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The Role of Decoupling Capacitor ESR in Resonance Suppression

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Executive Summary

At resonance, the impedance of a printed circuit board's power bus can increase dramatically, possibly leading to signal integrity and radiated emissions problems. One way to mitigate this problem is to add loss to the power bus by placing lossy components on the edges of the printed circuit board [1]. The optimum value of resistance for these lossy components depends on the number of components applied and the amount of connection inductance. General guidelines for selecting an optimum component equivalent series resistance (ESR) based on the equivalent series inductance (ESL) of the component are provided. Numerical simulations provide validation of the guidelines.

1 Introduction

At resonance, the power bus impedance of printed circuit boards with solid copper planes can be relatively high. Adding loss to the system, thereby lowering the quality factor of the power bus, can lower the power bus impedance at resonance. Many loss mechanisms exist within a printed circuit board power bus, including radiation loss, dielectric loss, conduction loss and component loss. On a typical PCB, radiation loss and dielectric loss are not great enough to completely eliminate resonances. Conduction loss can completely damp power bus resonances if the power planes are closely spaced (e.g. <0.1 mm [2]), but manufacturing boards with closely spaced power planes is expensive. Additionally, if the power planes are widely spaced (e.g. >0.1 mm [2]), conduction loss will not be great enough to completely dampen resonant peaks in the impedance. Optimizing component loss however, can be a cost-effective alternative to optimizing conduction or dielectric loss. Methods for adding component loss into the power bus structure have been developed [1] and have been shown to be effective at dampening power bus resonances. However, the effects of the equivalent series inductance (ESL) of the lossy component's connection to the power bus have not been considered in previous studies.

The goal of this work is to investigate a strategy that will help eliminate power bus resonance through the careful selection and placement of lossy decoupling capacitors along the edge of a printed circuit board. To determine the optimum value for the lossy capacitor's ESR, the power bus is modeled as several one-dimensional transmission lines in parallel; each one terminated with a lossy capacitor. The optimum value of the ESR is calculated by minimizing the reflection coefficient at the termination. Using this method, the effect of component ESL is modeled. To verify that the optimum value of ESR calculated for a one-dimensional transmission line model is also optimum for printed circuit boards, results are compared to simulations employing a two-dimensional transmission line grid model.

2 One-Dimensional Transmission Line Model

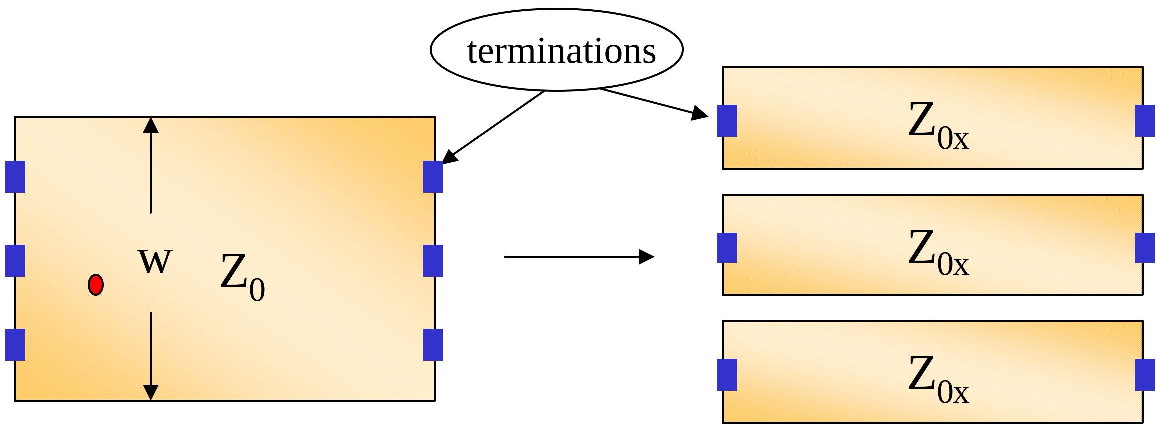
To determine the optimum resistance of the lossy components, the two-dimensional power bus of a PCB is represented as several one-dimensional transmission lines in parallel. If the entire power bus were represented as a one-dimensional transmission line, the characteristic impedance of that transmission line, Z_0, would be:

$$Z_0 = \frac{hd}{w} \tag{1}$$

where w is the cross-sectional width of the transmission line, d is the height of the dielectric between the planes and h is the intrinsic impedance of the dielectric material given by:

$$h = \frac{377}{\sqrt{e_r}} \text{ Ohms} \quad (2)$$

The number of one-dimensional transmission lines is set equal to the number of lossy components placed on the periphery of the board. For instance, if N components were placed uniformly along an edge of the power bus, then the characteristic impedance of each transmission line, Z_{0x}, would be equal to N * Z_0. This concept is illustrated in Fig. 1.



$$Z_{0x} = 3 \cdot Z_0$$

Fig. 1. Example of a power bus split into 3 one-dimensional transmission lines.

3 Series R Terminations on the Board Edge

To maximize the loss dissipated in each one-dimensional transmission line, the reflection coefficient at the termination must be minimized. The expression for the reflection coefficient on a transmission line is given as:

$$|\Gamma| = \left| \frac{Z_t - Z_{0x}}{Z_t + Z_{0x}} \right| \quad (3)$$

where Z_t is the impedance of the termination. Maximum power is dissipated in a termination when the termination impedance is equal to Z_{0x} ($|\Gamma| = 0$) or the reflection coefficient is minimized. If each lossy component is modeled as a simple resistive termination with resistance ESR, then the optimum value for ESR is equal to Z_{0x}. Fig. 2 shows the relationship between the reflection coefficient and the value of ESR in a 1.2-ohm transmission line for a purely resistive termination. Note how the reflection coefficient is equal to zero when ESR=Z_{0x}.

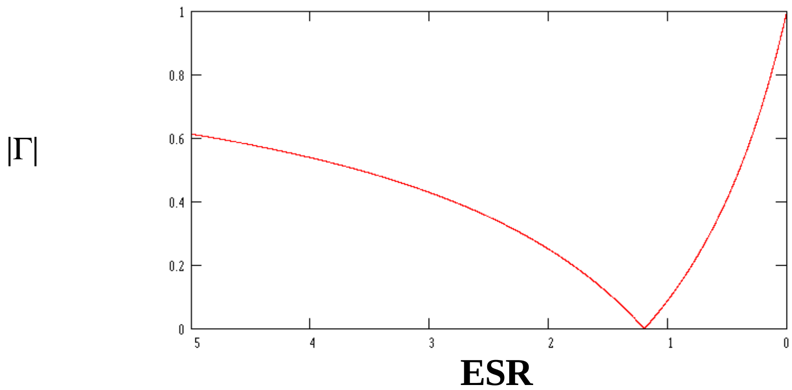


Fig. 2. The reflection coefficient plotted as a function of the termination ESR.
ESL = 0 nH, Z_{0x} = 1.2 Ohms.

4 Series R-L Terminations on the Board Edge

Any lossy component used on a printed circuit board will have some connection inductance. Because of this, the termination cannot be modeled simply as a resistor, but must generally be modeled as a resistance and inductance in series. Additionally, the component's connection inductance, ESL, will change the optimum value of ESR.

If the termination is modeled as a series R-L circuit, then from (3), the reflection coefficient becomes:

$$\Gamma = \frac{ESR + jwESL - Z_{0x}}{ESR + jwESL + Z_{0x}} \qquad (4)$$

If the ESR and ESL are normalized to Z_{0x}, then (4) becomes:

$$\Gamma = \frac{r + jz - 1}{r + jz + 1} = \frac{r - 1 + jz}{r + 1 + jz} \qquad (5)$$

where

$$r = \frac{ESR}{Z_{0x}} \text{ and } z = \frac{wESL}{Z_{0x}} \tag{6}$$

To find the optimum value of ESR, the magnitude of the reflection coefficient must be minimized.

$$\Gamma = \frac{r-1+jz}{r+1+jz} \cdot \frac{r+1-jz}{r+1-jz} = \frac{(r-1)(r+1)+z^2+j2z}{(r+1)^2+z^2} \tag{7}$$

$$|\Gamma| = \left| \frac{(r-1)(r+1)+z^2+j2z}{(r+1)^2+z^2} \right| = \sqrt{\frac{\left((r-1)(r+1)+z^2\right)^2+(2z)^2}{\left((r+1)^2+z^2\right)^2}} \tag{8}$$

$$|\Gamma| = \sqrt{\frac{r^2-2r+z^2+1}{r^2+2r+1+z^2}} \tag{9}$$

Squaring (9) gives:

$$|\Gamma|^2 = \frac{r^2-2r+z^2+1}{r^2+2r+1+z^2} \tag{10}$$

To find the value of ‘r’ that minimizes $|\Gamma|^2$, the first derivative of (10) is taken with respect to ‘r’ and set equal to zero.

$$\frac{\partial}{\partial r}\left(|\Gamma|^2\right) = 0 = 4\frac{r^2-1-z^2}{\left(r^2+2r+1+z^2\right)^2} \tag{11}$$

Equation (11) is equal to zero when:

$$r^2 = 1+z^2 \tag{12}$$

Using (6),

$$ESR^2 = Z_{0x}^2 + (wESL)^2 \tag{13}$$

Therefore, the optimum value for the ESR is:

$$ESR = \sqrt{Z_{0x}^2 + (wESL)^2} \tag{14}$$

or

$$ESR = \left| Z_{0x} + jwESL \right| \qquad (15)$$

As (15) indicates, the optimum value of ESR is dependent on the frequency, the value of the ESL, and the characteristic impedance Z_{0x}. In Fig. 3 and 4, the reflection coefficient is shown as a function of the ESR with two different values of ESL. In Fig. 3, the value of $w \cdot ESL$ is small relative to Z_{0x}. In Fig. 4, the value of $w \cdot ESL$ is large relative to the value of the ESR. In both figures, the reflection coefficient is shown at three frequencies. Values of $w \cdot ESL$ at different frequencies are given in Table 1.

The printed circuit board modeled was 10 cm by 6 cm, the relative dielectric constant, e_r, was equal to 4, and the dielectric height was 5 mils. If six terminations where placed on the two 6-cm sides of the board, then Z_{0x} is equal to 2.4 ohms. Table 1 and Table 2 show the values of $|w \cdot ESL|$ and $|jw \cdot ESL + Z_{0x}|$ for different values of ESL and frequency.

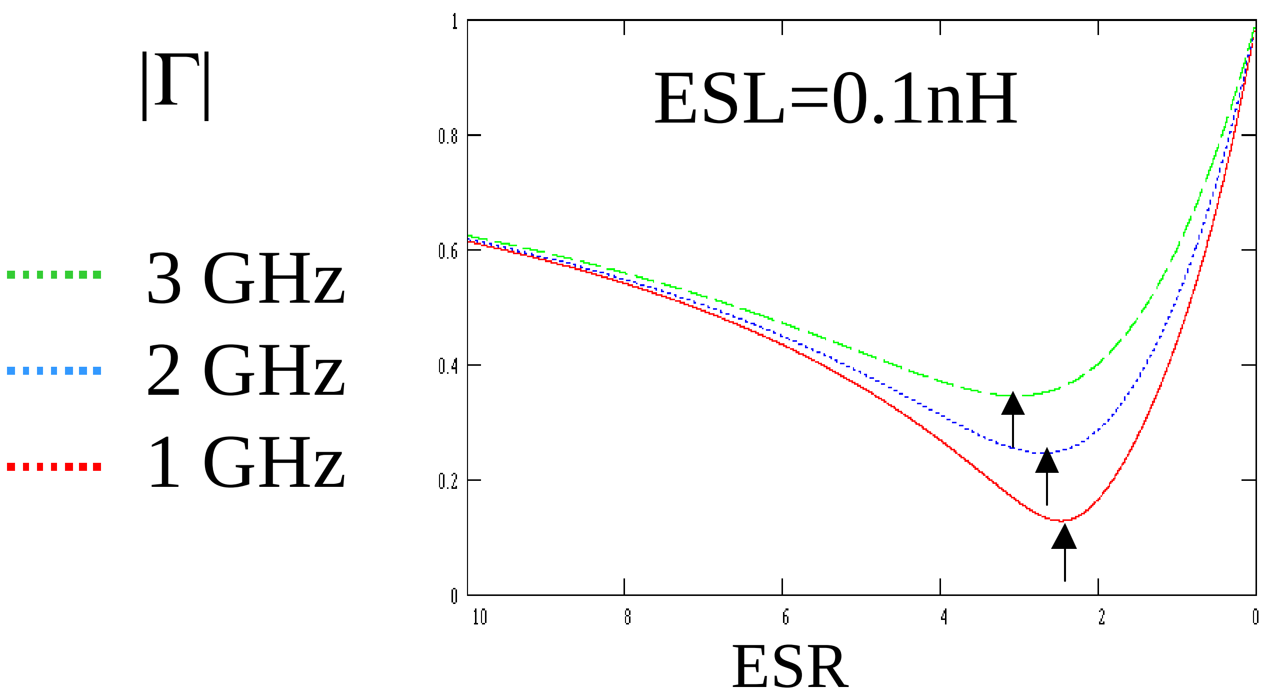


Fig. 3. The reflection coefficient of a termination with ESL and ESR. Arrows indicate optimum values for the ESR. ESL = 0.1 nH, Z_{0x} = 2.4 ohms.

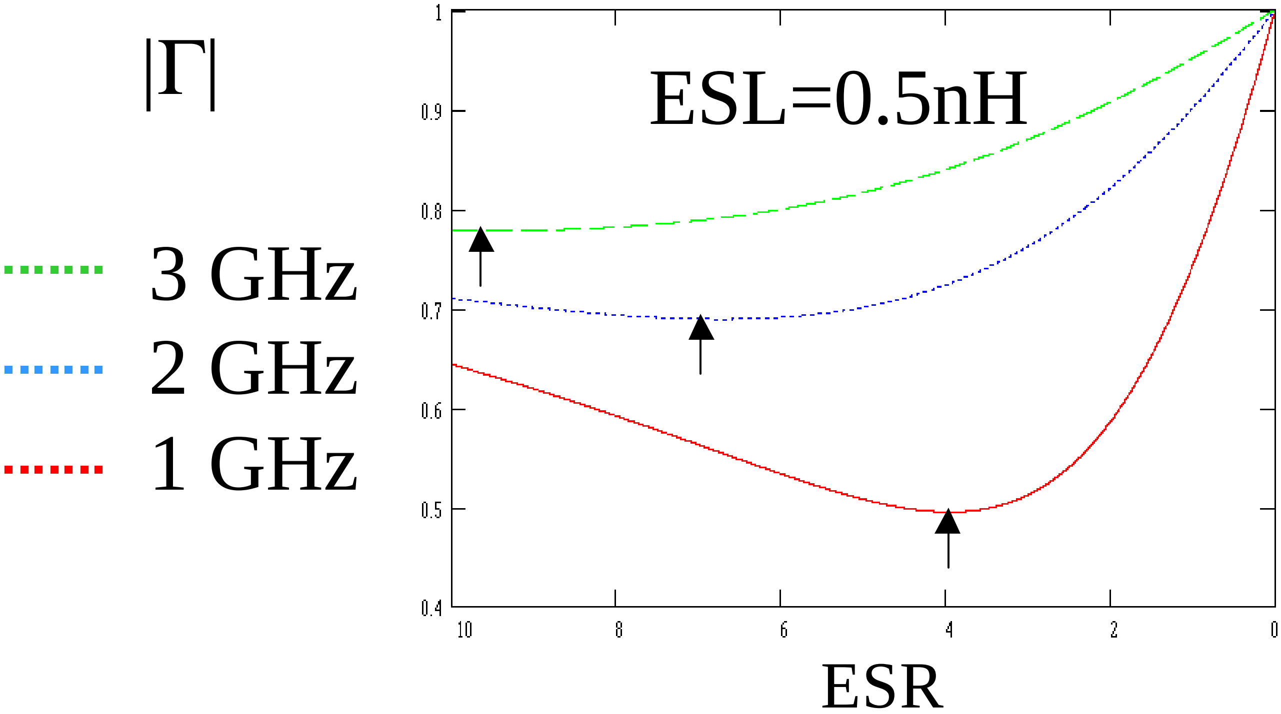


Fig. 4. The reflection coefficient of a termination with ESL and ESR. Arrows indicate optimum values for the ESR. ESL = 0.5 nH. Z_{0x} = 2.4 ohms.

Table 1: $\left| jw \cdot ESL \right|$ (Ohms)

	ESL=1nH	ESL =0.5nH	ESL =0.1nH	ESL =0.01nH
f=1GHz	6.2	3.1	0.63	0.06
f=2GHz	12.5	6.2	1.3	0.12
f=3GHz	18.8	9.4	1.9	0.18

Table 2: $\left| jw \cdot ESL + Z_{0x} \right|$ (Ohms)

	ESL =1nH	ESL =0.5nH	ESL =0.1nH	ESL =0.01nH
f=1GHz	6.7	4.0	2.5	2.4
f=2GHz	12.8	6.7	2.7	2.4
f=3GHz	19	9.7	3.0	2.4

5 Validation with the Two-Dimensional Model

To validate the conclusions drawn from the one-dimensional transmission line method, a two-dimensional transmission line model was used. In this model, the power bus is represented as a grid of small, lossy transmission line segments [1]. Each segment of the grid contains resistive elements, representing conductive and dielectric loss. Components connected to the power bus, (e.g. decoupling capacitors and integrated circuits) can be included in the model by adding the component's admittance to the appropriate grid point of the model. Conductive, dielectric and component loss can be modeled accurately with this technique. Radiation loss is not modeled, but this loss mechanism is typically insignificant compared to the other loss mechanisms in a printed circuit board [2], [4].

Two-dimensional simulations were run for 4 different values of termination ESL (1 nH, 0.5 nH, 0.1 nH, and 0.01 nH) to verify the optimum values of ESR calculated in Table 2. Fig. 5 through Fig. 8 show the power bus impedance at different values of ESR for a fixed value of ESL. The values of ESR were chosen such that the values calculated in Table 2 at 2.2 GHz could be verified.

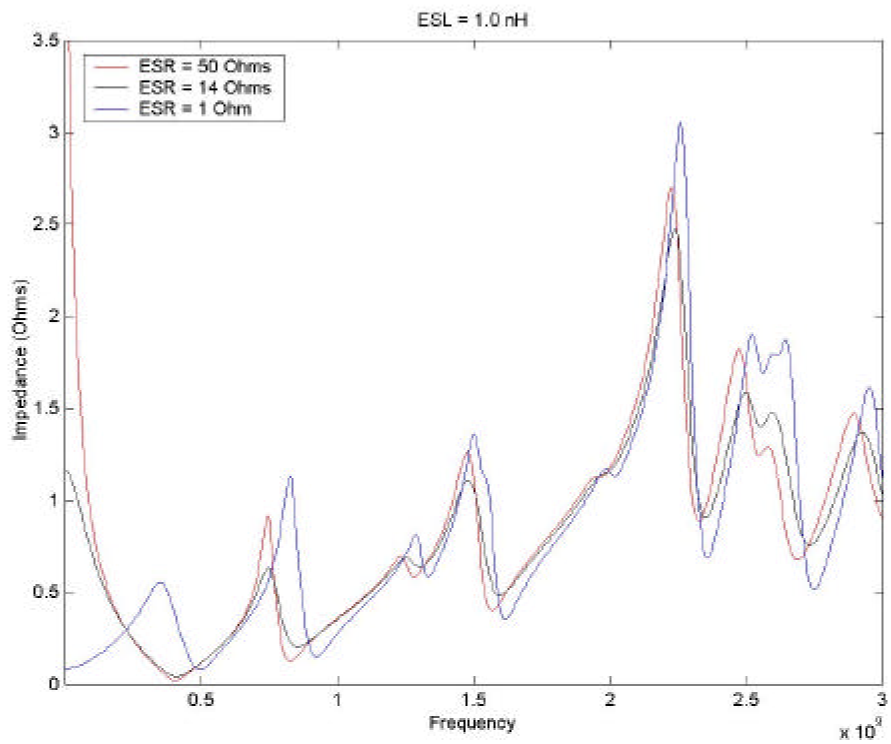


Fig. 5. The power bus impedance calculated with the 2-D model. ESL = 1.0 nH, Z_{0x} = 2.4 ohms.

In Fig. 5 (ESL = 1 nH), the ESR is set equal to 50, 14 and 1 ohm. When the ESR is equal to 50 ohms, a value much higher than $w \cdot ESL$, very little power is dissipated in the terminations, and the power bus impedance is high at the resonant frequencies of the board. When the termination ESR is 14 ohms, which is the optimum value for the ESR given $ESL = 1\,nH$, Z_{0x} = 2.4 ohms and f = 2.2 GHz, the impedance at 2.2 GHz is at its lowest level. When the ESR was lowered to 1 ohm, the impedance at 2.2 GHz is higher than it is with the ESR equal to 14 ohms.

In Fig. 6 (ESL = 0.5 nH), the ESR was set equal to 50, 7 and 1 ohm. Similar to Fig. 5, the power bus impedance is lowest (at 2.2 GHz) when the value of ESR calculated in Table 2 is used (ESR = 7 ohms).

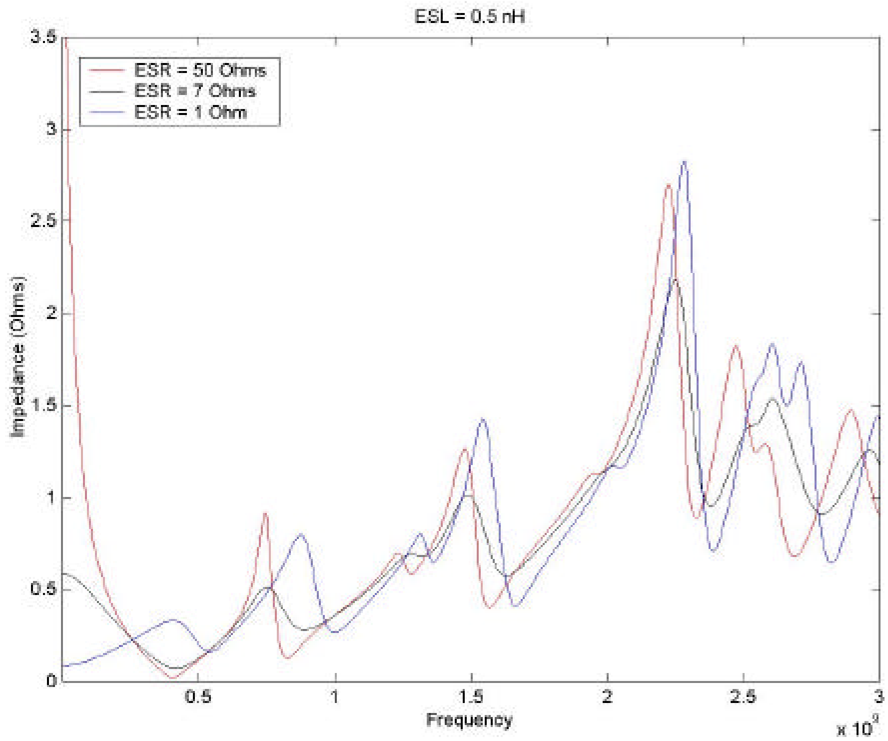


Fig. 6. The power bus impedance calculated with the 2-D model.
ESL = 0.5 nH, Z_{0x} = 2.4 ohms.

Fig. 7 (ESL = 0.1 nH) and Fig. 8 (ESL = 0.01 nH) show the power bus impedance when the ESR is equal to 50, 2.5 and 1 ohms. In these figures, the optimum value for the ESR is around 2.5 ohms, since Z_{0x} is equal to 2.4 ohms and $w \cdot ESL$ is rather small in comparison.

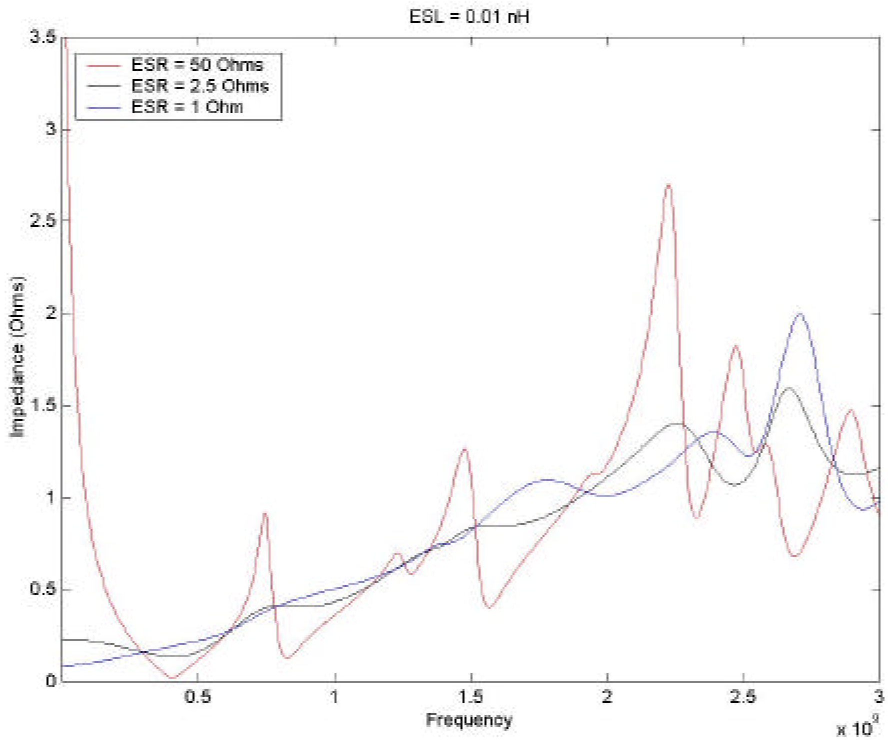


Fig. 7. The power bus impedance calculated with the 2-D model.
ESL = 0.1 nH, Z_{0x} = 2.4 ohms

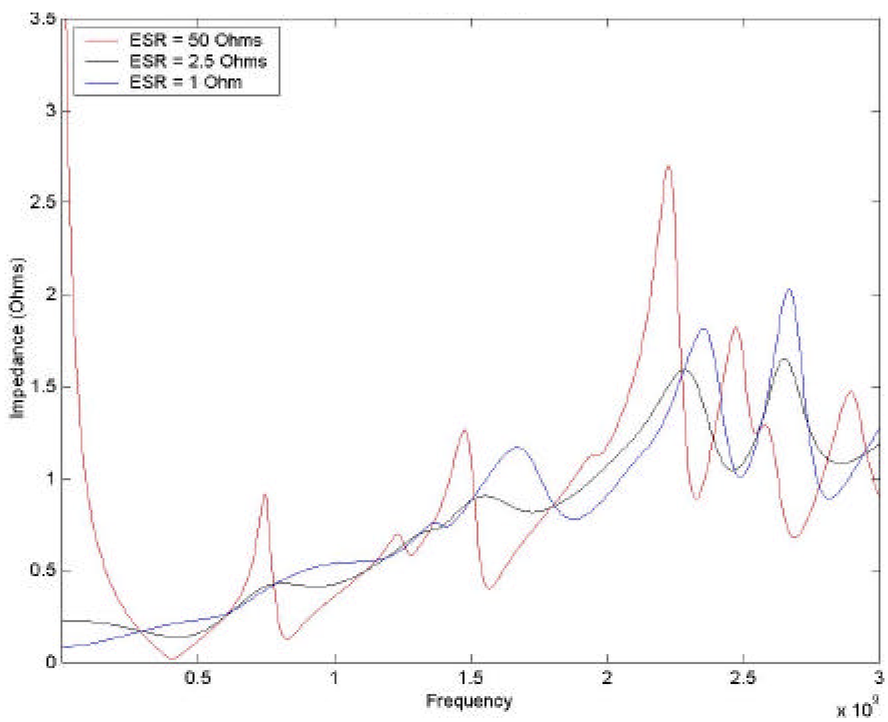


Fig. 8. The power bus impedance calculated with the 2-D model.
ESL = 0.01 nH, Z_{0x} = 2.4 ohms

6 Conclusions

Using a one-dimensional transmission line model, an optimum termination ESR, that will reduce the power bus impedance at resonance, can be found. Frequency dependant parasitics like the ESL of the termination affect the optimum value of the ESR, but this effect can be accounted for with this method. Numerical simulations with a two-dimensional transmission line grid model validate the results found with the one-dimensional analysis.

In general, if $w \cdot ESL$ is low relative to Z_{0x} , then the optimum value for the termination ESR is close to Z_{0x} . However, if the parasitic ESL of the termination is large relative to Z_{0x}, then the optimum value for the termination ESR will be close to $w \cdot ESL$. Since $w \cdot ESL$ is frequency dependent, it is not possible to damp resonances over a wide frequency range unless the ESL impedance is small relative to Z_{0x}. For a wide-band solution, the termination ESR should be selected to maintain a low reflection coefficient over the frequency range of interest. Selection of the optimum ESR over a wide frequency range can be done with the help of the one-dimensional transmission line model.

References

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[2] M. Xu, T. Hubing, J. Drewniak, T. Van Doren, R. DuBroff "Modeling Printed Circuit Boards with Embedded Decoupling Capacitance," *Proceedings of the 2001 IEEE International Symposium on Electromagnetic Compatibility,* Zurich, Switzerland, Feb. 2001

[3] K. Lee and W. Chen, *Advances in Microstrip and Printed Antennas*, New York, John Wiley & Sons, 1997, Chapter 5.