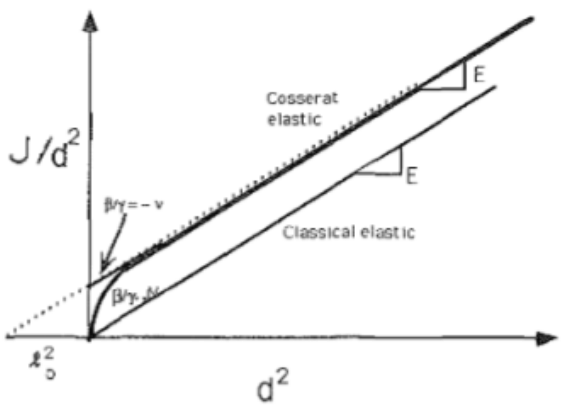
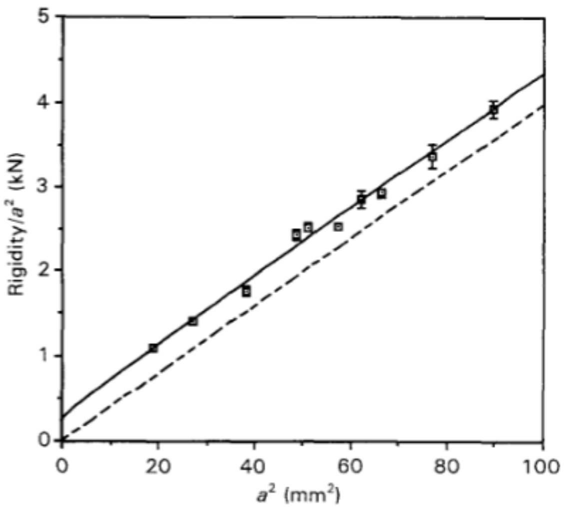


Fig. 2 Extraction of elastic constants from size effect data in torsion of a circular cylindrical rod. Rigidity/diameter squared *versus* diameter squared

Theoretical Graph (Lakes, 91)



Actual Experimental Results (Lakes, 94)

Detailed Derivation of Micropolar Mat'l Properties

- Equivalent energy
- Correct micropolar properties mean
  - lattice energy = continuum energy
  - Equivalent strain fields
    - Requires explicit definition of strain fields
    - Works for indistinguishable node points
  - Material properties from derivatives of strain energy with respect to strain

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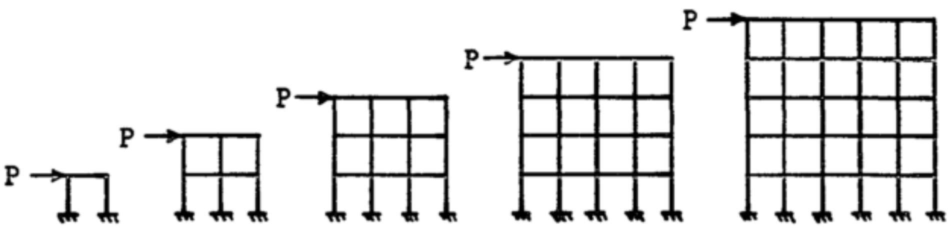
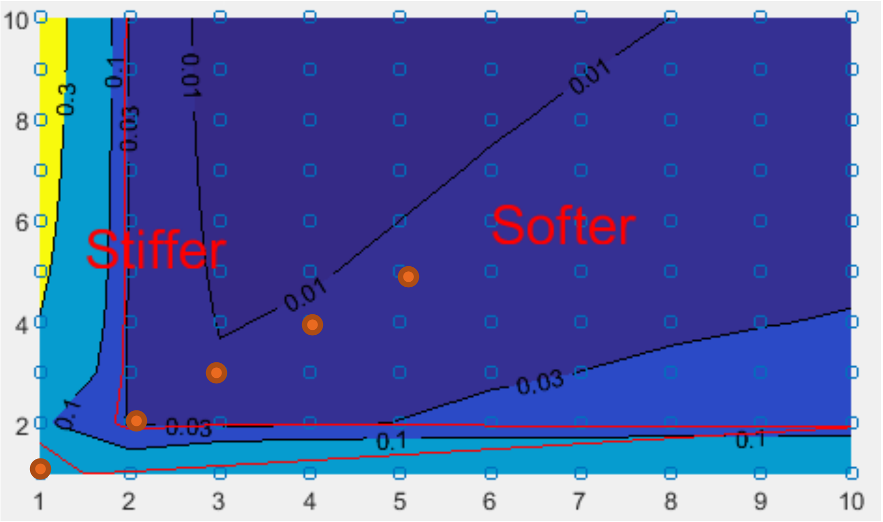
Verification-Perano

- Perano
  - Direct comparison of a lattice and continuum
    - Similar methods to mine
    - His results show higher accuracy than mine
      - Probably due to force controlled boundary conditions
  - My results suggest that his choice of simulations might overstate the general accuracy
  - Did not explain reasons for error



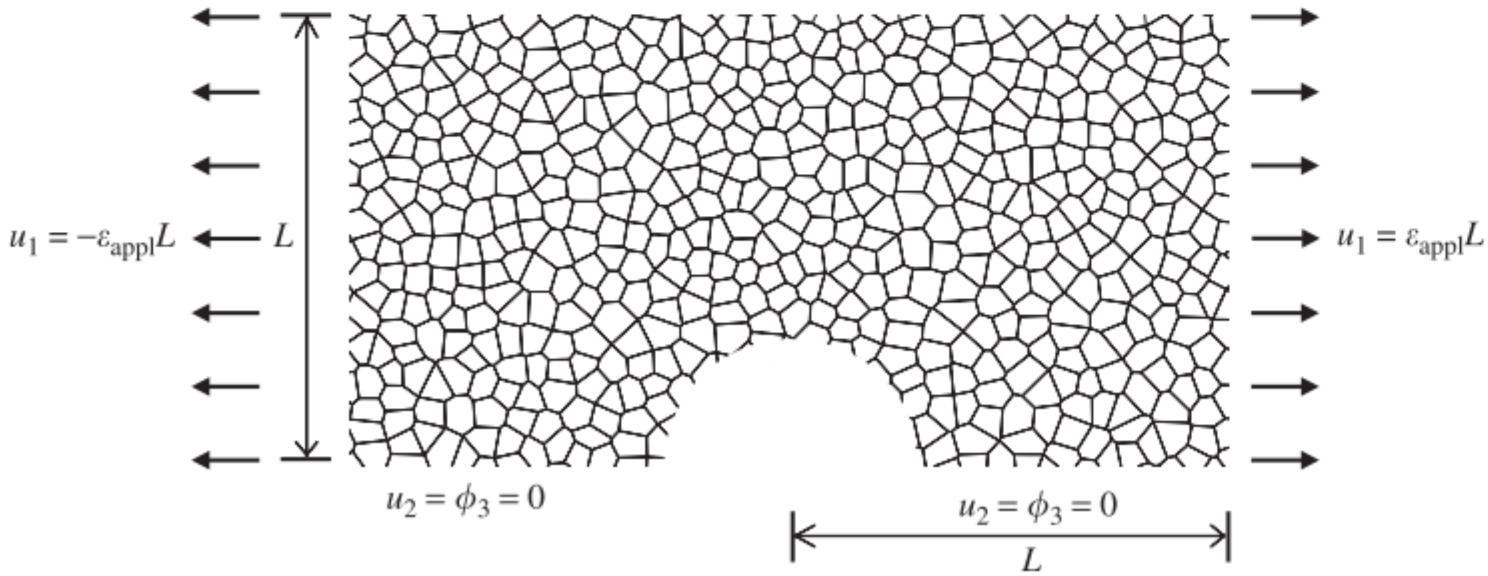
Perano's square lattices



My most equivalent results

Verification-Tekoğlu

- Compared lattice to continuum with random foams in shear
  - Optimized material properties
  - Limited set of boundary conditions
  - Random foams limited ability to explain reasons for error



Possible Cause of Error – Scaling Effects

- Classical Elasticity shows scaling solutions
  - Changing size of a part changes the size of a stress pattern
- Micropolar elastic solutions have both scaling and non-scaling components.
  - Lame’s constants and κ have units of pressure.
  - γ has units of pressure-length
    - Related to non-scaling compontents
- Run a number of simulations with increasing size.
- Figure on next slide looks at the stress patterns in red boxes

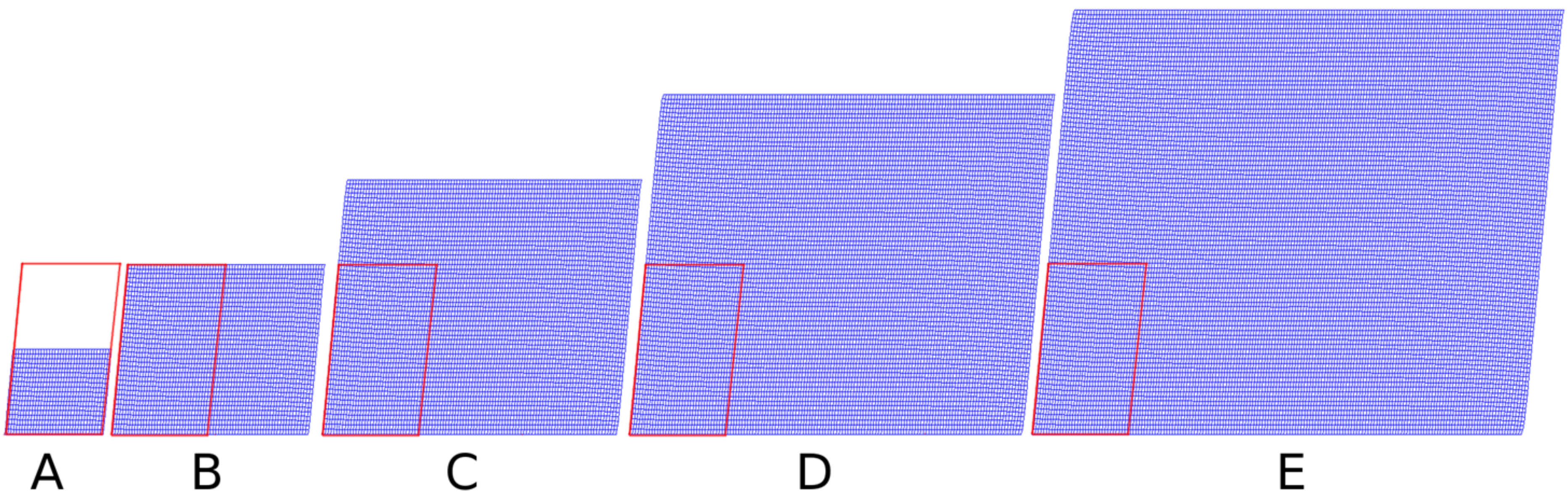


Illustration of scaling and non-scaling