EMC APPLICATIONS OF EMAP-2: A 3D FINITE ELEMENT MODELING CODE

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Abstract

EMAP-2 is a 3D finite element modeling code that can be used to model simple printed circuit board configurations. It is a true full-wave field solver designed to be used for education or research. EMAP-2 is written in the *C* programming language and the source code is available at no charge. This paper describes the code, its capabilities, and its limitations. Two examples illustrate how the code may be applied to problems of particular interest to EMC engineers.

Introduction

EMAP-2 (ElectroMagnetic Analysis Program, version 2) is a three-dimensional finite element modeling code that can solve for the field distribution in a variety of bounded, 3-dimensional geometries. It is relatively easy to learn to use and it is distributed in source code form. Although EMAP was designed as an education and research tool, it also has practical applications for the EMC engineer.

EMAP-2 is not intended to compete with commercial finite element modeling codes. It does not have a sophisticated mesh generator, graphical output, or unlimited technical support. Its primary strengths are ease-of-use, modest resource requirements, and accurate modeling of simple three-dimensional configurations over a wide range of frequencies.

EMAP-2 employs the Galerkin finite element formulation described in papers by Paulsen and Lynch [1,2]. It is written in the *C* programming language and can be compiled and run on PCs, workstations, or mainframes. There is also a parallel version of EMAP that is designed to run on the Intel hypercube platform. EMAP-2 is available by anonymous ftp from *emclab.ee.umr.edu*. It can be freely copied or distributed in unmodified form. A description and comparison of the EMAP-2 code with EMAP-1 (a similar code employing a variational method) can be found in [3].

Code Structure

EMAP-2 is a nodal finite element modeling code that uses first-order tetrahedral elements. Nodal finite element techniques generate smaller solution matrices and are easier to work with in terms of input and output than vector finite element techniques. On the other hand, nodal techniques require special handling at the interface between two media, which is unnecessary when using vector finite element techniques employing edge elements.

EMAP-2's finite element mesh is defined on a cartesian grid using brick-shaped elements (hexahedra). The EMAP code automatically subdivides each hexahedron into five tetrahedra as shown in Figure 1 [4]. Linear functions of electric field strength with unknown coefficients at each node are the basic building blocks of the finite element solution. A matrix equation relating constants derived from the fixed (or constrained) node coefficients to linear functions of the unconstrained node coefficients is derived using the procedure outlined in the following section.

A flowchart outlining the basic structure of the EMAP-2 code is provided in Figure 2. Specific subroutines are assigned to each task in the chart. A brief description of each subroutine is provided below:

- Read_InputData(): Reads the data input file and initializes variables.
- Count_HexaHedron_Number(): Calculates the number of hexahedra and number of global nodes.
- Assign_Global_Coordinates(): Assigns to each node a global coordinate (x,y,z) and stores the result in the array MeshNodeCord[].
- Assign_Hexahedron_NodeNumber(): Assigns a unique global node number to each vertex of every hexahedra and stores the result in the array HexHed[].
- HexaHedra_SubDivision_1(): Divides a hexahedra into 5 tetrahedra and assigns node numbers based on connectivity. Node numbers are stored in the array tetmod[].
- Find_Volume_Determinant(): Calculates the volume determinant of the specified tetrahedron
- Find_CoFactor(): Calculates the cofactors of the specified tetrahedron matrix
- TetraHedron_Submatrix(): Calculates the tetrahedral matrix coefficients (12 x 12 matrix).
- HexaHedron_Matrix(): Assembles the hexahedron matrix.
- HexaHedron_Link_1(): Defines how the hexahedra nodes are connected.
- Find_Global_Matrix(): Assembles the global matrix exploiting the sparsity to reduce the memory required.
- Partition_Global_Matrix(): Partitions the global matrix and applies boundary conditions. Prepares a matrix equation of the form Ax=B.
- Band_Matrix_Solver(): Solves the linear system of equations. Currently this is done using an LU-decomposition technique optimized for sparse, banded matrices..
- Print_Output(): Formats and prints the electric field coefficients at each node.

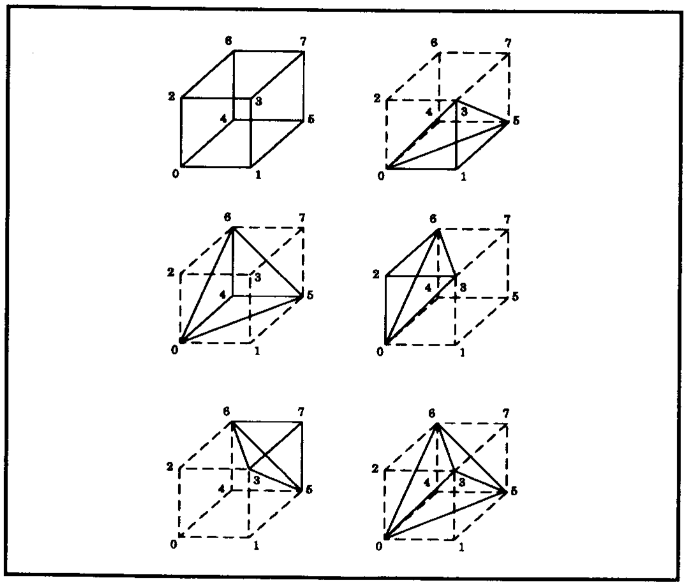

Figure 1: Division of hexahedron into 5 tetrahedra

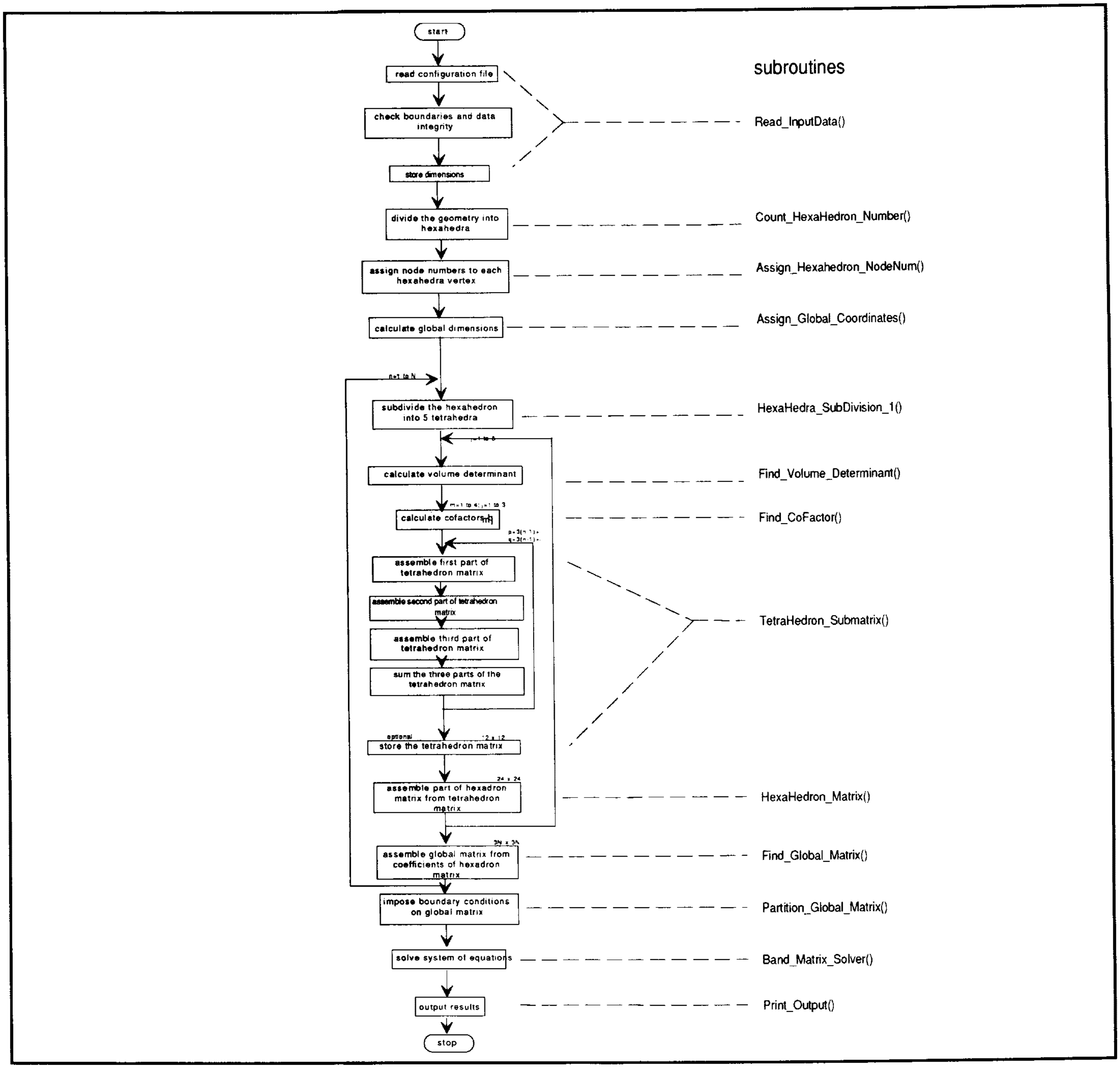
start
read configuration file
check boundaries and data integrity
store dimensions
divide the geometry into hexahedra
assign node numbers to each hexahedra vertex
calculate global dimensions
n=1 to N
subdivide the hexahedron into 5 tetrahedra
i=1 to 5
calculate volume determinant
m=1 to 4; j=1 to 3
calculate cofactors h_{mj}
p=3(n-1)+
q=3(n-1)+
assemble first part of tetrahedron matrix
assemble second part of tetrahedron matrix
assemble third part of tetrahedron matrix
sum the three parts of the tetrahedron matrix
optional
12 x 12
store the tetrahedron matrix
24 x 24
assemble part of hexadron matrix from tetrahedron matrix
3N x 3N
assemble global matrix from coefficients of hexadron matrix
impose boundary conditions on global matrix
solve system of equations
output results
stop
subroutines
Read_InputData()
Count_HexaHedron_Number()
Assign_Hexahedron_NodeNum()
Assign_Global_Coordinates()
HexaHedra_SubDivision_1()
Find_Volume_Determinant()
Find_CoFactor()
TetraHedron_Submatrix()
HexaHedron_Matrix()
Find_Global_Matrix()
Partition_Global_Matrix()
Band_Matrix_Solver()
Print_Output()

Figure 2: Flowchart for the EMAP-2 code

EMAP-2 Formulation

EMAP-2 does not employ a variational method like most finite element modeling techniques. Instead it applies a Galerkin method to solve a modified form of the vector Helmholtz equation introduced by Paulsen and Lynch [3]. The modified form includes a term which explicitly enforces the condition that $\nabla\cdot\boldsymbol{E}$ must be 0 in a homogeneous medium,

$$\nabla\times\left(\frac{1}{j\omega\mu}\nabla\times\boldsymbol{E}\right)-\nabla\left(\frac{1}{j\omega\mu\varepsilon}\nabla\cdot\varepsilon\boldsymbol{E}\right)+j\omega\varepsilon\boldsymbol{E}=0 \qquad (1)$$

The expanded weak form of this equation used by the EMAP-2 code can be written as,

$$<\frac{1}{j\omega\mu}(\nabla\times\boldsymbol{E})\times\nabla\varphi_i> + <\frac{1}{j\omega\mu\varepsilon}(\nabla\cdot\varepsilon\boldsymbol{E})\,\nabla\varphi_i> + <j\omega\varepsilon\boldsymbol{E}\,\varphi_i> = 0 \qquad (2)$$

where < and > indicate integration over the problem domain. φ_i are the basis functions associated with the Galerkin procedure (in this case the linear field distribution associated with each finite element). Expanding **E** in terms of weighting functions φ_j such that,

$$\boldsymbol{E}=\sum_{j=1}^{N}E_j\,\varphi_j \qquad (3)$$

and substituting for the electric field in Equation 1, results in an expression that can be written in matrix form as $A_{ij}\,E_j = 0$ where,

$$A_{ij}=\begin{bmatrix} <\frac{\delta\varphi_j}{\delta x}\frac{\delta\varphi_i}{\delta x}+\frac{\delta\varphi_j}{\delta y}\frac{\delta\varphi_i}{\delta y}+\frac{\delta\varphi_j}{\delta z}\frac{\delta\varphi_i}{\delta z}-k^2\varphi_j\varphi_i> & <-\frac{\delta\varphi_j}{\delta x}\frac{\delta\varphi_i}{\delta y}+\frac{\delta\varphi_j}{\delta y}\frac{\delta\varphi_i}{\delta x}> & <-\frac{\delta\varphi_j}{\delta x}\frac{\delta\varphi_i}{\delta z}+\frac{\delta\varphi_j}{\delta z}\frac{\delta\varphi_i}{\delta x}> \\ <-\frac{\delta\varphi_j}{\delta y}\frac{\delta\varphi_i}{\delta x}+\frac{\delta\varphi_j}{\delta x}\frac{\delta\varphi_i}{\delta y}> & <\frac{\delta\varphi_j}{\delta x}\frac{\delta\varphi_i}{\delta x}+\frac{\delta\varphi_j}{\delta y}\frac{\delta\varphi_i}{\delta y}+\frac{\delta\varphi_j}{\delta z}\frac{\delta\varphi_i}{\delta z}-k^2\varphi_j\varphi_i> & <-\frac{\delta\varphi_j}{\delta y}\frac{\delta\varphi_i}{\delta x}+\frac{\delta\varphi_j}{\delta z}\frac{\delta\varphi_i}{\delta y}> \\ <-\frac{\delta\varphi_j}{\delta z}\frac{\delta\varphi_i}{\delta x}+\frac{\delta\varphi_j}{\delta x}\frac{\delta\varphi_i}{\delta z}> & <-\frac{\delta\varphi_j}{\delta z}\frac{\delta\varphi_i}{\delta y}+\frac{\delta\varphi_j}{\delta y}\frac{\delta\varphi_i}{\delta z}> & <\frac{\delta\varphi_j}{\delta x}\frac{\delta\varphi_i}{\delta x}+\frac{\delta\varphi_j}{\delta y}\frac{\delta\varphi_i}{\delta y}+\frac{\delta\varphi_j}{\delta z}\frac{\delta\varphi_i}{\delta z}-k^2\varphi_j\varphi_i> \end{bmatrix} \tag{6}$$

The entries in this matrix can be expressed as functions of the tetrahedral element volume (V) and the corresponding cofactors (bl,cl,dl) [6] resulting in the expression below,

$$A_{ij}=\begin{bmatrix} \frac{b_lb_l^T+c_lc_l^T+d_ld_l^T}{36V}-k^2V\begin{cases}1/10 & i=j\\ 1/20 & i\neq j\end{cases} & \frac{-b_lc_l^T+c_lb_l^T}{36V} & \frac{-b_ld_l^T+d_lb_l^T}{36V} \\ -\frac{c_lb_l^T+b_lc_l^T}{36V} & \frac{b.b_l^T+c_lc_l^T+d_ld_l^T}{36V}-k^2V\begin{cases}1/10 & i=j\\ 1/20 & i\neq j\end{cases} & \frac{-c_ld_l^T+d_lc_l^T}{36V} \\ \frac{-d_lb_l^T+b_ld_l^T}{36V} & \frac{-d_lc_l^T+c_ld_l^T}{36V} & \frac{b_lb_l^T+c_lc_l^T+d_ld_l^T}{36V}-k^2V\begin{cases}1/10 & i=j\\ 1/20 & i\neq j\end{cases} \end{bmatrix} \tag{7}$$

These tetrahedron submatrices are then assembled into hexahedron submatrices, which are subsequently assembled to form a global matrix. The global matrix at this point is singular. The system of equations cannot be solved until a sufficient set of boundary conditions is applied. EMAP-2 requires that the tangential components of the electric field on every outer boundary node be constrained. This is usually accomplished by enclosing the entire configuration in a metallic enclosure. Sources are included in the model by constraining the field at some nodes to take on a fixed non-zero value.

After imposing boundary conditions, the matrix equation can be put in the form **Ax=B** where x is a vector of coefficients describing the electric field at each unforced node. The coefficients of **A** and **B** are known functions of the problem geometry, mesh geometry and boundary conditions. Solving the matrix equation for x yields the coefficients necessary to describe the field distribution throughout the problem region.

At a dielectric boundary, the normal component of the electric field is discontinuous. The EMAP-2 code recognizes and enforces this condition. The procedure used is similar to that described by Paulsen [7]. Nodes on a dielectric interface are effectively split in two during the initial stages of the algorithm. Each half of the split node is in a different dielectric. During the final stages of the global matrix assembly, the halves are recombined and the discontinuity in the normal electric field is explicitly enforced.

Input and Output

The EMAP-2 code reads the input configuration in as an ASCII text file and provides output in the same form. This allows the source code to remain free of non-standard graphical I/O commands or routines. As a result, the EMAP-2 code is highly portable and can be run on a variety of platforms without modification.

Because it can be difficult and time consuming to create an error-free data input file for 3D configurations, a translation code has been written that allows the user to draw the input configuration using a commercial computer aided drawing program. DesignCAD 3-D [8] is a powerful, but relatively easy-to-learn CAD program that can be used to generate fairly complex 3D geometries. The translation code reads the DesignCAD output file and creates an EMAP data input file. The translation code associates DesignCAD grid points with mesh nodes. Colors in the CAD drawing are used to represent different material properties and arrows represent excitation sources [9].

It is also possible to develop translation codes for other CAD programs running on other platforms. In many cases, the effort involved in writing a new translation code is less than the effort required to learn to use a non-standard graphical interface. There is currently an effort in progress to develop a translation code that will allow EMAP-2 to be used with any 3D computer drawing software that is capable of generating IGES standard graphics description files.

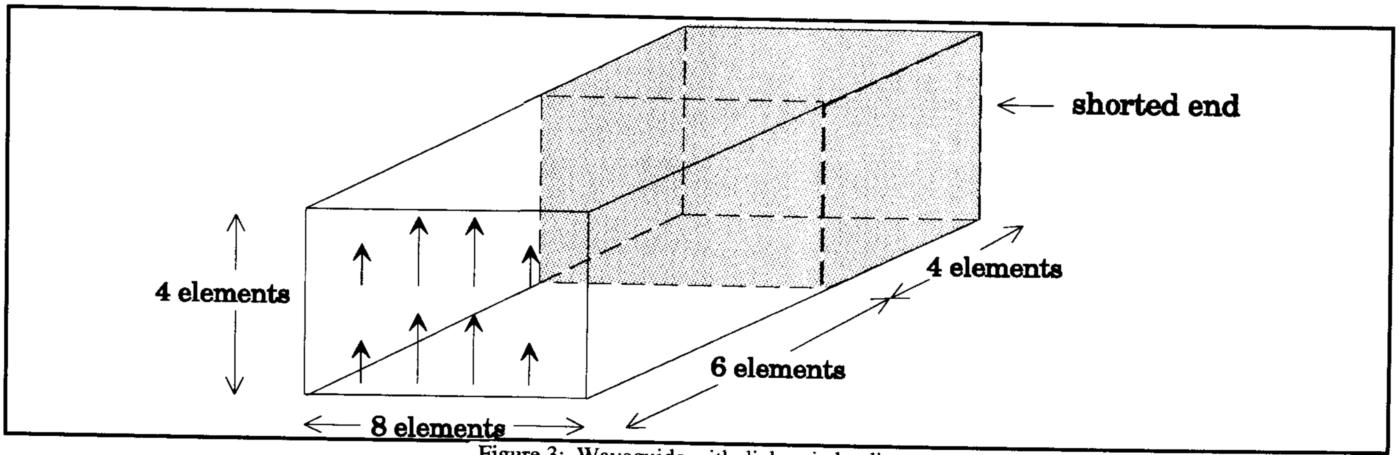

Figure 3: Waveguide with dielectric loading

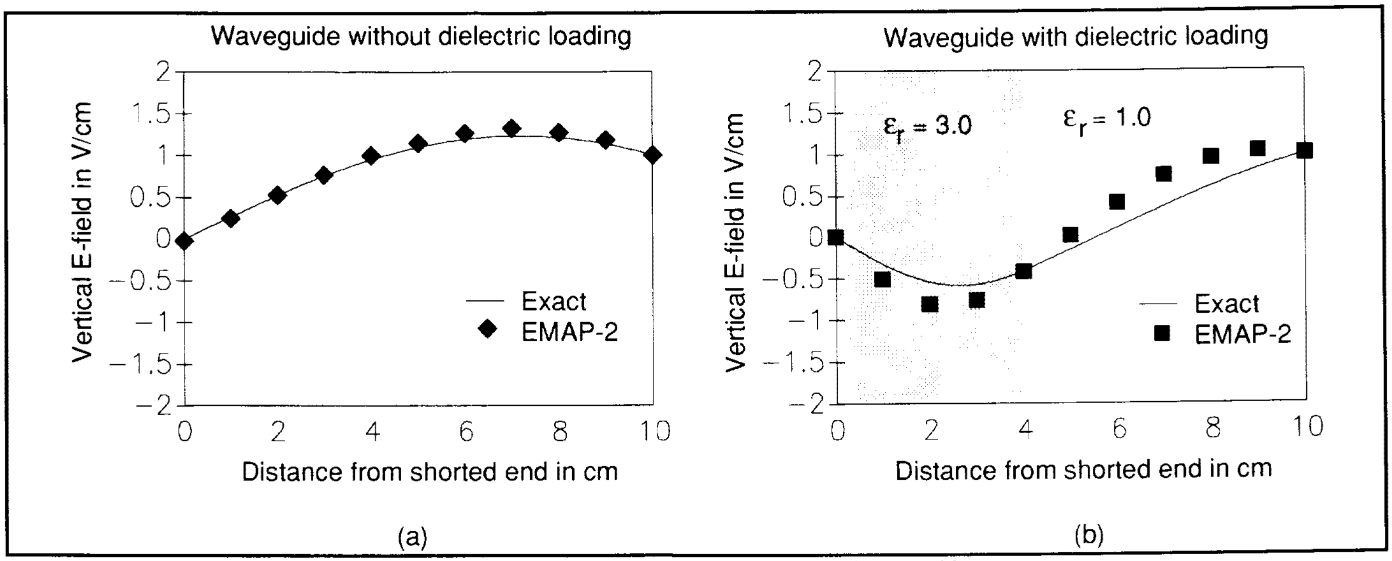

Figure 4: Vertical E-fieldin dielectric loaded waveguide

Output from the EMAP code consists of a listing of the node coordinates and the electric field strength at each node. This data can be easily read by a variety of spreadsheet programs and rearranged, plotted, or manipulated as desired.

Examples

As an illustration of how EMAP-2 can be used, two sample configurations are analyzed below. The first configuration is the dielectric loaded waveguide illustrated in Figure 3. This waveguide is shorted at one end and driven in the TE_{10} mode. The vertical electric field strength is plotted as a function of position along the center of the waveguide in Figure 4. With or without the dielectric loading, there is very good agreement between the finite element model and analytical results calculated using standard transmission line theory.

A simple configuration resembling a printed circuit board in an enclosure is illustrated in Figure 5. It consists of a rectangular ring on a dielectric slab in a metal box. The ring is driven by a 5 volt/cm electric field at one end. Figure 6 shows the amplitude of the x-component of the electric field in the plane of the ring at three frequencies. The ring behaves like a shorted transmission line. At most frequencies the field is largely confined to the interior of the ring and the expected standing wave pattern is observed (Figure 6a. and 6b.). However, at frequencies that excite a resonance of the enclosure, the field outside the ring can be much stronger than the field within the ring (Figure 6c.).

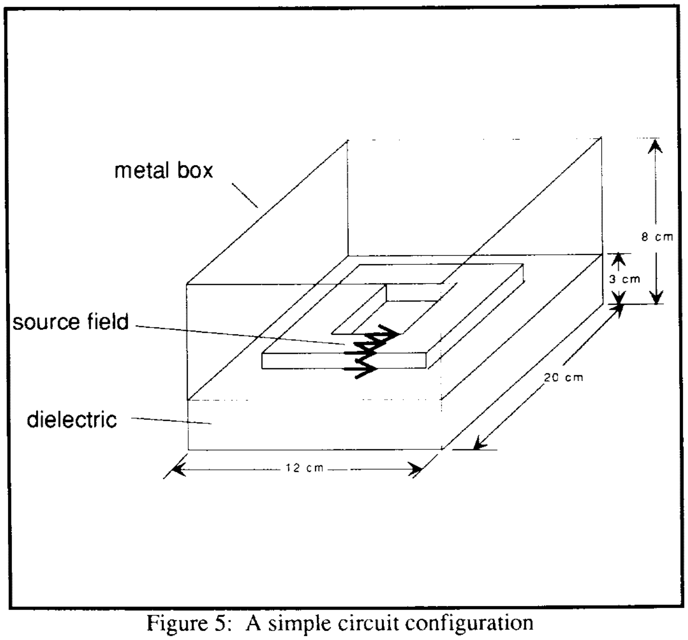

Figure 5: A simple circuit configuration

Summary

EMAP-2 is a relatively easy-to-use 3D finite element modeling code. It is primarily intended for educational and research applications, but it is capable of modeling configurations of interest to EMC engineers. EMAP-2 is available by anonymous ftp from *emclab.ee.umr.edu.* It is free-of-charge, free-of-technical-support, and free-of-warranty. Nevertheless, it is a powerful electromagnetic modeling code and it is capable of analyzing useful, three-dimensional configurations that are beyond the scope of many commercial finite element software packages.

Acknowledgement

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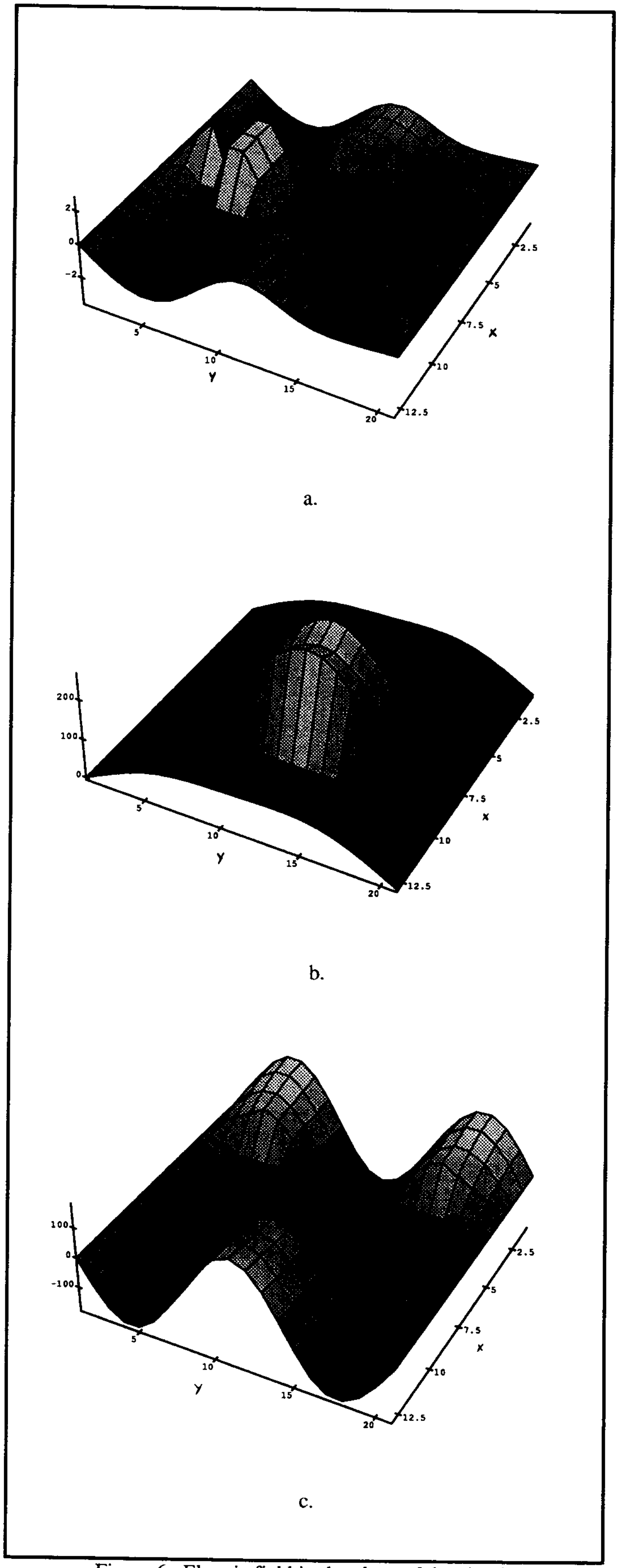

a.

b.

c.

Figure 6: Electric field in the plane of the circuit

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