EMC APPLICATIONS OF THE EMAP5 HYBRID FEM/MOM CODE

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Abstract--EMAP5 is a numerical software package designed to model electromagnetic problems. It employs the finite element method (FEM) to analyze three-dimensional volumes, and the method of moments (MOM) to analyze the current distribution on the surface of these volumes. The two methods are coupled through the fields on the dielectric surface. This paper describes the formulation of the EMAP5 code and demonstrates how it can be used to analyze simple printed circuit board configurations.

I. INTRODUCTION

EMAP5---**E**lectro**M**agnetic **A**nalysis **P**rogram Version **5** is a 3D numerical electromagnetic modeling code developed at the University of Missouri-Rolla (UMR). The code can be freely downloaded from the world wide web at http://www.emclab.umr.edu/emap5. EMAP5 is a hybrid FEM/MOM code designed primarily to simulate electromagnetic interference (EMI) sources at the printed circuit board level.

Generally, printed circuit boards and their components are composed of many different materials with arbitrary shapes. Thus, the Method of Moments (MOM) does not model this kind of problem efficiently. A hybrid method combining FEM with MOM has been proposed by many researchers [1][2][3] as a means of modeling structures that are both open (i.e. unbounded) and complex. FEM is applied to model the fields within a fictitious boundary containing regions of high complexity, while MOM is used to model the fields on the fictitious boundary. The two methods are coupled by the fields at the dielectric boundary.

Details of the hybrid formulation for EMAP5 are presented in [4][5]. This paper demonstrates how EMAP5 can be used to model 3D microstrip configurations.

II. FORMULATION

The general structure of interest is shown in Figure 1. A dielectric volume V_2 has electrical properties (ϵ_2, μ_2). It is enclosed by a surface S_2. A conductive volume V_3 is enclosed by a conductive surface S_c. The fields within V_3 vanish. V_1, which denotes the volume outside of V_2 and V_3, is assumed to be free space, hence has electrical properties (ϵ_0, μ_0). (E_1, H_1) and (E_2, H_2) denote the electric and magnetic fields in V_1 and V_2, respectively. The unit normal vectors for S_2 and S_c are defined pointing outward
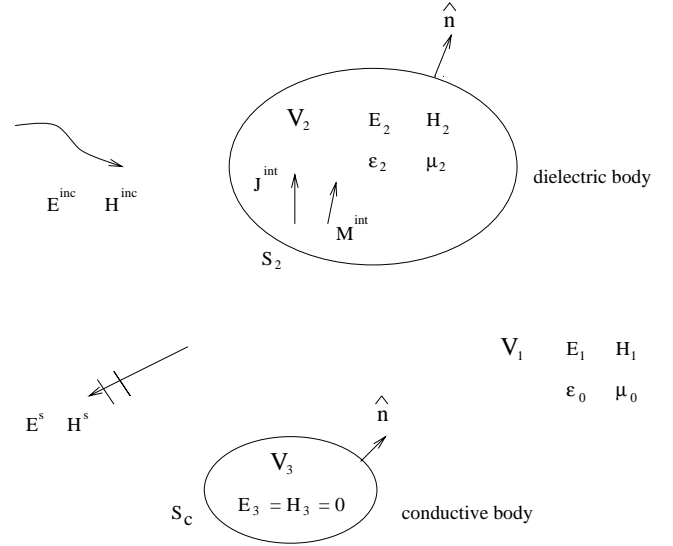

Fig. 1. A dielectric obect and a conductive object illuminated by $\mathbf{E}^i$, $\mathbf{H}^i$ or $\mathbf{J}^{int}$, $\mathbf{M}^{int}$.

toward V_1. The structure is excited by an incident wave ($\mathbf{E}^i$, $\mathbf{H}^i$) or impressed sources ($\mathbf{J}^{int}$, $\mathbf{M}^{int}$). The scattered electric and magnetic fields are ($\mathbf{E}^s$, $\mathbf{H}^s$). The dielectric surface S_d, is defined as S_2 if the conductive body is not adjacent to the dielectric body; Otherwise, $S_d = S_2 - (S_2 \cap S_c)$. The objective is to solve for the scattered fields ($\mathbf{E}^s$, $\mathbf{H}^s$) or the surface electric current density on S_c.

A. Discretization of FEM

From the double curl equation in terms of **E,** the weak form of FEM equation can be expressed as:

$$\int_{V_2} \left[\left(\frac{\nabla \times \mathbf{E}(\mathbf{r})}{j\omega\mu_0\mu_r} \right) \bullet (\nabla \times \mathbf{w}(\mathbf{r})) + j\omega\epsilon_0\epsilon_r \mathbf{E}(\mathbf{r}) \bullet \mathbf{w}(\mathbf{r}) \right] dV$$

$$= \int_{S_2} (\hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r})) \bullet \mathbf{w}(\mathbf{r}) \, dS - \int_{V_2} \left[\mathbf{J}^{int}(\mathbf{r}) + \frac{\nabla \times \mathbf{M}^{int}(\mathbf{r})}{j\omega\mu_0\mu_r} \right] \bullet \mathbf{w}(\mathbf{r}) \, dV \qquad (1)$$

where $\mathbf{w}(\mathbf{r})$ is a weighting function.

Tetrahedral elements are used to discretize the volume V_2. Basis functions proposed by M. L. Barton and Z. J. Cendes [6] are chosen here. Each basis function is defined within a tetrahedron and is associated with one of the six edges. The electric field **E** within volume V_2 can be expanded as:

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$$\mathbf{E(r)} = \sum_{n=1}^{N_v} E_n\, \mathbf{w_n(r)}$$

where N_v is the total number of interior edges and boundary edges on the dielectric surface, and $\{E_n\}$ is a set of unknown complex scalar coefficients. The weighting functions chosen are $\mathbf{w_n(r)}$, n=1,...N_v. After the weighting functions are applied to Eq. (1), Eq.(1) can be discretized as:

$$\begin{bmatrix} A_{ii} & A_{id} \\ A_{di} & A_{dd} \end{bmatrix}\begin{bmatrix} E_i \\ E_d \end{bmatrix} = [B_{dd}][J_d] + [g^{int}] \qquad (2)$$

where $\{E_i\}$ are the unknown electric fields within the FEM volume. $\{E_d\}$ are the unknown equivalent surface magnetic currents on S_d. $\{J_d\}$ are the equivalent electric currents on the dielectric surface, which is discussed in later section. $\{g^{int}\}$ are the impressed current source terms. $\{A_{ii}\}$, $\{A_{id}\}$, $\{A_{di}\}$, $\{A_{dd}\}$ and $\{B_{dd}\}$ are the coefficient matrices.

B. Discretization of MoM

The MOM surface integral equation is [7]:

$$\mathbf{E^{inc}(r)} = \frac{1}{2}\mathbf{E(r)} + \int_S \{ \mathbf{M(r')} \times \nabla' G_0(\mathbf{r},\mathbf{r'})$$

$$+ j k_0 \eta_0 \mathbf{J(r')} G_0(\mathbf{r},\mathbf{r'}) - j\frac{\eta_0}{k_0} \nabla' \bullet \mathbf{J(r')}\, \nabla' G_0(\mathbf{r},\mathbf{r'})\} dS' \qquad (3)$$

where $\mathbf{r} \in S$, $S = S_c \cup S_2$, η_0 and k_0 are the intrinsic impedance and wavenumber in free space, respectively, and

$$G_0(\mathbf{r},\mathbf{r'}) = \frac{e^{-j k_0 |\mathbf{r} - \mathbf{r'}|}}{4\pi |\mathbf{r} - \mathbf{r'}|}$$

is the Green's function in free space. The equivalent surface electric and magnetic currents are defined as:

$$\mathbf{J(r')} = \hat{\mathbf{n}} \times \mathbf{H(r')} \qquad \mathbf{r'}\ \text{on } S$$

$$\mathbf{M(r')} = \mathbf{E(r')} \times \hat{\mathbf{n}} \qquad \mathbf{r'}\ \text{on } S_d$$

$\mathbf{M(r')}$ vanishes on S_c. $\mathbf{J(r')}$ and $\mathbf{M(r')}$ can be approximated by using the triangular basis function $\mathbf{f_n(r)}$ proposed by S. M. Rao *et al.* [8]. On surface S_d, the MOM basis function $\mathbf{f_n(r)}$ and the FEM basis function $\mathbf{w_n(r)}$ are related by:

$$\mathbf{w_n(r)} = \hat{\mathbf{n}} \times \mathbf{f_n(r)} \qquad \mathbf{r} \in S_d$$

$\mathbf{J(r')}$, $\mathbf{M(r')}$ can be expanded as:

$$\mathbf{J(r')} = \hat{\mathbf{n}} \times \mathbf{H(r')} = \sum_{n=1}^{N_s} J_n \mathbf{f_n(r')}$$

$$\mathbf{M(r')} = \mathbf{E(r')} \times \hat{\mathbf{n}} = \sum_{n=1}^{N_d} E_n \mathbf{w_n(r')} \times \hat{\mathbf{n}} = \sum_{n=1}^{N_d} E_n \mathbf{f_n(r')}$$

where N_s is the total number of edges on the surface S, and N_d is the total number of edges on the surface S_d. $\{J_n\}$ and $\{E_n\}$ are unknown complex scalar coefficients. The weighting functions chosen are $\mathbf{f_n(r)}$, n=1, ... N_s. After the weighting functions are applied to Eq. (3), Eq. (3) can be discretized into,

$$\begin{bmatrix} C_{dd} & C_{dc} \\ C_{cd} & C_{cc} \end{bmatrix}\begin{bmatrix} J_d \\ J_c \end{bmatrix} = \begin{bmatrix} D_{dd} & 0 \\ D_{cd} & 0 \end{bmatrix}\begin{bmatrix} E_d \\ 0 \end{bmatrix} + \begin{bmatrix} F_d^i \\ F_c^i \end{bmatrix} \qquad (4)$$

where $\{J_d\}$ and $\{J_c\}$ are the unknown equivalent surface electric currents on S_d and S_c respectively. $\{E_d\}$ are the unknown equivalent surface magnetic currents on S_d. $\{C_{dd}\}$, $\{C_{dc}\}$, $\{C_{cd}\}$, $\{C_{cc}\}$,$\{D_{dd}\}$ and $\{D_{cd}\}$ are coefficient matrices. $\{F_i^d\}$ and $\{F_c^i\}$ are forcing terms due to the incident wave.

The FEM Eq. (2) and the MOM Eq. (4) are coupled through $\{E_d\}$ and $\{J_d\}$. By solving the two equations, all fields within the FEM region and the surface equivalent currents can be obtained. Details of how to construct the matrix equation and solve it are provided in [4][5].

III.   NUMERICAL RESULTS

Two numerical examples are provided in this section to demonstrate the application of the EMAP5 code to printed circuit board geometries.

A. Determining the Characteristic Impedance of a Microstrip Geometry

This example simulates a microstrip line on a printed circuit board. The geometry of the structure is shown in Figure 2. The board is made of a dielectric with ε_r=4.0. The trace is excited by a 1V source at one end, and is terminated by a resistor at the other end. To determine the characteristic impedance Z_0 of the transmission line, we need to determine the input impedance when the load side is shorted or open, respectively. The input impedance Z_{in} of a transmission line is given by,

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan(\beta l)}{Z_0 + j Z_L \tan(\beta l)}$$

where Z_L is the load impedance; β is the wavenumber; l is the length of the transmission line. When the load side is shorted, the input impedance is given by,

$$(Z_{in})_{short} = j Z_0 \tan(\beta l)$$

when the load side is open, the input impedance is given by,

$$(Z_{in})_{open} = -j \frac{Z_0}{\tan(\beta l)}$$

Thus, the characteristic impedance is given by,

$$Z_0 = \sqrt{(Z_{in})_{short}(Z_{in})_{open}}$$

Since the source is electrically short and small, it can be modeled as a current filament [9]. The source can be expressed as,

$$\mathbf{J}^{int} = I\, \delta(x - x_f)\, \delta(y - y_f)\, \hat{\mathbf{z}}$$

where (x_f, y_f) specifies its position, I denotes the electric current magnitude, and $\delta(x)$ is the Dirac delta function. $\mathbf{J}^{int}$ is the impressed current source that is included in Eq. (1) as the source term. After the $\mathbf{E}$ fields along the source edges are obtained, the voltage drop along the current filament can be calculated. Thus, the input impedance Z_{in} can be obtained.
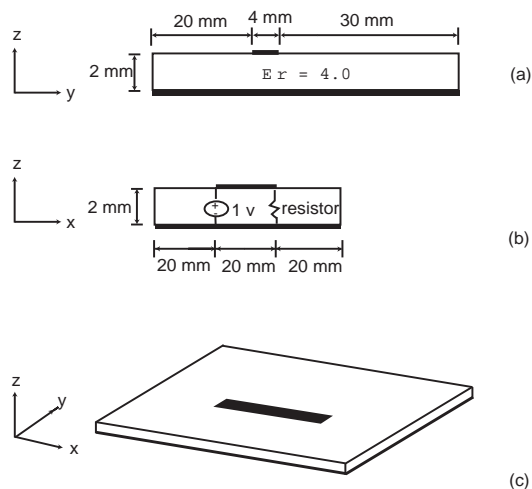
Fig. 2. Grounded printed circuit board with an active trace.
(a) y-z plane view.
(b) x-z plane view. and (c) 3D view.

Table 1 shows inductance and capacitance obtained by EMAP5 when the load side is shorted or open, respectively. The characteristic impedance Z_0 of the trace then can be determined. Three frequencies have been investigated. At each frequency, the calculated value of Z_0 is 56.4 Ω. We can put a 56.4 Ω resistor at the load side to terminate the transmission line. Theoretically, there should be no reflection if Z_0 is 56.4 Ω. Figure 3 shows the numerical results obtained by EMSIM when the trace is terminated with a 56.4 Ω resistor. It is evident that the transmission line is almost perfectly matched.

B. Determining the Input Impedance of a Microstrip Line with a Resistive Load.

In this example, the configuration is the same as shown in Figure 2. Now however, the load is a 50 Ω resistor. A load Z_L can be modeled as an element with finite conductivity
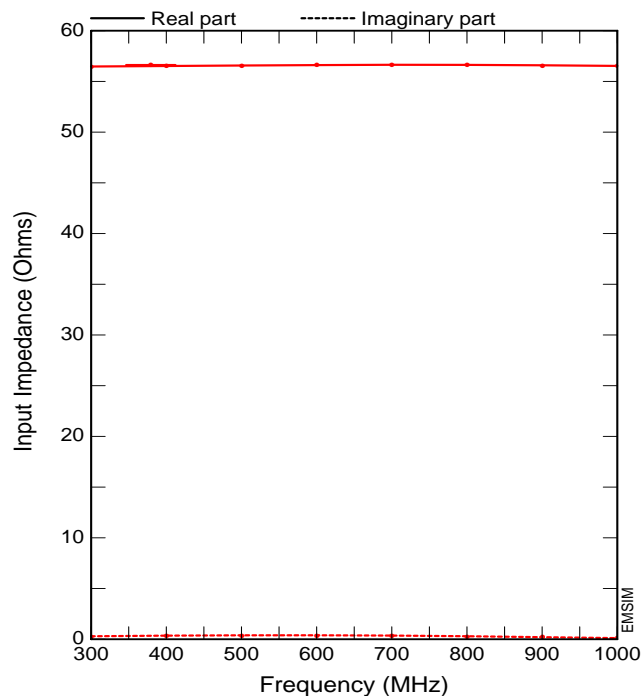

Fig. 3. The input impedance obtained by EMSIM when the trace in Fig. 2. is terminated by a 56.4 Ω resistor.

given by $\sigma = l/(Z_L S)$, where l is its length, and S is the cross section. If the load is treated as a lumped element, its contribution to the finite element matrix is as follows [8]:

$$\left[K^e\right] = \frac{l^2}{Z_L} \iint \{w\}\{w\}^T \delta(x - x_L)(y - y_L)\, dx\, dy$$

where (x_L, y_L) is the position of the load impedance. Only edges coinciding with the load are affected by the load.

Figure 4 shows the impedance obtained by EMAP5 and compares them with results obtained by EMSIM. Since the characteristic impedance of the microstrip line is about 56.4 Ω, the 50 Ω load does not match the microstrip line perfectly. As shown in Figure 4, the input impedance is not exactly 50 Ω due to the mismatch. The EMAP5 results agree very well with the EMSIM results for this example.

Table 1.   Characteristic impedance obtained by EMAP5.

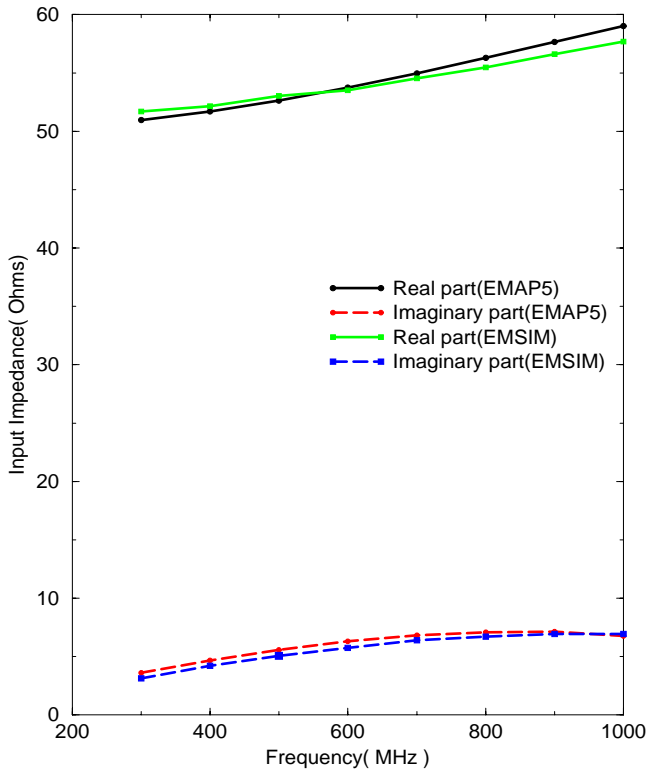| Source Frequency (MHz)	$(Z_{in})_{open}$ (Ω)	$(Z_{in})_{short}$ (Ω)	Z_0 (Ω)
300	-j187	+j17	56.4
500	-j106	+j30	56.4
700	-j69	+j46	56.3

Fig. 4. The input impedance of the structure in Fig 2. when the trace is terminated by a 50 Ω resistor.

IV. CONCLUSION

This paper introduces the hybrid FEM/MOM code called EMAP5. Examples of the application of EMAP5 to printed circuit geometries were presented and good agreement was achieved with theoretical results and results obtained using EMSIM.

REFERENCES

[1] A. J. Sangster and Hanyang Wang, "A Combined FEM/MoM Technique of Coupling Radiating Apertures in Rectangular Waveguide," *IEEE Trans. on Magnetics*, vol. 31, pp. 1554-1557, May 1995.

[2] Jian-Ming Jin and John L. Volakis, "Electromagnetic Scattering by and Transmission Through a Three-Dimensional Slot in a Thick Conducting Plane," *IEEE Trans. on Antennas and Propagat.*, vol. 39, pp. 543-550, April 1991.

[3] X. C. Yuan, D. R. Lynch, and J. W. Strohbehn, "Three-Dimensional Finite, Boundary, and Hybrid Element Solutions of the Maxwell's Equations for Lossy Dielectric Media," *IEEE Trans. on Microwave Theory and Tech.*, vol. 36, pp. 682-693, April 1988.

[4] M. W. Ali, T. H. Hubing and J. L. Drewniak, "A Hybrid FEM/MOM Technique for Electromagnetic Scattering and Radiation from Dielectric Objects with Attached Wires," *IEEE Trans. on Electromagnetic Compatibility*, vol. 39, pp. 304-314, November 1997.

[5] Mohammad W. Ali, "Development of a Hybrid 3D Numerical Modeling Technique for Analyzing Printed Circuit Models with Attached Wires," Ph.D. dissertation, University of Missouri-Rolla, December 1996.

[6] M. L. Barton and Z. J. Cendes, "New Vector Finite Elements for Three-Dimensional Magnetic Field Computation," *Journal of Applied Physics*, vol. 61, pp. 3919-3921, April 1982.

[7] Johnson J. H. Wang, *Generalized Moment of Methods in Electromagnetics*, pp. 171-174, New York: John Wiley & Sons, 1990.

[8] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic Scattering by Surfaces of Arbitrary Shape," *IEEE Trans. on Antennas and Propagat.,* vol. 30, pp. 409-418, May 1982.

[9] Jianming Jin, *The Finite Element Method in Electromagnetics*, pp. 324-325, New York: John Wiley & Sons Inc, 1993.

[10] EMSIM is a 3D numerical electromagnetic modeling code marketed by Pacific Numerix, Scottsdale, AZ, http://www.pnc.com.