Finite Element Modeling of Patch Antenna and Cavity Sources

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Abstract: This paper examines two different approaches that can be used to model patch antennas and cavities fed by a coaxial cable. The probe model represents the feed as a current filament along the center conductor of the coaxial cable. The coaxial-cable model enforces the analytical field distribution at the cable opening. These two models have been implemented in a hybrid FEM/MoM code. A power bus structure and a cavity geometry with coaxial-cable feeds are investigated. Numerical results obtained for these two examples are compared with measurements. It is shown that the probe model should only be applied to electrically short feeding structures, while the coaxial cable model can be applied to both electrically short and electrically long feeding structures.

I. INTRODUCTION

At high frequencies, the power bus structure in a printed circuit board can behave like a microstrip patch antenna. Radiation from the power bus can be a significant problem above 1 GHz for boards tens of square inches or larger. Shielding enclosures are also of interest in EMI modeling because cavity resonances can result in radiated emission peaks.

Integral equation (IE) methods are the most widely used numerical methods for modeling patch antennas. However, IE methods are generally formulated based on approximate Green's functions for specific geometries, or based on the assumption of an infinite substrate and/or an infinite return plane. These assumptions can lead to inaccuracies, especially in EMI models because finite return planes will resonate at certain frequencies, which can have a big impact on the radiated fields. Hybrid FEM/MoM models employs an integral equation to model the exterior equivalent problem, and the finite element method to model the interior problem. These two techniques are coupled by enforcing boundary conditions [1]. Full-wave FEM/MoM modeling codes do not make assumptions about the size or shape of the ground plane and substrate and thus can generate accurate results for modeling patch antennas or printed circuit boards.

It is important to properly represent sources when a hybrid FEM/MoM method is employed to model patch antennas or cavities. When making measurements, these structures are often driven with a coaxial cable. The outer conductor is bonded to one metal wall and the center conductor extends through to the opposite wall. The reference plane of measurement is normally calibrated to the cable opening, where the center conductor begins to extend beyond the outer conductor. The sources are normally modeled with the finite element portion of the hybrid FEM/MoM method. This paper examines two different approaches that can be used to model sources. The probe model represents the feed structure as a current filament along the center conductor. The coaxial-cable model enforces the analytical field distribution at the cable opening. These two models has been implemented in EMAP5, a hybrid FEM/MoM code developed at University of Missouri-Rolla [2]. Numerical results and measurements are presented to compare and validate these two models.

II. THE PROBE MODEL

Figure 1 illustrates the cross-section of a coaxial cable, which has a center conductor, an outer conductor, and dielectric filling between the two conductors. The probe model uses an impressed electric current to model the source [1], [3]. An infinitesimally thin current filament is assumed to flow along the portion of the center conductor that extends between the planes of a patch antenna or into the interior of a cavity. The perfect-electric-conductor (PEC) boundary condition along the center conductor is not enforced. The cable opening in the wall of the patch antenna or cavity is modeled as a PEC. This approach has been adopted by Pozar in the context of MoM [4], and by Jin and Volakis in the context of FEM [5].

The weak form of the vector Helmholtz equation is as follows,

$$\int_{V_2} \left[j\omega\varepsilon_0\varepsilon_r \mathbf{E(r)} \bullet \mathbf{w(r)} + \frac{(\nabla\times\mathbf{E(r)}) \bullet (\nabla\times\mathbf{w(r)})}{j\omega\mu_0\mu_r} \right] dV$$

$$= \int_{S_2} (\hat{n}\times\mathbf{H(r)}) \bullet \mathbf{w(r)}\, dS$$

$$- \int_{V_2} \left[\mathbf{J}^{int}(\mathbf{r}) + \frac{1}{j\omega\mu_0\mu_r}\nabla\times\mathbf{M}^{int}(\mathbf{r}) \right] \bullet \mathbf{w(r)}\, dV \tag{1}$$

An impressed current source along the z-axis can be expressed as,

$$\mathbf{J}^{int} = I_I\, \delta(x-x_f)\delta(y-y_f)\hat{z} \tag{2}$$

where (x_f, y_f) specifies its position in the Cartesian system, I_I denotes the electric current magnitude, and $\delta(x)$ is the Dirac delta function. The source term in the FEM equation is then given by,

$$\left[g^{int}\right]^e = I_I\, l_I \tag{3}$$

where l_1 is the edge length. The input impedance can be calculated as follows,

$$Z_{in} = \frac{V_1}{I_1} = \frac{E_1\, l_1}{I_1} \tag{4}$$

where E_1 is the electric field along the source edge. It is a common practice to model the source using one edge. However, it is possible to model the source using several edges. In that case, the voltage along the probe is the sum of the voltage along the source edges.
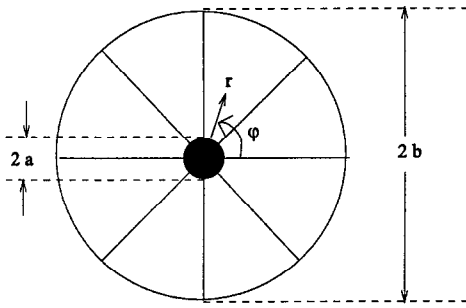

Figure 1. Cross-section of a coaxial cable

III. THE COAXIAL-CABLE MODEL

The probe model generates satisfactory results for electrically short feeding structures [4], [5]. However, it has been criticized for its simple assumptions. First, the current is uniformly distributed along an infinitesimal edge. Second, the PEC boundary condition along the center conductor is not strictly enforced. Third, the cable opening is modeled as PEC. All of these assumptions are unrealistic. A more accurate model was proposed by Aberle and Pozar. Their model includes the effects of the finite center conductors and current variations along the probe in the context of MoM [6].

This paper considers a coaxial cable model for FEM proposed by Gong and Volakis [7]. This model assumes a TEM mode field distribution at the cable opening,

$$\mathbf{E} = \frac{e_0}{r}\hat{r}, \tag{5}$$

$$\mathbf{H} = \frac{h_0}{r}\hat{\phi} \tag{6}$$

where e_0 and h_0 are parameters satisfying

$$e_0 = \frac{I_1 Z_{c1}}{2\pi\sqrt{\varepsilon_{rc}}}(1+\Gamma), \tag{7}$$

$$h_0 = \frac{I_1}{2\pi}(1-\Gamma), \tag{8}$$

$$h_0 = -\frac{\sqrt{\varepsilon_{rc}}}{Z_{c1}}e_0 + \frac{I_1}{\pi} \tag{9}$$

where I_1 is the *incident* current in the cable, ε_{rc} is the relative permittivity of the dielectric inside the cable, Γ is the reflection coefficient, and Z_{c1} is the characteristic impedance of the cable. The equipotential condition is enforced at the cable opening as follows,

$$\Delta V = E_i(b-a) = e_0\ln(\frac{b}{a}) \tag{10}$$

$$E_i \in \{\text{unknowns at the cable junction}, i = 1,...,N\}$$

where a and b are the radii of the center and outer conductors, N is the total number of unknowns on the cable interface. In [7], the cable excitation was derived from the FEM formulation based on the variational method, which is equivalent to the FEM formulation based on the weak form and Galerkin's method. Analytical evaluation of the functional f_i^c at the cable opening S_c is given by [7],

$$f_i^c = \int_{S_c} (\hat{n}\times\mathbf{H(r)}) \bullet \mathbf{E(r)}\, dS = C_i E_i - f_i \tag{11}$$

where

$$C_i = \frac{2\pi\sqrt{\varepsilon_{rc}}(b-a)^2}{N\ln(b/a)\eta_0}, \tag{12}$$

$$f_i = \frac{2(b-a)I_1}{N}. \tag{13}$$

Differentiation is performed on Eq. (11) to minimize the functional. Thus, C_i is added to the diagonal entries corresponding to the cable edges in the FEM matrix, and f_i is added to the FEM source entries corresponding to the cable edges. After E_i is solved, the input admittance is given by,

$$Y_{in} = \frac{2I_1}{V_1} - \frac{1}{Z_{c1}}, \tag{14}$$

where V_1 is the voltage along the cable edges. In the coaxial-cable model, the PEC boundary condition along the center conductor is strictly enforced. The dielectric opening at the coaxial cable is modeled using FEM. Therefore this model does not have the shortcomings of the probe model.

IV. NUMERICAL AND EXPERIMENTAL RESULTS

Two geometries are investigated to validate and compare the two source models. Figure 2 shows the geometry of a patch antenna. A coaxial cable feeds the structure at the location designated as Port 1. An impedance analyzer is used to measure the input impedance at the cable opening. Numerical results are obtained using EMAP5. Figure 3 shows the calculated and measured magnitude of the input impedance. Figure 4 shows the input reactance. It is evident that the measurements and the numerical results, obtained using the probe model and the coaxial cable model, agree very well up to 1.0 GHz. The discrepancies after 1.0 GHz may be due to the difficulty of measuring the input impedance at high frequencies. Both the probe and coaxial-cable models generate satisfactory results for modeling this thin feed structure.
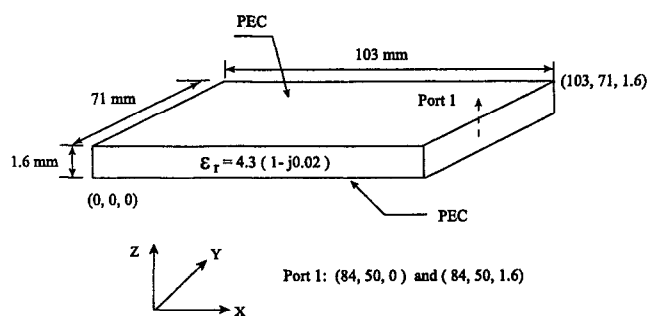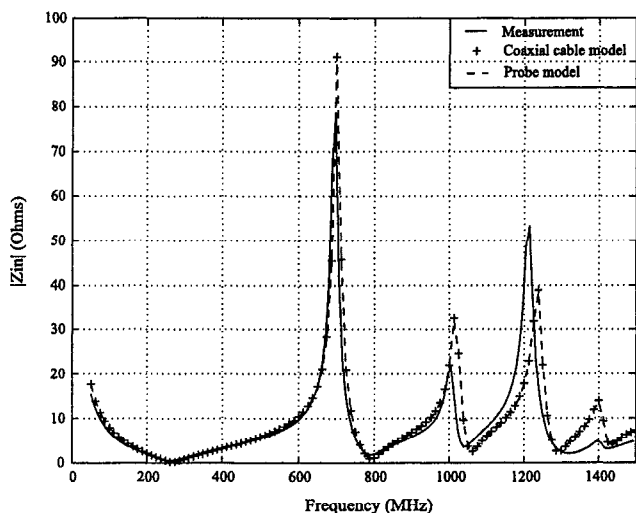

Figure 2. Geometry of a patch antenna

Figure 3. Magnitude of the input resistance of the patch antenna
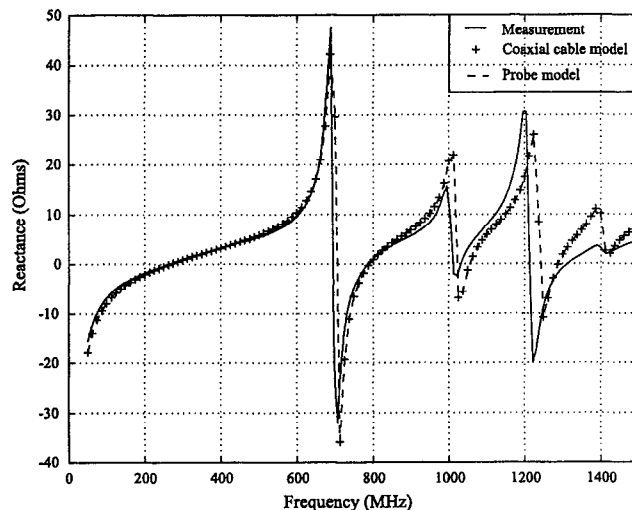

Figure 4. The input reactance of the patch antenna

The second geometry is a PEC cavity shown in Figure 5. The cavity is fed by a coaxial cable. A 13-cm wire linking the center conductor of the cable is connected to the bottom of the cavity through a 47-ohm resistor. This problem was investigated by Li *et al.* using an FDTD technique [8]. Both the probe and coaxial-cable models are applied to model this geometry using a same mesh file. In the probe model, a 1.0-cm segment of the wire connecting to the coaxial cable is modeled as a current source. The rest of the 12-cm wire is modeled as PEC. In the coaxial cable model, the entire length of the wire is modeled as PEC thus the boundary condition is strictly enforced. The power dissipated by the cavity is calculated as follows,

$$P_{dissipated} = \frac{V^2}{2|Z_{in} + Z_s|^2} \text{Real}(Z_{in}) \qquad (15)$$

where Z_{in} is the input impedance of the cavity, V is the peak value of the source and Z_s is the source impedance of 50 ohms. The maximum power is dissipated by the cavity when $Z_{in} = Z_s$,

$$P_{max} = \frac{V^2}{8Z_s} \qquad (16)$$

Therefore, the normalized dissipated power is given by,

$$P_{normalized} = \frac{4Z_s}{|Z_{in} + Z_s|^2} \text{Real}(Z_{in}) \qquad (17)$$

Because this geometry is a bounded problem, only the FEM portion of the hybrid FEM/MoM code is used. Figure 6 shows the measured and numerical results. The probe model failed to generate satisfactory results while good agreement has been achieved between the measurement and numerical results obtained using the coaxial cable model.
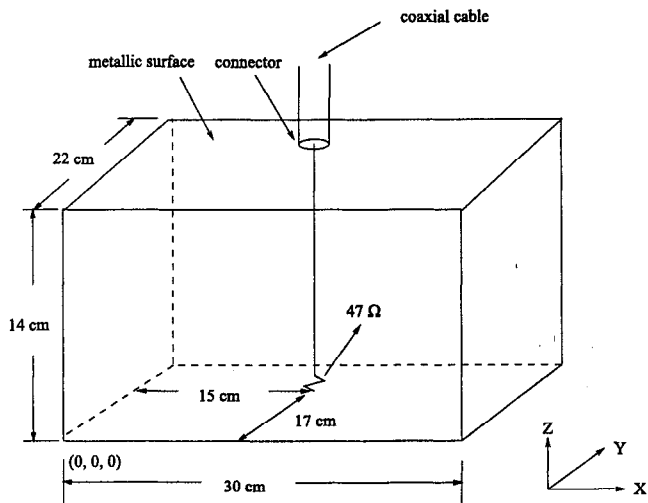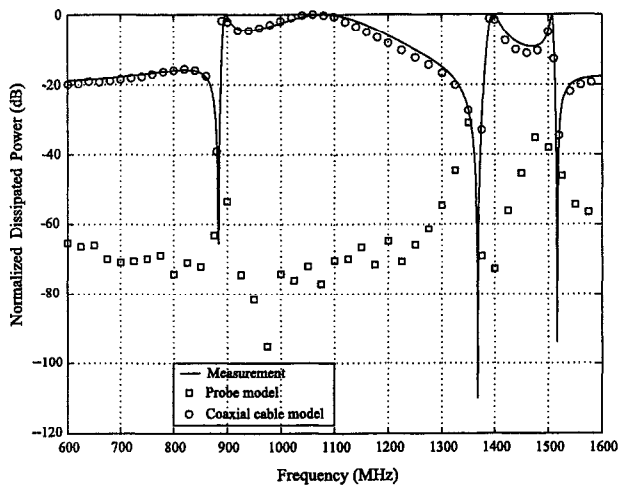
Figure 5. Geometry of a cavity

Figure 6. Normalized power dissipated in the cavity

V. CONCLUSION

Two different approaches can be employed to model coaxial cable feeds used in patch antennas and cavities. The probe model is easy to implement and is very effective for modeling thin feeding structures. The probe model has three limitations. First, the current is uniformly distributed along an infinitesimal edge (i.e. the radius of the center conductor is not considered). Second, the PEC boundary condition along the center conductor is not enforced. Third, the cable opening is modeled as PEC.

The coaxial model assumes a TEM field distribution at the cable opening. The boundary term is analytically evaluated and the equipotential condition is enforced to extract the

cable excitation. The boundary conditions along the center conductor and at the cable opening are strictly enforced.

Numerical results demonstrate that the probe model generates satisfactory results for the patch antenna with a short feed considered here. There are no apparent differences between the results obtained using the probe and coaxial-cable models. However, the probe model is much simpler to implement because it does not require meshing at the cable opening. For long feeding structures, the probe model fails to work but the coaxial cable model generates satisfactory results.

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