Short Papers

Determining the Maximum Allowable Power Bus Voltage to Ensure Compliance With a Given Radiated Emissions Specification

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***Abstract*—It is generally understood that excessive power bus noise voltage on a printed circuit board will result in unacceptable radiated emissions levels, but how much noise is *too much*? In this paper, an analysis based on a lossy cavity model is performed to determine the maximum possible radiated field corresponding to a given power bus noise voltage. A closed-form expression relating the maximum power bus noise voltage to the radiation peaks is then derived. This expression is solved in reverse to determine the minimum power bus voltage that is necessary to generate a radiated field above a specified limit. When troubleshooting a radiated emissions problem, this expression can be applied to measured values of power bus noise voltage to determine whether radiation directly from the power bus is potentially the emissions source.**

***Index Terms*—Maximum radiated emissions, power bus noise voltage, printed circuit board (PCB).**

I. Introduction

Switching noise currents on the power pins of digital integrated circuits can be a significant source of radiated emissions. These switching currents cause voltage fluctuations on the power bus due to the nonzero high-frequency impedance of the bus [1]. The radiation directly from the planes of a printed circuit board (PCB) can be numerically calculated using full-wave simulation software. However, this type of simulation requires extensive computational resources and yields results that are sensitive to the exact board parameters provided. Expert system algorithms employ closed-form calculations to estimate the maximum radiated emissions from PCBs due to many possible sources [2]–[4]. In this way, sources and coupling mechanisms that cannot significantly contribute to radiated emissions can be systematically eliminated and attention can be focused on only those features of a given design that are capable of producing electromagnetic interference (EMI) problems.

Leone [5] introduced a closed-form expression relating source currents to the radiation from a rectangular power bus based on the cavity model. Deng *et al*. [4] developed an expression for estimating the maximum radiated emissions from a cable attached to a PCB due to power bus noise.

This paper develops a closed-form expression for the maximum possible radiation corresponding to a given power bus noise voltage, and conversely, the minimum power bus noise voltage required to exceed a given radiated emission limit due to radiation coming directly from the power planes.

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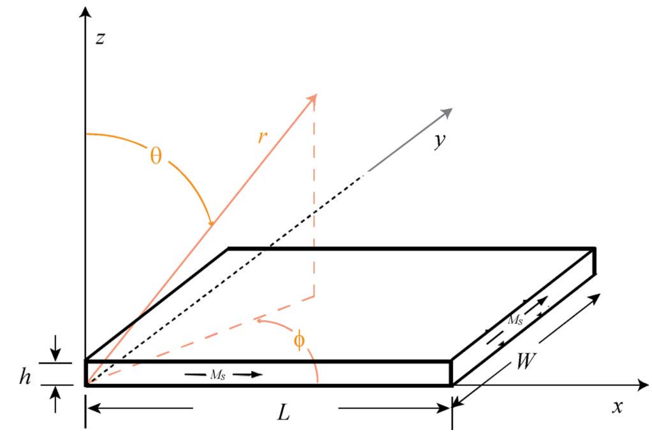

Fig. 1. Power bus structure configuration.

II. Far-Field Calculation

The power bus structure under consideration is illustrated in Fig. 1. The power planes have a length L, width W $(L \geq W)$, and are separated by a dielectric with a thickness h. The length and width are much greater than the thickness. The tangential magnetic field and normal electric field on each edge are approximately zero. The fields inside the cavity can be expressed as a summation of 2-D TMz modes [6]. The voltage distribution $V(x, y)$ due to an impressed current I_0 at the location (x_0, y_0) with source dimensions dx_0, dy_0 is given by [7]

$$\frac{V(x,y)}{I_0} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{j\omega\mu h}{ab} \frac{k_{mn}}{k_{xm}^2 + k_{yn}^2 - k^2} \times \cos(k_{xm}x_0)\cos(k_{yn}y_0)\cos(k_{xm}x)\cos(k_{yn}y) \times \text{sinc}\left(\frac{k_{xm}dx_0}{2}\right)\text{sinc}\left(\frac{k_{yn}dy_0}{2}\right) \quad (1)$$

with $k_{xm} = m\pi/L$, $k_{yn} = n\pi/W$. K_{mn} and k^2 are given by

$$K_{mn} = \begin{cases} 1, & m = n = 0 \\ 2, & m = 0 \text{ or } n = 0 \\ 4, & mn \neq 0 \end{cases} \quad (2)$$

$$k^2 = \omega^2 \varepsilon_r \varepsilon_0 \mu \left(1 - j\left(\tan\delta + \frac{\delta_s}{h}\right)\right) \quad (3)$$

where δ_s is the skin depth of the plane conductors, $\tan\delta$ is the loss tangent of the dielectric substrate, and ε_r is the relative permittivity of the dielectric substrate.

The maximum power bus noise voltage due to a current source I_0 located at (x_0, y_0) is given by

$$V_{\max}(\omega) = \max\left(\frac{V(x,y)}{I_0(x_0,y_0)}\right) \cdot I_0(x_0,y_0). \quad (4)$$

Analytical expressions for the radiated electric field from a rectangular power bus structure driven by a current source can be found in [5] and [8]. These expressions can be useful for modeling the effect of a known source; however, in many situations, the current driving the power bus is not readily obtainable. For boards that have already been built, it is generally easier to measure power bus voltages than source currents. The following section derives an expression for the maximum radiated field strength from a rectangular power bus structure as a function of the maximum voltage on the edge of the planes.

III. Maximum Radiation Estimation

To relate radiated field strengths to the voltages at the edge of a power bus structure, Huygen's Principle can be employed [5], [6]. The radiated electric field is calculated using an equivalent magnetic current density as the radiation source

$$\vec{E} = j\omega\eta\, \vec{e}_r \times \overleftarrow{F} \tag{5}$$

$$\vec{F} \approx \frac{\varepsilon_0 e^{-jk_0 r}}{4\pi r} \iint_s \vec{M}_s(r') e^{jk_0 \vec{r'}\vec{e_r}}\, ds, \tag{6}$$

where $\omega = 2\pi f$, η is the intrinsic impedance, k_0 is the wavenumber in free space, $\vec{F}$ is the electric vector potential, and s is the sidewall surface of the board. The relationship between the tangential electric field on the edges and the corresponding equivalent magnetic current density is given by

$$\vec{M}_s = -\vec{n} \times \vec{E}_z. \tag{7}$$

Based on the assumption that the spacing h is much smaller than the wavelength, the integration on the sidewall surface in (6) can be simplified by integrating along the periphery l of the planes

$$\vec{F} \approx \frac{\varepsilon_0 e^{-jk_0 r}}{4\pi r} \int_l V(r')(\vec{e_Z} \times \vec{n}) e^{jk_0 \vec{r'}\vec{e_r}}\, dl \tag{8}$$

with $k_0 \vec{r'}\vec{e_r} = xk_x + yk_y$, where $k_x = k_0 \sin\theta\cos\phi$, and $k_y = k_0 \sin\theta\sin\phi$, and $V(r') = \vec{E}(r')\cdot\hat{z}$ is the voltage between the planes. The resulting expression can be written as

$$\vec{F} \approx \frac{\varepsilon_0 e^{-jk_0 r}}{4\pi r} \times \begin{pmatrix} \vec{e}_x \int_{x=0}^{L} [V(x,y=0)+V(x,y=W)e^{jk_y W}] e^{jk_x x}\, dx \\ +\vec{e}_y \int_{y=0}^{W} [V(x=L,y)e^{jk_x L}+V(x=0,y)] e^{jk_y y}\, dy \end{pmatrix}. \tag{9}$$

In (9), the electric vector potential $\vec{F}$ reaches an upper limit when the power bus noise voltage $V(x,y)$ is set to its maximum value $V_{\max}$ everywhere along the periphery of the board planes. This limit is given by

$$\vec{F}_{\max} \approx \frac{j\varepsilon_0 e^{-jk_0 r} V_{\max}}{4\pi r k_0} \times \begin{pmatrix} \vec{e}_x \dfrac{[1+e^{jk_0 \sin\theta\sin\phi\cdot W}][1-e^{jk_0 \sin\theta\cos\phi\cdot L}]}{\sin\theta\cos\phi} \\ +\vec{e}_y \dfrac{[1+e^{jk_0 \sin\theta\cos\phi\cdot L}][1-e^{jk_0 \sin\theta\sin\phi\cdot W}]}{\sin\theta\sin\phi} \end{pmatrix}. \tag{10}$$

Substituting (10) into (5), the following expressions for E_θ and E_ϕ are obtained:

$$E_\theta \approx \frac{-e^{-jk_0 r} V_{\max}}{4\pi r} \times \begin{pmatrix} \dfrac{\sin\phi[1+e^{jk_0 \sin\theta\sin\phi\cdot W}][1-e^{jk_0 \sin\theta\cos\phi\cdot L}]}{\sin\theta\cos\phi} \\ -\dfrac{\cos\phi[1+e^{jk_0 \sin\theta\cos\phi\cdot L}][1-e^{jk_0 \sin\theta\sin\phi\cdot W}]}{\sin\theta\sin\phi} \end{pmatrix} \tag{11}$$

$$E_\phi \approx \frac{-e^{-jk_0 r} V_{\max}}{4\pi r} \times \begin{pmatrix} \dfrac{\cos\theta[1+e^{jk_0 \sin\theta\sin\phi\cdot W}][1-e^{jk_0 \sin\theta\cos\phi\cdot L}]}{\sin\theta} \\ +\dfrac{\cos\theta[1+e^{jk_0 \sin\theta\cos\phi\cdot L}][1-e^{jk_0 \sin\theta\sin\phi\cdot W}]}{\sin\theta} \end{pmatrix}. \tag{12}$$

Because $V_{\max}$ is a constant, the maximum E-field $E_{\max}$ is found by setting $\theta = 0$ and is given by

$$|E_{\max}| = \frac{k_0 V_{\max}}{2\pi r}\sqrt{L^2+W^2}. \tag{13}$$

The upper bound in (13) occurs when the voltages on opposite sides, such as $V(x, y = 0)$ and $V(x, y = W)$, are constant and 180° out of phase (resulting in equivalent magnetic currents on opposite edges that are in phase). This occurs for the TM_{m0} and TM_{0n} modes, where m and n are odd integers. The radiation is also proportional to frequency, so the higher order TM_{m0} and TM_{0n} modes radiate more effectively. When both m and n are nonzero, adjacent lobes in the field distribution along both edges are out of phase. In this case, the higher order modes do not radiate any more efficiently than the lower order modes, so we can cap our maximum emissions estimate at the value calculated for the TM_{11} mode as long as we are sure that no TM_{m0} or TM_{0n} modes ($m,n > 2$) occur at frequencies higher than the TM_{11} cutoff.

In a relatively square board, TM_{m0} and TM_{0n} modes are not likely to be excited efficiently when m or n is much greater than one. In a slim board where the length is p times the width, the TM_{p0} mode is the highest order mode that is likely to be excited, but this will occur at a frequency below the TM_{11} cutoff. Thus, relatively square boards are unlikely to exhibit a maximum radiation greater than the maximum level calculated for the TM_{11} mode, because the modes capable of producing this radiation are not likely to be excited. Slim boards are unlikely to exhibit maximum radiation greater than the maximum level calculated for the TM_{11} mode, because the TM_{11} mode occurs at a higher frequency than the TM_{n0} modes that are likely to be excited. Therefore, regardless of the shape of the board, the maximum level calculated for the TM_{11} mode represents a good upper-bound estimate for the board emissions. Therefore, the expression for the maximum radiated field in (13) can be capped at the TM_{11}-mode radiation level for frequencies at or above the TM_{11} frequency.

Furthermore, the radiation of the short edge is dominant at frequencies below the TM_{01}-mode frequency. Therefore, the $\sqrt{L^2+W^2}$ in (13) can be replaced by W at frequencies below the TM_{01} cutoff. With these enhancements, the expression in (13) becomes

$$|E_{\max}| = \begin{cases} \dfrac{k_0 V_{\max}}{2\pi r} W, & f < f_{c1} \\ \dfrac{k_0 V_{\max}}{2\pi r} L, & f_{c1} \le f < f_{c2} \\ \dfrac{k_{c2} V_{\max}}{2\pi r}\sqrt{L^2+W^2}, & f \ge f_{c2} \end{cases} \tag{14}$$

where nominally, f_{c1} is the cutoff frequency of the TM_{01} mode, f_{c2} is the cutoff frequency of the TM_{11} mode, and k_{c2} is the value of the wavenumber at the TM_{11} cutoff frequency. However, the peaks occurring at a board resonance have a finite bandwidth, so the expression in (14) may underestimate the amplitude of the radiated field at frequencies just below the first cutoff frequency. To compensate for this, we define a transition frequency f_{t1} that occurs midway between adjacent resonances instead of at f_{c1}. Rewriting (14) in terms of the

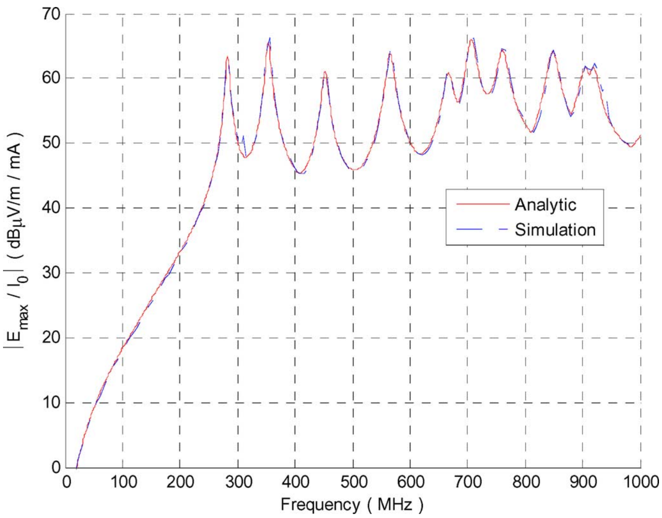

Fig. 2. Maximum radiation due to a 1-mA current source located at one corner of a board with $W = 20$ cm, $L = 25$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, $\tan\delta = 0.02$, and $r = 3$ m.

board parameters and modified transition frequency, we obtain

$$|E_{\max}| = \begin{cases} \dfrac{f\sqrt{\mu\varepsilon_0}V_{\max}W}{r}, & f < f_{t1} \\ \dfrac{f\sqrt{\mu\varepsilon_0}V_{\max}L}{r}, & f_{t1} \le f < f_{c2} \\ \dfrac{f_{c2}\sqrt{\mu\varepsilon_0}V_{\max}\sqrt{L^2+W^2}}{r}, & f \ge f_{c2} \end{cases} \quad (15)$$

where

$$f_{t1} = \left(\frac{1}{2\sqrt{\mu\varepsilon_r\varepsilon_0}}\frac{1}{W} + \frac{1}{2\sqrt{\mu\varepsilon_r\varepsilon_0}}\frac{m}{L}\right)\frac{1}{2}. \quad (16)$$

The first part of the expression for f_{t1} is the cutoff frequency of the TM_{01} mode. The second part is the cutoff frequency of the TM_{m0} mode that occurs at a frequency closest to, but lower than, the TM_{01} mode. For example, $m = 1$ for a 20 cm × 25 cm × 0.1 cm board. The second cutoff frequency f_{c2} is

$$f_{c2} = \frac{1}{2\sqrt{\mu\varepsilon_r\varepsilon_0}}\sqrt{\left(\frac{1}{L}\right)^2 + \left(\frac{1}{W}\right)^2} \quad (17)$$

which is the cutoff frequency of the TM_{11} mode.

Solving (15) for $V_{\max}$, a closed-form expression can be derived to calculate the minimum voltage required between two planes to generate an electric field exceeding a specified radiated emissions limit $|E|_{\mathrm{Limit}}$

$$V_{\min} = \begin{cases} \dfrac{|E|_{\mathrm{Limit}}\, r}{f\sqrt{\mu\varepsilon_0}W}, & f < f_{t1} \\ \dfrac{|E|_{\mathrm{Limit}}\, r}{f\sqrt{\mu\varepsilon_0}L}, & f_{t1} \le f < f_{c2} \\ \dfrac{|E|_{\mathrm{Limit}}\, r}{f_{c2}\sqrt{\mu\varepsilon_0}\sqrt{L^2+W^2}}, & f \ge f_{c2}. \end{cases} \quad (18)$$

IV. Validation

The closed-form expression (15) for estimating the maximum radiated field is validated by comparing with analytic calculations of the field strengths radiated by the equivalent magnetic current derived from the cavity model. The analytic calculations were validated by comparing to full-wave results obtained using a finite-difference time-domain (FDTD) code [9]. Fig. 2 shows analytic and FDTD results for the maximum radiation 3 m from a PCB excited by a 1-mA current source located at one corner of the board. The board parameters are: $W = 20$ cm, $L = 25$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, and $\tan\delta = 0.02$. Fig. 3 shows the corresponding maximum power bus noise voltage around the periphery of the board. Fig. 4 shows the radiated electric field strength (in the direction of maximum radiation) divided by the maximum power bus noise voltage as well as the maximum possible value as determined by (15). Similar calculated and estimated maximum radiation results for boards with various dimensions are shown in Fig. 5 ($W = 40$ cm, $L = 45$ cm), Fig. 6 ($W = 10$ cm, $L = 25$ cm), Fig. 7 ($W = 5$ cm, $L = 5$ cm), and Fig. 8 ($W = 5$ cm, $L = 25$ cm). Fig. 9 shows the results obtained for two 10 cm × 25 cm × 0.1 cm boards with different dielectric permittivities.

In order to include a sufficient number of resonant peaks, the larger boards in this study were evaluated up to 1 GHz and the smaller boards were analyzed up to 6 GHz. The plots in Figs. 4–8 demonstrate that the closed-form expression (15) provides an accurate upper bound for the

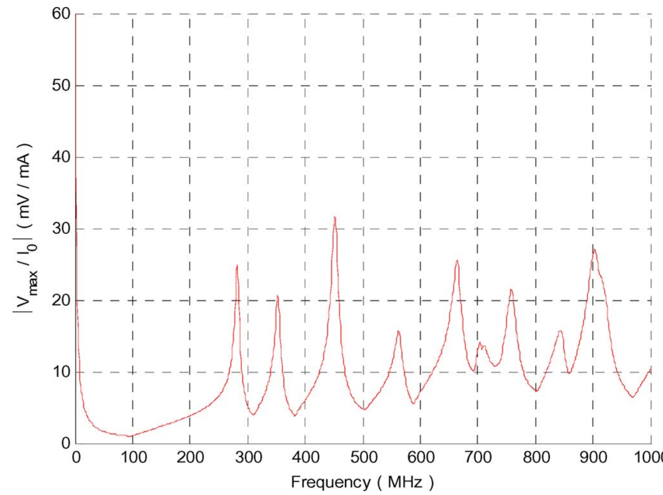

Fig. 3. Maximum power bus noise voltage due to a 1-mA current source located at one corner of a board with $W = 20$ cm, $L = 25$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, and $\tan\delta = 0.02$.

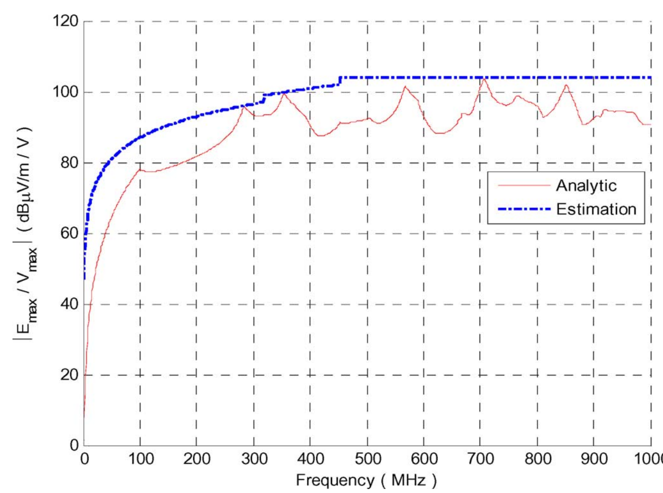

Fig. 4. Maximum radiation due to a given power bus noise voltage of a board with $W = 20$ cm, $L = 25$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, $\tan\delta = 0.02$, and $r = 3$ m.

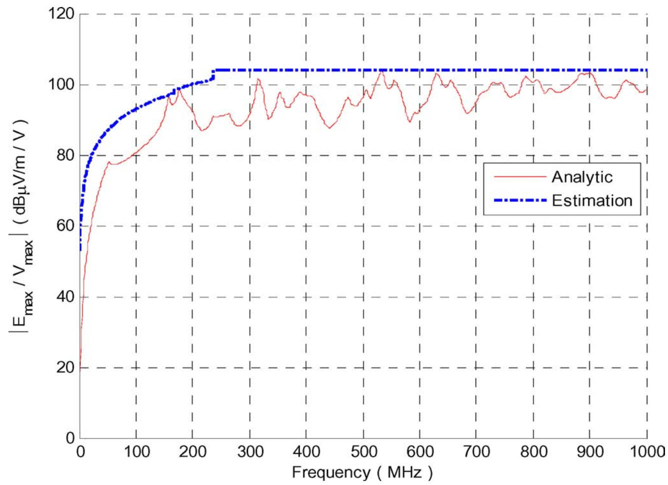

Fig. 5. Maximum radiation due to a given power bus noise voltage of a board with $W = 45$ cm, $L = 50$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, $\tan\delta = 0.02$, and $r = 3$ m.

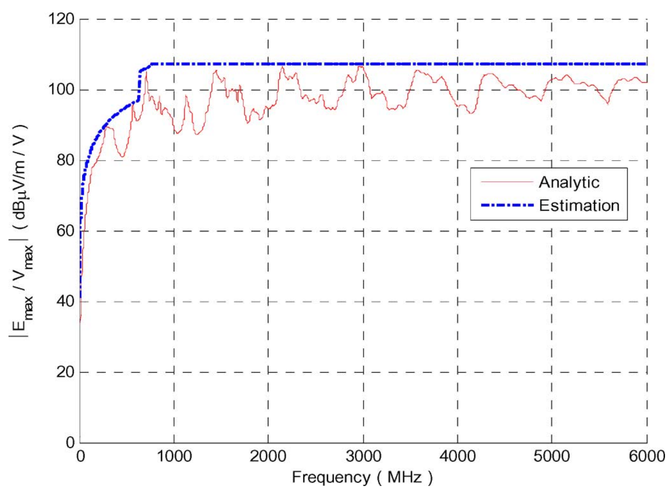

Fig. 6. Maximum radiation due to a given power bus noise voltage of a board with $W = 10$ cm, $L = 25$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, $\tan\delta = 0.02$, and $r = 3$ m.

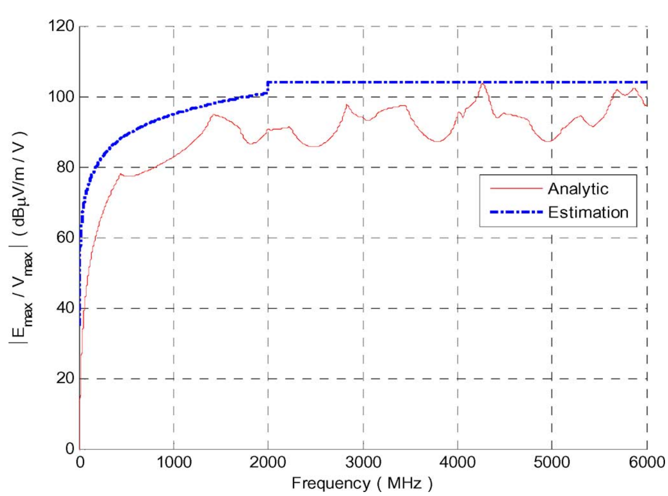

Fig. 7. Maximum radiation due to a given power bus noise voltage of a board with $W = 5$ cm, $L = 5$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, $\tan\delta = 0.02$, and $r = 3$ m.

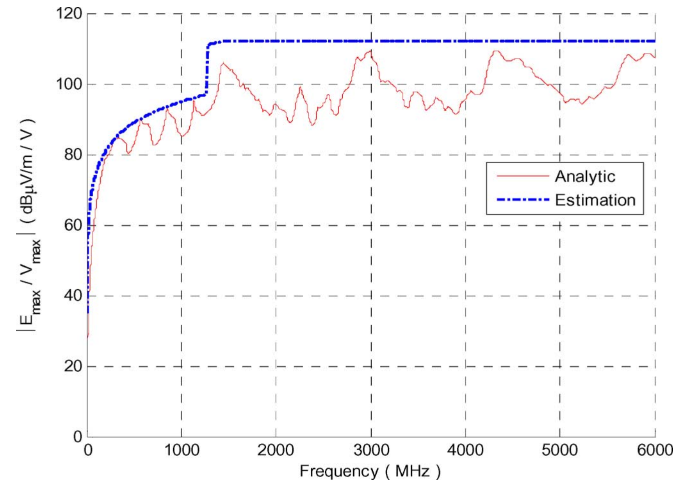

Fig. 8. Maximum radiation due to a given power bus noise voltage of a board with $W = 5$ cm, $L = 25$ cm, $h = 0.1$ cm, $\varepsilon_r = 4.5$, $\tan\delta = 0.02$, and $r = 3$ m.

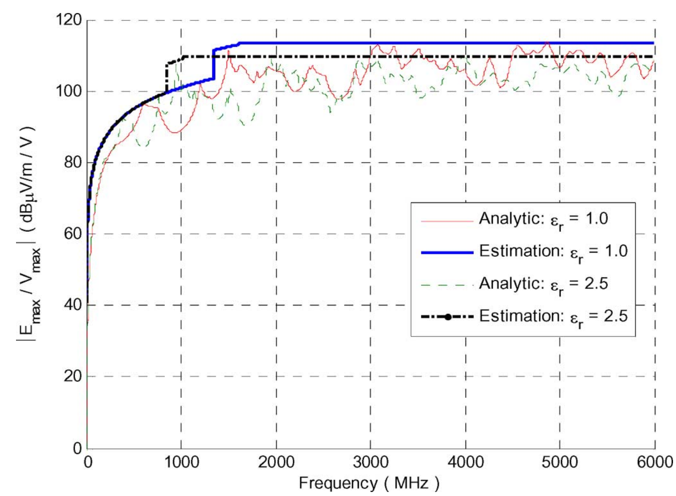

Fig. 9. Maximum radiation due to a given power bus noise voltage of a board with $W = 10$ cm, $L = 25$ cm, $h = 0.1$ cm, $\tan\delta = 0.02$, and $r = 3$ m for two values of ε_r.

amplitude of the radiation peaks from a board with a given maximum voltage between the planes.

Table I summarizes the results of a similar analysis performed on a variety of boards of different sizes. The board dimensions ranged from 5 cm to 50 cm. The permittivity of the substrate was 4.5. Peak 1 corresponds to the TM_{10}-mode frequency, peak 2 corresponds to the TM_{01} mode, and peak 3 corresponds to the frequency at or above the TM_{11} cutoff frequency (within the calculation frequency range) where the field-strength-to-voltage ratio is the highest. For every board evaluated, the actual value was no higher than the estimated upper bound. However, as indicated by the results shown in the table, the peak emissions come within a few decibels of the estimate for each of the boards evaluated.

If (15) is an upper bound on the emissions for a given peak power bus noise voltage, then (18) can be used to determine the minimum voltage required to generate a field exceeding a given limit (e.g., a maximum radiated field specification). In other words, if the peak voltage on the perimeter of the power bus is below the level calculated by (18), then the power bus will not radiate above the given limit. Fig. 10 shows the

TABLE I
DIFFERENCE BETWEEN THE UPPER BOUND ESTIMATE AND THE PEAK EMISSIONS FOR VARIOUS BOARD DIMENSIONS

Board Dimension $W \times L$ (cm)	Estimate – Actual Field Strength in dB					
	Peak 1		Peak 2		Peak 3	
5×5	TM_{10}	3.0	TM_{01}	3.0	TM_{31}	0.2
5×10	TM_{10}	0.02	TM_{01}	6.0	TM_{13}	0.4
5×25	TM_{10}	0.06	TM_{01}	5.9	TM_{42}	3.0
10×15	TM_{10}	0.15	TM_{01}	0.4	TM_{24}	0.1
10×25	TM_{10}	0.03	TM_{01}	0.9	TM_{23}	0.9
10×35	TM_{10}	0.06	TM_{01}	2.9	TM_{12}	1.7
15×20	TM_{10}	0.09	TM_{01}	0.2	TM_{02}	0.3
15×35	TM_{10}	0.58	TM_{01}	1.0	TM_{12}	1.7
15×40	TM_{10}	0.03	TM_{01}	1.5	TM_{12}	1.6
20×25	TM_{10}	0.07	TM_{01}	0.25	TM_{02}	0.5
20×50	TM_{10}	0.08	TM_{01}	1.1	TM_{12}	1.5
25×25	TM_{10}	3.0	TM_{01}	3.0	TM_{03}	0.4
25×35	TM_{10}	0.09	TM_{01}	0.2	TM_{02}	0.9
25×45	TM_{10}	0.09	TM_{01}	0.9	TM_{02}	0.5
30×40	TM_{10}	0.09	TM_{01}	0.2	TM_{02}	0.4
45×50	TM_{10}	0.21	TM_{01}	0.25	TM_{03}	0.0
50×50	TM_{10}	3.09	TM_{01}	3.09	TM_{16}	1.0

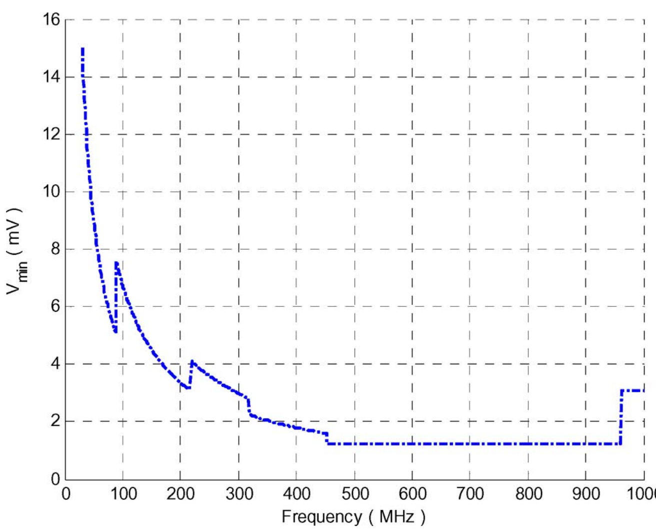

Fig. 10. Minimum voltage required to generate fields exceeding the FCC class B limit.

minimum voltage between the planes of a 20 cm × 25 cm × 0.1 cm board with $\varepsilon_r = 4.5$ and $\tan\delta = 0.02$, required to generate a field exceeding the FCC class B limit as calculated using (18). It shows that, at 50 MHz, approximately 9.0 mV is required to exceed the radiated emission specification. However at 500 MHz, as little as 1.2 mV is sufficient to cause a failure. A board of this size exhibiting less power bus noise than that indicated in Fig. 10 is not capable of exceeding the FCC class B limit due to radiation directly from the planes.

V. CONCLUSION

An expression for the maximum radiated field from a rectangular parallel-plate power bus structure exhibiting a given maximum power bus voltage has been developed. The equation can be solved in reverse to determine the minimum voltage required for a given board to radiate above a specified limit. The expression was validated by comparing it with analytic and full-wave model results for boards of various sizes. The maximum radiation-to-voltage ratio estimates are within a few decibels of the calculated peaks for all of the boards evaluated in this study.

While the expression provided in (15) is expected to perform well for the vast majority of board geometries, there are a couple of special situations worth noting. First, the derivation of (15) implicitly assumes that the first resonant peak is the TM_{10} resonance. It is possible (though not likely) that resonances involving components mounted to the board could induce enough voltage in the planes to cause a significant radiated peak at a frequency below the TM_{10} resonance. If this were to occur the radiated field due to the long sides of the board would dominate and (15) would underestimate the radiated field. If this is a concern, the W in the first term of (15) should be replaced by an L, thus making the first and second terms identical, and thereby eliminating the transition between them.

A second special situation would be a nearly square board exhibiting resonances associated with highly asymmetric modes. If, for example, the TM_{09} mode was somehow excited efficiently in a square board, this board could radiate more than estimated by (15), which assumes that no TM_{m0} or TM_{0n} modes are efficiently excited at frequencies above the TM_{11} cutoff. This situation is unlikely to occur in typical PCB structures, but it could be an issue in boards with highly repetitive structures.

Finally, it should be noted that many PCBs are not rectangular, and therefore, do not have a well-defined L and W. As long as the shape is basically a polygon, it should be possible to replace the L, W, and $\sqrt{L^2 + W^2}$ in (15) with a single value, D, equal to the length of the longest board dimension. This will be slightly less accurate than (15) at the lower frequencies, but should still provide an effective upper bound estimate.

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