

**TECHNICAL REPORT: CVEL-13-044**

**Analysis of the Electromagnetic Shielding Mechanisms of Plane Waves in  
Generally Lossy Materials**

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## Abstract

The well-known equations for electromagnetic shielding effectiveness (EM SE) are based on an assumption that the shielding material is a good conductor. With the introduction of new composite materials consisting of non-conductive substrates loaded with carbon nano-particles, it has become necessary to calculate the EM SE of materials that are not particularly good conductors. Although these materials have moderate conductivities, they can still provide significant levels of shielding. The report develops expressions for calculating the plane-wave shielding effectiveness of linear, homogeneous materials with arbitrary levels of conductive and dielectric loss.

## 1. Introduction

Electromagnetic shielding is often an important requirement for protecting susceptible electronics from electromagnetic coupling [1, 2]. Advances in nanotechnology are producing new engineering materials that offer advantages such as low density, corrosion-resistance, high flexibility, and better processability [1, 3-6]. However, a proper assessment of the shielding performance of these materials can be difficult, due to the intrinsic complexities of the shielding measurements, which require specific sample sizes and shapes, and employ relatively expensive and specialized measuring equipment/facilities. Thus, researchers often rely on electromagnetic shielding effectiveness (EM SE) predictions based on the equations that assume the shielding materials have a high conductivity. Unfortunately, many composite materials have moderate conductivities that are not consistent with the assumptions used to derive these equations. An analysis of the shielding mechanisms in a moderately lossy material can be useful for analyzing these composite materials. In this report a derivation of the equations for the shielding effectiveness of generally lossy materials is presented. The equations presented allow the shielding effectiveness to be divided into two components: a reflection loss and an absorption loss.

## 2. Shielding Mechanisms for the Generally Lossy Case

For linear, isotropic, and generally lossy materials, the AC electrical transport has two components: (1) the free electron/hole transport brought about by the conductivity,  $\sigma = \sigma' - j\sigma''$ , and (2) the bound electron dielectric displacement implied in the permittivity,  $\varepsilon = \varepsilon' - j\varepsilon''$  [2]. In this report,  $j = \sqrt{-1}$ . The real component of permittivity,  $\varepsilon'$ , is related to the level of polarization that the applied electric field confers to the molecules and atoms, whereas the imaginary component,  $\varepsilon''$ , to losses associated with the dielectric damping that the bound electrons in the dipoles undergo due to the varying electric field at an angular frequency,  $\omega$  [7]. Hence, the current density,  $J$ , set by an electric field,  $E$ , will be  $J = j\omega \left( \varepsilon' - j\varepsilon'' - j\frac{\sigma'}{\omega} + \frac{\sigma''}{\omega} \right) E$  [2]. However, the imaginary conductivity (or susceptibility),  $\sigma''$ , is generally negligible at frequencies below 300 GHz, so we will assume that  $\sigma' = \sigma_{DC} = \sigma$  [2]. Thus,  $J = j\omega \left( \varepsilon' - j \left( \varepsilon'' + \frac{\sigma}{\omega} \right) \right) E$ , where the term  $\left( \varepsilon'' + \frac{\sigma}{\omega} \right)$  linearly scales with that of the “total effective conductivity” [8].

The intrinsic impedance of a material,  $\eta$ , is,

$$\eta = \sqrt{\frac{\mu}{\varepsilon' - \left( \varepsilon'' + \frac{\sigma}{\omega} \right) j}} \quad (1)$$

where  $\mu$  is the magnetic permeability of the material.

For a generally lossy medium, the expression for the attenuation constant,  $\alpha$ , as a function of the electrical, dielectric and magnetic losses, can be derived from the propagation constant,  $\gamma$ , definition [1]:

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \quad (2)$$

Thus, combining (2) with the definition of complex permittivity:

$$\gamma^2 = j\omega\mu(\sigma + j\omega(\varepsilon' - j\varepsilon'')) = j\omega\mu\{(\sigma + \omega\varepsilon'') + j\omega\varepsilon'\} \quad (3)$$

The propagation constant is also defined in terms of its complex components, i.e., the attenuation,  $\alpha$ , and phase,  $\beta$ , constants as,

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) = (\alpha + j\beta)^2 \quad (4)$$

Expanding the right term in (4):

$$(\alpha + j\beta)^2 = \alpha^2 + j2\alpha\beta - \beta^2 = (\alpha^2 - \beta^2) + j2\alpha\beta \quad (5)$$

Then,

$$(\alpha^2 - \beta^2) + j2\alpha\beta = j\omega\mu(\sigma + \omega\varepsilon'') - \omega^2\mu\varepsilon' \quad (6)$$

Thus, equating the corresponding terms in (6), the following system of simultaneous equations is obtained,

$$2\alpha\beta = \omega\mu(\sigma + \omega\varepsilon'') \quad (7-a)$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon' \quad (7-b)$$

Solving for  $\beta$  in (7-a),

$$\beta = \frac{\omega\mu(\sigma + \omega\varepsilon'')}{2\alpha} \quad (8)$$

Substituting (8) in (7-b),

$$\alpha^2 - \left[ \frac{\omega\mu(\sigma + \omega\varepsilon'')}{2\alpha} \right]^2 = -\omega^2\mu\varepsilon' \quad (9)$$

$$\alpha^4 + \omega^2\mu\varepsilon'\alpha^2 - \left[ \frac{\omega\mu(\sigma + \omega\varepsilon'')}{2} \right]^2 = 0 \quad (10)$$

Solving for  $\alpha^2$ , using the quadratic formula,

$$\alpha^2 = \frac{-\omega^2\mu\varepsilon' + \sqrt{(\omega^2\mu\varepsilon')^2 + 4\left[\frac{\omega\mu(\sigma + \omega\varepsilon'')}{2}\right]^2}}{2} \quad (11)$$

$$\alpha^2 = \frac{-\omega^2\mu\varepsilon' + \sqrt{(\omega^2\mu\varepsilon')^2 + [\omega\mu(\sigma + \omega\varepsilon'')]^2}}{2} \quad (12)$$

$$\alpha^2 = -\frac{\omega^2\mu\varepsilon'}{2} + \frac{1}{2}\omega\mu\sqrt{\omega^2\varepsilon'^2 + [(\sigma + \omega\varepsilon'')]^2} \quad (13)$$

$$\alpha^2 = -\frac{\omega^2\mu\varepsilon'}{2} + \frac{1}{2}\omega^2\mu\varepsilon'\sqrt{1 + \left[\frac{(\sigma + \omega\varepsilon'')}{\omega\varepsilon'}\right]^2} \quad (14)$$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon'}{2} \left( -1 + \sqrt{1 + \left[\frac{(\sigma + \omega\varepsilon'')}{\varepsilon'}\right]^2} \right) \quad (15)$$

Finally, solving for the real component of  $\alpha$ ,

$$\alpha = \omega \sqrt{\frac{\mu \varepsilon'}{2} \left( \sqrt{1 + \left[ \frac{(\sigma + \varepsilon'')}{\varepsilon'} \right]^2} - 1 \right)} . \quad (16)$$

Figure 1 is a schematic representation of how the plane wave, in terms of electric field, interacts with an isotropic conductive medium with finite thickness. The incident field,  $E_o$ , travels in air with an intrinsic impedance  $\eta_o = 120\pi \Omega$ . When the field strikes the surface, the first reflection,  $E_{R_1}$ , occurs due to the mismatch in impedance, because the wave impedance (ratio of E to H) becomes  $\eta$ . Part of this field is transmitted through the thickness of the specimen,  $t$ , and as it travels, it is attenuated,  $e^{-\gamma t}$  [1]. When the field reaches the second surface, a similar phenomenon takes place, leading to the transmitted field,  $E_{T_1}$ . The part that is reflected back into the material experiences a series of subsequent partial reflections and transmissions while being attenuated along the path.

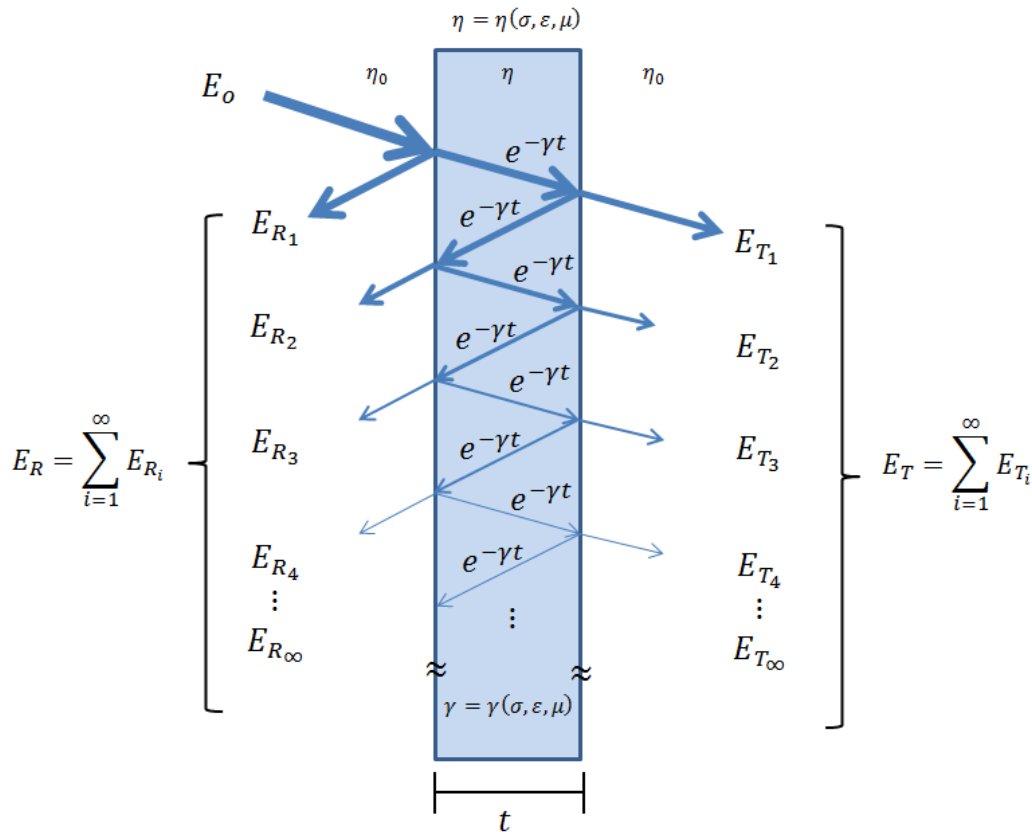


Fig. 1. Schematic representation of the shielding mechanisms in terms of the electric field,  $E$ , in a generally lossy isotropic specimen. For the sake of clarity, the incident plane wave, which propagates perpendicular to the sample surface, has been depicted with oblique incidence, so that the multiple reflections can be readily represented.

Figure 2 displays a matrix-like summary of the definitions for the transmission and reflection coefficients which accounts for how much of the wave is either reflected or transmitted when the wave reaches each surface.  $T_\eta = \frac{2\eta}{\eta_o + \eta}$  and  $\Gamma_\eta = \frac{\eta - \eta_o}{\eta + \eta_o}$  are respectively the transmission and reflection coefficients for the plane wave traveling in the vacuum  $\eta_o$  and striking on a surface of impedance  $\eta$ ;

whilst,  $T_{\eta_o} = \frac{2\eta_o}{\eta_o + \eta}$  and  $\Gamma_{\eta_o} = \frac{\eta_o - \eta}{\eta_o + \eta}$  are respectively the transmission and reflection coefficients for the plane wave traveling in the material with impedance  $\eta$  and re-entering the air  $\eta_o$  [1].

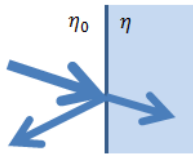
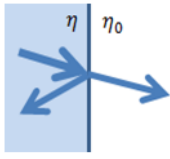
		
<i>Transmission Coefficients:</i>	$T_{\eta} = \frac{2\eta}{\eta_o + \eta}$	$T_{\eta_o} = \frac{2\eta_o}{\eta_o + \eta}$
<i>Reflection Coefficients:</i>	$\Gamma_{\eta} = \frac{\eta - \eta_o}{\eta + \eta_o}$	$\Gamma_{\eta_o} = \frac{\eta_o - \eta}{\eta_o + \eta}$

Fig. 2. Summary of the transmission and reflection coefficients definitions.

Adding the single transmissions,  $E_{T_i}$ , across the specimens, the total transmitted field,  $E_T$ , is obtained:

$$E_T = \sum_{i=1}^{\infty} E_{T_i} = E_{T_1} + E_{T_2} + E_{T_3} + \dots \quad (17)$$

Figure 3 displays only the parts of the wave that are transmitted forward through the barrier, which are the components of the total transmitted field,  $E_T$ . Analyzing the forward transmissions in terms of their attenuation,  $e^{-\gamma t}$ ,

$$E_{T_1} = E_o T_{\eta} e^{-\gamma t} T_{\eta_o} \quad (18)$$

$$E_{T_2} = E_o T_{\eta} e^{-\gamma t} \Gamma_{\eta} e^{-\gamma t} \Gamma_{\eta} e^{-\gamma t} T_{\eta_o} \quad (19)$$

$$E_{T_3} = E_o T_{\eta} e^{-\gamma t} \Gamma_{\eta} e^{-\gamma t} \Gamma_{\eta} e^{-\gamma t} \Gamma_{\eta} e^{-\gamma t} \Gamma_{\eta} e^{-\gamma t} T_{\eta_o} \quad (20)$$

A recurrent pattern for the terms in this mathematical succession can be identified, thus equation (17) becomes:

$$E_T = E_o T_{\eta} e^{-\gamma t} T_{\eta_o} \left\{ 1 + (\Gamma_{\eta} e^{-\gamma t})^2 + (\Gamma_{\eta} e^{-\gamma t})^4 + \dots \right\} = E_{T_1} \left\{ \sum_{i=0}^{\infty} (\Gamma_{\eta} e^{-\gamma t})^{2i} \right\} \quad (21)$$

Equation (21) is an infinite series of a geometric succession of the form:

$$\sum_{j=0}^{\infty} r^j = \frac{(1 - r^{n-1})}{(1 - r)} \quad (22)$$

This series is convergent if  $r < 1$ . Replacing  $r$  with  $(\Gamma_\eta e^{-\gamma t})^2 < 1$ , yields,

$$\sum_{i=0}^n (\Gamma_\eta e^{-\gamma t})^{2i} = \frac{1 - (\Gamma_\eta e^{-\gamma t})^{2n+2}}{1 - (\Gamma_\eta e^{-\gamma t})^2} \quad (23)$$

Taking the limit as  $n \rightarrow \infty$ , (23) becomes:

$$\sum_{i=0}^{\infty} (\Gamma_\eta e^{-\gamma t})^{2i} = \frac{1}{1 - (\Gamma_\eta e^{-\gamma t})^2} \quad (24)$$

Therefore, the total transmitted field is:

$$E_T = \frac{E_{T_1}}{1 - (\Gamma_\eta e^{-\gamma t})^2} \quad (25)$$

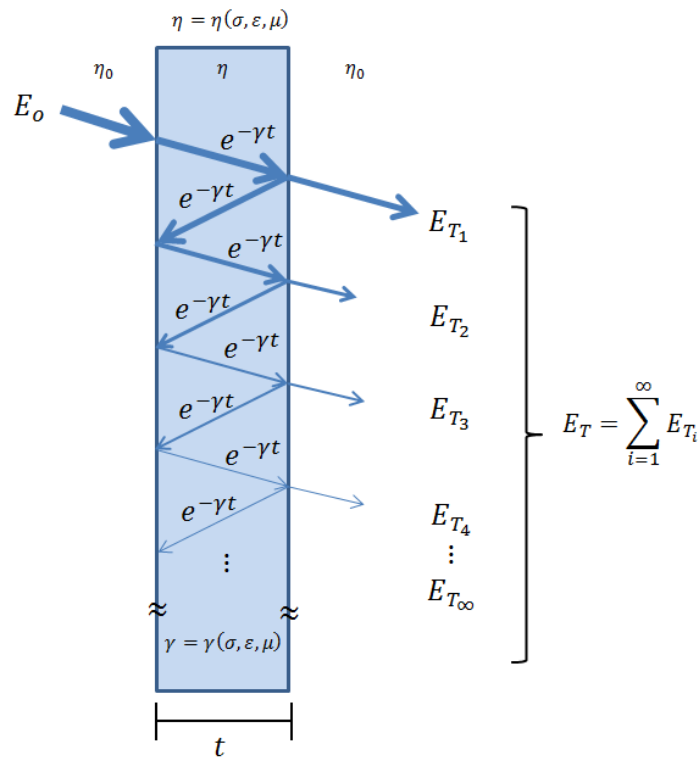


Fig. 3. Representation of the forward transmission of the electric field through the material barrier.

The total reflection,  $E_R$ , that is composed of all the single reflections,  $E_{R_i}$ , that come back from the barrier are represented in Figure 4,

$$E_R = \sum_{i=1}^{\infty} E_{R_i} = E_{R_1} + E_{R_2} + E_{R_3} + \dots, \quad (26)$$

where the first four terms in the series are,

$$E_{R_1} = E_o \Gamma_{\eta_0} \quad (27)$$

$$E_{R_2} = E_o T_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} T_{\eta_o} \quad (28)$$

$$E_{R_3} = E_o T_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} T_{\eta_o} \quad (29)$$

$$E_{R_4} = E_o T_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} T_{\eta_o} . \quad (30)$$

Once again, a recurrent pattern for the terms in the succession can be identified, thus equation (26) becomes:

$$E_R = E_o \Gamma_{\eta_o} + E_o T_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} T_{\eta_o} \left\{ 1 + (\Gamma_\eta e^{-\gamma t})^2 + (\Gamma_\eta e^{-\gamma t})^4 + \dots \right\} = E_{R_1} + E_{R_2} \left\{ \sum_{i=0}^{\infty} (\Gamma_\eta e^{-\gamma t})^{2i} \right\} \quad (31)$$

which is also a convergent geometric succession. Thus, the total reflected electric field becomes:

$$E_R = E_{R_1} + \frac{E_{R_2}}{1 - (\Gamma_\eta e^{-\gamma t})^2} \quad (32)$$

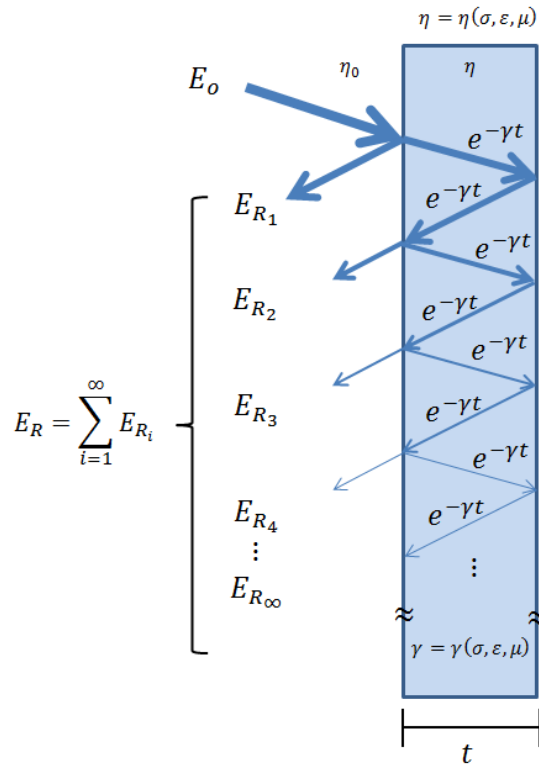


Fig. 4. Representation of the backward reflection of the electric field from the material barrier.

Thus, the total transmitted field,  $E_T$ , and total reflected field,  $E_R$ , can be expressed as:

$$E_T = \sum_{i=1}^{\infty} E_{T_i} = \frac{E_{T_1}}{1 - (\Gamma_\eta e^{-\gamma t})^2} = \frac{E_o T_\eta e^{-\gamma t} T_{\eta_o}}{1 - (\Gamma_\eta e^{-\gamma t})^2} \quad (33)$$

$$E_R = \sum_{i=1}^{\infty} E_{R_i} = E_{R_1} + \frac{E_{R_2}}{1 - (\Gamma_\eta e^{-\gamma t})^2} = E_o \Gamma_{\eta_o} + \frac{E_o T_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} T_{\eta_o}}{1 - (\Gamma_\eta e^{-\gamma t})^2} \quad (34)$$



The wave power density is proportional to the square of the electric field, i.e.,  $P \propto E^2$  [1]. The transmittance,  $T$ , and reflectance,  $R$ , are defined as the ratio of the transmitted power density,  $P_T$ , and reflected power density,  $P_R$ , to that of the incident power density,  $P_o$  [1, 6], and can be expressed in terms of their corresponding scattering parameters,  $S_{21}$  and  $S_{11}$ , respectively:

$$T = |S_{21}|^2 = \left| \frac{P_T}{P_o} \right| = \left| \frac{E_T}{E_o} \right|^2 \quad (35)$$

$$R = |S_{11}|^2 = \left| \frac{P_R}{P_o} \right| = \left| \frac{E_R}{E_o} \right|^2 \quad (36)$$

Dividing equations (33) and (34) by the incident electric field  $E_o$  and using the transmission and reflection coefficient substitutions:

$$\frac{E_T}{E_o} = \frac{T_\eta e^{-\gamma t} \Gamma_\eta \eta_o}{1 - (\Gamma_\eta e^{-\gamma t})^2} = \frac{\frac{2\eta}{\eta_o + \eta} e^{-\gamma t} \frac{2\eta_o}{\eta_o + \eta}}{1 - \left( \frac{\eta - \eta_o}{\eta + \eta_o} \right)^2 e^{-2\gamma t}} \quad (37)$$

$$\frac{E_R}{E_o} = \Gamma_\eta + \frac{T_\eta e^{-\gamma t} \Gamma_\eta e^{-\gamma t} \Gamma_\eta \eta_o}{1 - (\Gamma_\eta e^{-\gamma t})^2} = \frac{\eta_o - \eta}{\eta_o + \eta} + \frac{\frac{2\eta}{\eta_o + \eta} e^{-\gamma t} \frac{\eta - \eta_o}{\eta + \eta_o} e^{-\gamma t} \frac{2\eta_o}{\eta_o + \eta}}{1 - \left( \frac{\eta - \eta_o}{\eta + \eta_o} \right)^2 e^{-2\gamma t}} \quad (38)$$

Based on the definitions of total shielding (dB) [6],

$$EM SE_{dB} = -10 \log T = -10 \log \left| \frac{E_T}{E_o} \right|^2 \quad (39)$$

and reflective shielding (dB) [6],

$$EM SE_{R, dB} = -10 \log(1 - R) = -10 \log \left( 1 - \left| \frac{E_R}{E_o} \right|^2 \right) \quad (40)$$

and respectively substituting (37) and (38) into (39) and (40), the following expressions are obtained for the total shielding and reflective shielding components, respectively:

$$EM SE_{dB} = -10 \log \left| \left\{ \frac{\frac{4\eta\eta_o}{(\eta_o + \eta)^2} e^{-\gamma t}}{1 - \left( \frac{\eta - \eta_o}{\eta + \eta_o} \right)^2 e^{-2\gamma t}} \right\}^2 \right| \quad (41)$$

$$EM SE_{R, dB} = -10 \log \left| 1 - \left\{ \frac{\eta_o - \eta}{\eta_o + \eta} + \frac{\frac{4\eta\eta_o}{(\eta_o + \eta)^2} e^{-2\gamma t} \frac{\eta - \eta_o}{\eta + \eta_o}}{1 - \left( \frac{\eta - \eta_o}{\eta + \eta_o} \right)^2 e^{-2\gamma t}} \right\}^2 \right| \quad (42)$$

The absorptive component of shielding is then readily obtained as:

$$EM SE_{A, dB} = EM SE_{dB} - EM SE_{R, dB} \quad (43)$$

Moreover, since only magnitudes are needed in these calculations, and for a complex number  $|e^Z| = |e^{a+bj}| = |e^a e^{bj}| = e^a$ , the terms  $|e^{-2\gamma t}|$  and  $|e^{-\gamma t}|$  in these equations can be replaced by  $|e^{-2\alpha t}|$  and  $|e^{-\alpha t}|$ , respectively. Therefore, equations (41) and (42) along with the intrinsic impedance  $\eta$  (1) and attenuation constant  $\alpha$  (16) expressions can be used to estimate the plane wave shielding of an intermediately lossy, isotropic and homogeneous material of thickness,  $t$ , based on the electrical conductivity,  $\sigma$ , complex permittivity,  $\epsilon$ , and magnetic permeability,  $\mu$ , over the frequency range of interest.

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### 3. Conclusion

An analysis of the plane-wave propagation in generally lossy materials was used to develop a set of equations to calculate the EM SE and its components from the fundamental transport properties of the material. The derivation accounts for the multiple reflections that occur in materials with moderate-to-low loss. Expressions for the attenuation constant and intrinsic impedance that account for the electrical, magnetic and dielectric properties were also derived. Thus, a set of closed-form equations for the estimating the reflection loss, absorption loss, and overall EM SE are proposed in this report. These equations enable quick and accurate estimations of the EM SE of diverse engineering materials based on their electrical properties.

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