Some Modifications to the Numerical Electromagnetics Code and their Effects

T. H. Hubing
University of Missouri-Rolla
Rolla, MO 65401

J. F. Kauffman
North Carolina State University
Raleigh, NC 27695

Abstract

The Numerical Electromagnetics Code (NEC2) does not always yield consistent or accurate results when analyzing configurations that contain thin conductive surfaces or configurations with sources located near a wire-surface attachment. This paper evaluates modifications to the code designed to "test the limits" of the NEC2 algorithm. These modifications can significantly increase the amount of time required for the algorithm to run and do not necessarily belong in a "general purpose" EM modeling code. Nevertheless, the modified code is able to analyze configurations that the unmodified algorithm is unable to analyze and the results obtained using the modified code can help the user to understand where certain types of error originate.

Introduction

The Numerical Electromagnetics Code [1] is a general-purpose moment-method computer program capable of analyzing the electromagnetic scattering properties of a variety of wire-surface geometries. Several other general-purpose moment-method codes are available, however NEC is unique in that it is the only widely-distributed, well-documented code that analyzes surfaces by solving a form of the magnetic field integral equation (MFIE). MFIE-based algorithms have certain computational advantages over algorithms that analyze surfaces by solving the electric field integral equation (EFIE). However, MFIE-based codes do not model electrically thin conductive surfaces efficiently. The inability of the Numerical Electromagnetics Code to analyze geometries with thin metal plates significantly limits its potential applications.

Another limitation of the NEC algorithm is its inability to accurately calculate the input impedance of sources located near a wire-surface attachment point. The method used by NEC2 to model wire-surface attachments analyzes simple configurations accurately and efficiently, however a more detailed wire-attachment model is required for many applications.

The purpose of this paper is to investigate the thin-surface and wire-attachment limitations of NEC2. In the course of this investigation, modifications to the algorithm will be introduced. These modifications expand the scope of the algorithm at the expense of requiring additional computation.

Thin Surfaces

The form of the MFIE used by NEC2 to analyze conductive surfaces is,

$$\boldsymbol{J}_S(x) = 2\hat{n}\times\boldsymbol{H}^i + 2\hat{n}\times\int_S \boldsymbol{J}_S(x')\times\nabla' G(x,x')\, ds \ . \qquad (1)$$

The $2\hat{n}\times\boldsymbol{H}^i$ term is the component of $\boldsymbol{J}_S(\mathrm{x})$ that is due directly to the incident magnetic field. The $2\hat{n}\times\int_S \boldsymbol{J}_S(x')\times\nabla' G(x,x')\, ds$ term is the component of $\boldsymbol{J}_S(\mathrm{x})$ induced by the fields from the induced surface currents at all points on the surface except the neighborhood of the point $x' = x$. A detailed derivation of Equation (1) can be found in [2].

Conductive surfaces are represented as a collection of surface "patches" in NEC2. The surface current on each patch that is not connected to a wire is described by two impulse functions. The two impulses represent components of current flowing in each of two orthogonal directions on the surface. This modeling technique is referred to as point matching since the boundary conditions are enforced at individual points on the surface. The surface integral on the right hand side of

Equation (1) is reduced to a simple multiplication, which significantly reduces the time required to compute the values of the impedance matrix.

The surface current integral in Equation (1) is only used to calculate off-diagonal terms of the impedance matrix. Large smooth surfaces result in well-conditioned diagonally dominant impedance matrices [3]. Small errors in the calculation of off-diagonal terms do not tend to significantly affect the accuracy of the result when analyzing large surfaces. Relatively small surfaces or thin plates, on the other hand, are more sensitive to errors in the calculation of the off-diagonal terms. The analysis of small or thin surfaces requires a more careful evaluation of the integral in Equation (1). In other words, a weighting function other than an impulse is needed. For our purposes, a pulse function was considered to be the best trade-off between an impulse function and more complex weighting functions that would require considerably more computation time. Flexibility and ease of data entry was a primary consideration.

When point matching is used, it is not necessary to know the exact shape of a patch or the location of its edges. Pulse weighting functions require the shape and location of the patch to be known but they do not require a knowledge of how a patch is oriented relative to the other patches. Switching from impulse to pulse weighting functions required two basic modifications to the algorithm:

1. Describing the patch geometry to the necessary subroutines
2. Using this data to evaluate the surface integral in Equation (1).

The original code accepted a variety of patch shapes and even allowed a patch to be defined with an arbitrary, unspecified shape. The only data used by the algorithm was the patch area and center point location. In order to facilitate both of the tasks listed above, only rectangular patches are permitted in the modified code.

The subroutine DATAGN was modified to read or calculate the corner points of each surface patch and store them in the common block, PCORNR. The single multiplication that used to perform the integration in Equation (1) was replaced by a double sum of the form,

$$\int_s J_s(x')\, G(x,x')\; ds \approx \frac{A}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} J(x_{ij})\, G(x,x_{ij}) \qquad (2)$$

where: J = constant pulse amplitude

A = area of surface patch

N^2 = # of subpatches

x_{ij} = center point of $(ij)^{th}$ subpatch

This simple method for estimating the value of the surface integral is referred to as the mid-point rule [4]. It is similar to trapezoidal rule integration in terms of efficiency and error but it avoids problems that can occur when the integrand has singularities at the end-points. Larger values of N result in more accurate evaluations of the integral at the expense of additional computation time. When N equals 1, this method is equivalent to the original point-matching technique.

Note that if G(x,x') is fairly constant over the surface of the patch, relatively small values of N are required. However, when G(x,x') is a strong function of position (as it is for example when evaluating the interaction between two patches on opposite sides of a thin plate), larger values of N are necessary to achieve a given level of accuracy. The NEC2 subroutine HINTG was modified to do the midpoint rule integration. The value of N is stored in the variable NPATCH at the beginning of the modified subroutine. The subroutine UNERE, which evaluates a form of the EFIE for

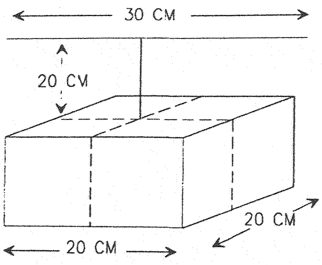

Figure 1: Wire-T Configuration

analyzing wire-to-surface interactions was also modified to perform a midpoint rule integration. The value of N used for this integration is stored in the variable NSTEP. NSTEP and NPATCH are independent of one another, since the accuracy of the surface integration required for the EFIE may not be the same as the accuracy required for evaluation of the MFIE.

In order to evaluate the effect of this modification to the NEC2 code, one of the example problems from the NEC2 User's Guide [1] was analyzed using different values for NPATCH and NSTEP. The configuration is illustrated in Figure 1. It consists of a 1 volt, 300 MHz source driving a wire-T relative to a 20x20x10 cm metal box resting on a ground plane.

As the User's Guide points out, a lossless structure over a ground plane should have an average power gain of 2.0. The calculated average power gain for this example however, is about 1.8 indicating that the calculated input impedance (about 181 + j218 ohms) may be inaccurate.

The User's Guide cites the relatively crude way that the patches are modeled as the probable source of the error, however a closer examination of this and other examples reveals another problem. During the course of this work, it was observed that inconsistent results were obtained whenever a voltage source was located on a wire segment with one end attached to a surface patch. The reason for this relates to the way that voltage sources are modeled by the program [3]. The error can usually be reduced by using shorter segment lengths in the vicinity of a patch attachment and placing at least one segment between the surface and the segment containing the source. This was done for the example in Figure 1 by subdividing the surface-attached wire segment into 3 smaller segments of equal length. The source was located on the center segment making it the same height above the surface.

The program was run again and the new value calculated for the input impedance was 176 - j413 ohms and the average power gain was 1.82. This was significantly different from the 181 + j218 ohm impedance that was originally calculated but even this value is not correct. Another modification to the NEC2 code intended to improve the accuracy of wire-to-surface attachment calculations was applied and the new calculated value of the input impedance became 195 + j260 ohms. (This modification is described in the next section.) Using this impedance as a starting point, Table 1 shows the effect of increasing the values of the variables NPATCH and NSTEP. Note that in this example, the results stabilize for values of NPATCH and NSTEP greater than 5.

Table 1: Effect of NPATCH and NSTEP on Wire-T

NSTEP	NPATCH	NINT	INPUT Z	AVE. PG
1	1	48	195+j260	1.81
5	1	48	188+j258	1.87
15	1	48	188+j258	1.87
15	5	48	174+j272	1.89
15	15	48	174+j272	1.89
5	15	48	174+j272	1.89
1	15	48	180+j274	1.84

Wire-to-Surface Attachments

Errors related to the use of point matching can generally be overcome without modifying the algorithm by simply using a larger number of patches to represent the surface when sufficient computing resources are available. However, there is another source of error that arises when using NEC2 to analyze small or thin surfaces with a wire-to-surface attachment point that cannot be compensated for without modifying the algorithm.

NEC2 allows wires to be attached to a surface at patch centers. The procedure for analyzing wire-surface attachments used by NEC2 is based on a technique used by Albertsen et al [5]. When a wire surface attachment is made, NEC2 divides the region near the attachment point into four subpatches as illustrated in Figure 2. The expansion of the surface current density in this region must satisfy the following condition,

$$\nabla_s \cdot \boldsymbol{J}_s(\xi,\eta) = \boldsymbol{J}_o(\xi,\eta) + I_o\,\delta(\xi,\eta) \qquad (3)$$

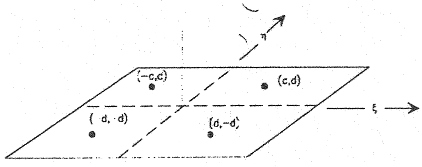

Figure 2: Wire Attachment Point

where ∇_s denotes surface divergence, $\boldsymbol{J}_o(\xi,\eta)$ is a continuous function in the region of the attachment, and I_o is the wire current flowing onto the surface. The expansion function used by NEC2 is,

$$\boldsymbol{J}_s(\xi,\eta) = I_o\,\boldsymbol{f}(\xi,\eta) + \sum_{j=1}^{4} g_j(\xi,\eta)\,[\boldsymbol{J}_j - I_o\,\boldsymbol{f}_j] \qquad (4)$$

where $\boldsymbol{f}(\xi,\eta) = \dfrac{\xi\hat{\xi} + \eta\hat{\eta}}{2\pi(\xi^2+\eta^2)}$

$$\boldsymbol{J}_j = \boldsymbol{J}_s(\xi_j,\eta_j)$$

$$\boldsymbol{f}_j = \boldsymbol{f}(\xi_j,\eta_j)$$

and ξ_j,η_j are the coordinates at the centers of patch j. The interpolation functions $g_j(\xi,\eta)$ used by NEC2 are:

$$g_1(\xi,\eta) = \frac{1}{4d^2}(d+\xi)(d+\eta) \qquad (5a.)$$

$$g_2(\xi,\eta) = \frac{1}{4d^2}(d-\xi)(d+\eta) \qquad (5b.)$$

$$g_3(\xi,\eta) = \frac{1}{4d^2}(d-\xi)(d-\eta) \qquad (5c.)$$

$$g_4(\xi,\eta) = \frac{1}{4d^2}(d+\xi)(d-\eta) \qquad (5d.)$$

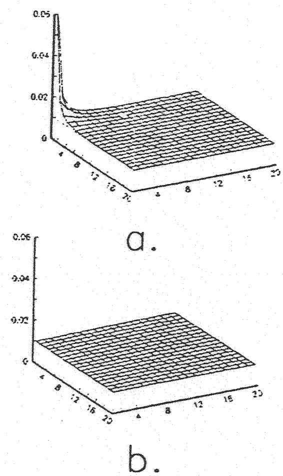

Figure 3: Current Expansion Functions for a Patch with a Wire Attachment

A plot of the magnitude of this current expansion function for the case $J_1 = J_2 = J_3 = J_4$ is shown in Figure 3a. The surface current expansion in Equation (4) is used *only when computing the electric field at the center of the attached wire segment due to the surface current on the four surrounding patches.* An impulse expansion of this patch current is used to calculate the fields at all of the other wire segment centers.

This technique for analyzing wire-surface attachments is relatively simple. It doesn't require excessive computation and it doesn't place severe restrictions on the size or placement of wire attachment regions. However the errors introduced by this simple technique may be unacceptable, particularly in situations where an accurate calculation of the input impedance of a source on a wire near a surface attachment point is desired.

One source of error arises from the fact that the surface current expansion, Equation (4), is defined over a square region centered at the wire attachment point even when the patch to which the wire is connected is not square. Since the unmodified NEC2 does not consider the shape of a patch when performing a surface integration, there would be little to gain by defining the wire attachment region to be a specific shape. However, with the modifications described in the previous section, surface integrations are performed using specific patch dimensions. If the wire attachment current expansion function is not modified, the surface integration would be performed twice in some areas and not at all in others.

Two modifications are required to make the program integrate the surface current expansion function over the correct patch surface. First, the limits of the integration in Equation (4) must be changed to correspond to the corners of the patch. This is done by modifying the subroutine PCINT. Second, the interpolation functions, Equations (5), must be modified so that the points ξ_j,η_j are at the centers of the four subpatches. The new interpolation functions can be written:

$$g_1(\xi,\eta) = \frac{1}{4d_1d_2}(d_1+\xi)\,(d_2+\eta) \tag{6a.}$$

$$g_2(\xi,\eta) = \frac{1}{4d_1d_2}(d_1-\xi)\,(d_2+\eta) \tag{6b.}$$

$$g_3(\xi,\eta) = \frac{1}{4d_1d_2}(d_1-\xi)\,(d_2-\eta) \tag{6c.}$$

$$g_4(\xi,\eta) = \frac{1}{4d_1d_2}(d_1+\xi)\,(d_2-\eta) \tag{6d.}$$

where d_1 and d_2 are the ξ and η components of the distance to the center of each subpatch. This modification to the interpolation functions was also made in the subroutine PCINT.

There is a second source of error with this wire-attachment method that is not quite as apparent. This error arises from the fact that two different current expansion functions are used to represent the current in the region of a wire attachment. The current expansion function in Equation (4) is used to calculate the electric field at the center of the *attached* wire segment due to the current on the four subpatches,

$$\boldsymbol{E}_{attached} = \int_s J_s(\xi,\eta)\; G(w,\xi,\eta)\;\, ds\;. \tag{7}$$

while the electric field at the center of *non-attached* wire segments is calculated using an impulse expansion of the current on the four subpatches,

$$\boldsymbol{E}_{non-attached} = \sum_{j=1}^{4} J_j\, A\, G(w,\xi,\eta)\;. \tag{8}$$

The variable w represents the coordinates at the center of the wire segment at which the field is evaluated in Equations (7) and (8) and A is the area of one subpatch.

Two types of error result from this approximation. One source of error is due to the fact that in many configurations, particularly those involving wire segments located close to a surface, $G(w,\xi,\eta)$ can vary significantly over the integration surface. However, even when $G(w,\xi,\eta)$ is relatively constant in the region of a wire attachment, error is introduced.

To illustrate this, assume for the moment that all of the wire segments except for the attached segment are relatively far from the attachment subpatches and the $\boldsymbol{G}(w,\xi,\eta)$ can be approximated as a constant, G, over the surface of integration, Equation (7) can be written,

$$\boldsymbol{E} = \boldsymbol{G}\int_s J_s(\xi,\eta)\;\, ds \tag{9}$$

substituting for $J_s(\xi,\eta)$ using Equation (4) and filling in the integration limits for the first of the four subpatches,

$$E_{attached} = G \int_0^{2d_1} \int_0^{2d_2} \left[I_o f(\xi,\eta) + \sum_{j=1}^{4} g_j(\xi,\eta)\,(J_j - I_o f_j) \right] ds \tag{10}$$

The electric field due to the current in the first subpatch calculated using Equation (8) making the same assumption is,

$$E_{non-attached} = G\, J_1\, A \quad . \tag{11}$$

In order to determine how good this approximation is, we can evaluate the integral of Equation (10) assuming for the moment that $J_1 = J_2 = J_3 = J_4$ and that $d_1 = d_2 = d$. The field due to the ξ component of the current is given below. By symmetry, a similar result can be obtained for the η component.

$$E_{attached} = G \int_0^{2d_1} \int_0^{2d_2} \left[I_o f(\xi,\eta) + \sum_{j=1}^{4} g_j(\xi,\eta)\,(J_j - I_o f_j) \right] ds \; = \; G\, [\, I_o \frac{d}{\pi} (3 - 2\sqrt{2} - 1) + J_1 A \,] \tag{12}$$

The term at the far right, $J_1 A$, is equivalent to the approximation of Equation (11). However, there is an additional term, which is a function of I_o, that the approximation neglects. Therefore even when **G** can be considered relatively constant over the integration, the two current expansion functions used to represent patches attached to wire segments are not equivalent.

Another way to view this is illustrated in Figure 3. The current expansion functions in Figure 3a. and Figure 3b. are both used to represent the current on the same patch for different calculations. However, the volume under each of these curves is not equal. The moment method program calculates a single value for **J** based on two inconsistent expansion functions. This introduces error into the calculated value of **J**.

The actual error term is highly dependent on the integration technique used in the vicinity of the wire attachment. When using midpoint rule integration, the calculated volume under the curve in Figure 3a. is very dependent on N because of the singularity at the origin. The value of N used for this integration is stored in the variable NINT in the subroutine PCINT. Larger values of N increase the accuracy of the calculation in Equation (10), but do not necessarily reduce the error term.

One way to eliminate this error would be to define the expansion function, Equation (4) and the integration technique so that the volume under each of the surface curves in Figure 3 is equal. This is difficult to do without restricting the shape of the wire-attachment region. Another approach that also eliminates the error due to the non-constant $G(w,\xi,\eta)$, is to use the same expansion function

Table 2: Effect of Modified CMSW on Wire-T

VERSION OF CMSW	NINT	INPUT Z	AVE. PG
ORIGINAL	10	161-j384	1.83
ORIGINAL	12	163-j295	1.82
ORIGINAL	24	173+j77	1.80
ORIGINAL	48	174+j136	1.80
MODIFIED	10	160-j298	1.91
MODIFIED	12	163-j199	1.91
MODIFIED	24	173+j206	1.89
MODIFIED	48	174+j272	1.89
MODIFIED	66	174+j272	1.89

for all of the calculations. The impulse expansion is too simplistic to provide an accurate representation of the patch currents in the area of a wire attachment, so the program was modified to use the expansion of Equations (4,6) to calculate all of the electric fields due to currents on patches connected to wire segments. This was done by modifying the subroutine CMSW. As expected, this significantly increased the amount of time required to calculate the values of the impedance matrix for configurations with many wire segments. The original impulse expansion function for the subpatches was still used to calculate patch-to-patch interactions, because it was felt that the increase in accuracy resulting from this change would not justify the increased amount of computation.

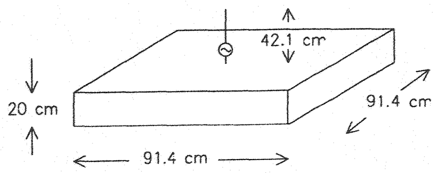

Figure 5: Wire Attachment Example

Table 2 gives the calculated values of input impedance and average power gain for the wire-T configuration in Figure 1 using both the modified and unmodified versions of the subroutine CMSW. The variable NINT represents the number of subpatches used to perform the integration in Equation (4). Note that both versions of the code stabilize for values of NINT somewhere between 25 and 48, but they result in two different solutions. The input impedance calculated with the unmodified algorithm corresponds to a reflection coefficient magnitude of 0.7 when attached to a 50-ohm cable. The input impedance calculated using the modified version of CMSW corresponds to a reflection coefficient magnitude of 0.85, which agrees with the measured value of reflection coefficient obtained using a network analyzer and a copper model of this configuration [3].

Figure 5 shows a configuration consisting of a 42.1 cm wire above a 91.4 x 91.4 x 20 cm conductive surface. The wire is driven by a voltage source located on the wire 8 cm above the surface. This configuration was analyzed using both the modified and unmodified versions of the NEC2 code in the frequency range 125 - 215 MHz. Both the top and bottom of the surface were divided into 25 square patches and the sides were each divided into 5 patches lengthwise. Since the conductive surface is neither small or thin at these frequencies, one might expect point matching to be sufficiently accurate. This is indeed the case as indicated by the plot in Figure 6, which shows that setting the variables NPATCH and NSTEP equal to 1 (as opposed to 15) had little effect on the calculated input conductance. However, the modified wire-attachment technique had a significant effect on the calculations as illustrated in Figure 7. This result shows how the error introduced by using two different expansion functions to represent the surface patches at a wire attachment can significantly affect the calculated input impedance near the attachment point.

Since one of the modifications made to the code was designed to correct a problem that can occur when using non-square patches, this configuration was analyzed again with the top and bottom

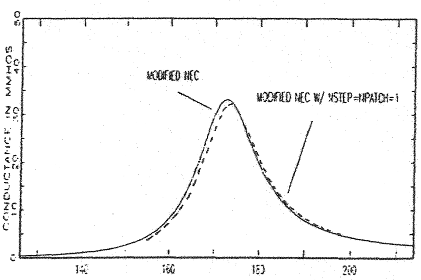

Figure 6: Effect of Point Matching

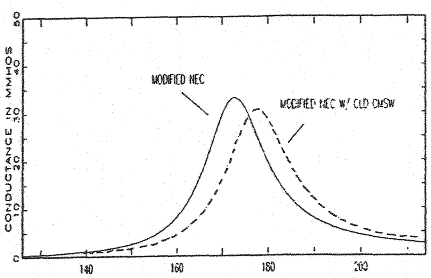

Figure 7: Effect of Modified Wire Attachment

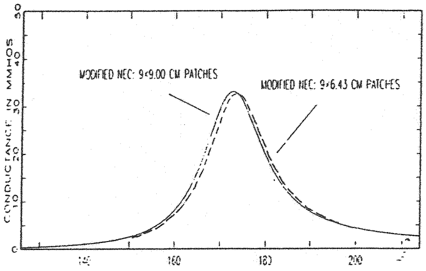

Figure 8: Effect of Patch Shape with Modified Code

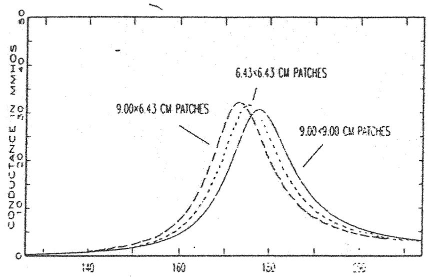

Figure 9: Effect of Patch Shape with Original Code

surfaces divided into 35 (9.00 x 6.43 cm) rectangular subpatches. The configuration being modeled was unchanged, so ideally there should be little change in the calculated results. This is indeed the case using the modified code as shown in Figure 8, however the results from the unmodified code are significantly different as shown in Figure 9.

Conclusions

The Numerical Electromagnetics Code (NEC2) is limited in its ability to analyze even moderately thin conductive surfaces and configurations with a source located near a wire-to-surface attachment. Modifications have been described that can improve the ability of the algorithm to model some types of configurations at the expense of requiring additional computation. In general, the improvement in accuracy applies only to specific types of configurations and the decreased efficiency due to these modifications can be significant.These modifications and their effects should be of interest to anyone trying to test the limits of NEC2 or model configurations that NEC2 isn't able to model accurately.

References

[1] G. J. Burke and A. J. Poggio, *Numerical Electromagnetics Code (NEC)-Method of Moments*, Naval Ocean Syst. Center, San Diego, CA, NOSC Tech. Document 116, Jan. 1981.

[2] A. J. Poggio and E. K. Miller, "Integral Equation Solutions of Three-Dimensional Scattering Problems", *Computer Techniques for Electromagnetics*, R. Mittra editor, Pergamon Press, 1973, Chapter 4.

[3] T. H. Hubing, "Modeling the Electromagnetic Radiation from Electrically Small Sources with Attached Wires", Ph. D. dissertation, North Carolina State University, May, 1988.

[4] A. Ralston and P. Rabinowitz, *A First Course in Numerical Analysis*, McGraw-Hill, 1978, p.432.

[5] N. C. Albertsen et al., "Computation of Radiation from Wire Antennas on Conducting Bodies", *IEEE Transactions on Antennas and Propagation*, vol. AP-22, Mar 1974, pp. 200-206.