



University of Missouri-Rolla

Electromagnetic Compatibility Laboratory

Title: **Survey of Numerical Electromagnetic
Modeling Techniques**

Report
Number: TR91-1-001.3

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Sponsor: Intel Corporation

Date: September 1, 1991

This report reviews and summarizes numerical electromagnetic modeling techniques that can be used to analyze sources of electromagnetic interference. Several references are cited to aid the reader who wishes to investigate a particular technique in more detail.

Introduction

Computer techniques have revolutionized the way in which electromagnetic problems are analyzed. Antenna and microwave engineers rely heavily on computer methods to analyze and help evaluate new designs or design modifications. Although most EM problems ultimately involve solving only one or two partial differential equations subject to boundary constraints, very few practical problems can be solved without the aid of a computer.

Computer methods for analyzing problems in electromagnetics generally fall into one of three categories, analytical techniques, numerical techniques, and expert systems. Analytical techniques make simplifying assumptions about the geometry of a problem in order to apply a closed-form (or table look-up) solution. Numerical techniques attempt to solve fundamental field equations directly, subject to the boundary constraints posed by the geometry. Expert systems do not actually calculate the field directly, but instead estimate values for the parameters of interest based on a rules database.

A number of computer programs based on analytical techniques are available to the EMC engineer. Some are very simple and run on personal computers. Others such as IEMCAP [1] are very elaborate. Analytical techniques can be a useful tool when the important EM interactions of the configuration can be anticipated. However, most EMC problems of interest are simply too unpredictable to be modeled using this approach.

Expert systems approach a problem in much the same way as a quick-thinking, experienced EM engineer with a calculator would approach it. As system design and board layout procedures become more automated, expert system EM software will certainly play an important role. Nevertheless, expert systems are no better than their rules database and it is unlikely that they will ever be used to model or understand the complex EM interactions that cause EMI sources to radiate.

Numerical techniques generally require more computation than analytical techniques or expert systems, but they are very powerful EM analysis tools. Without making a priori assumptions about which field interactions are most significant, numerical techniques analyze the entire geometry provided as input. They calculate the solution to a problem based on a *full-wave* analysis.

A number of different numerical techniques for solving electromagnetic problems are available. Each numerical technique is well-suited for the analysis of a particular type of problem. The

numerical technique used by a particular EM analysis program plays a significant role in determining what kinds of problems the program will be able to analyze.

This report reviews the strengths and limitations of different numerical techniques for analyzing electromagnetic configurations. A particular emphasis is placed on how these techniques could be applied to the analysis of electromagnetic interference (EMI) sources.

EMI Source Models

One problem with attempting to analyze sources of EMI on a computer is that it is difficult to predict what details of the EMI source need to be modeled. Generally, it is the *common-mode* currents in a system that have the biggest impact on the radiated EMI [2,3]. These currents are not intentionally generated and they cannot be predicted using simple lumped-element circuit-modeling techniques.

Sources of common-mode current are often difficult to locate. A single printed circuit card may contain many potential sources and coupling paths. The intentionally generated, *differential-mode* currents are generally orders of magnitude larger than the common-mode currents. Because of this, there has been a tendency for source models to focus on the differential-mode currents. Elaborate transmission line modeling techniques have been employed to calculate these currents with a high degree of accuracy. In practical situations however, these currents have been found to have little or no correlation with the radiated field strengths.

In order to be truly useful, any EMI source modeling technique must be able to model all aspects of the system that could affect the common-mode current levels. For systems with printed circuit cards, this means modeling the geometric details of configurations containing metal surfaces, wires, and dielectrics. In general, these configurations have no particular symmetry and they are neither electrically large nor small. This presents a significant challenge for a numerical modeling technique. So far, no one technique has been developed that is able to meet this challenge. As a result, computer techniques have not been utilized for EMI source modeling to the extent that they have for modeling antennas and microwave devices.

The state-of-the-art in numerical electromagnetic modeling is advancing at a rapid pace, however. Every year hundreds of papers are published describing new techniques, enhancements to existing techniques, new implementations of existing techniques, and new applications of

computer modeling. Sorting through this wealth of information in order to choose the computer technique that is best for a particular application can be overwhelming.

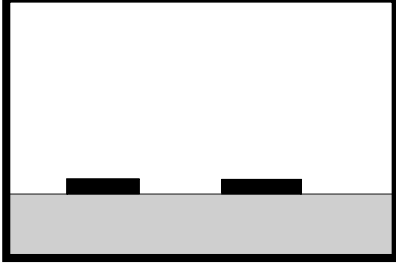
The following sections outline several general numerical modeling techniques that have been used to analyze EMI source configurations with some success. Each technique is best-suited to analyze different configurations. No one technique can be used to model all EMI sources, however each of these techniques can be applied to a number of EMI source configurations. Two or three of these techniques, collectively, represent a potentially powerful set of tools for the EMI engineer.

Finite Element Methods

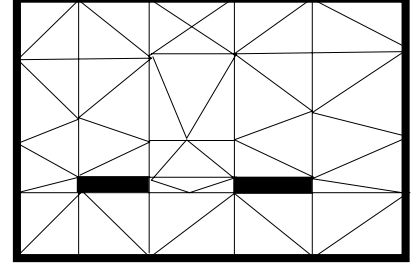
Scalar finite element methods are widely used by civil and mechanical engineers to analyze material and structural problems. Electrical engineers use finite element methods to solve complex, nonlinear problems in magnetics and electrostatics. Until recently however, very little practical modeling of 3-dimensional electromagnetic radiation problems was performed using this technique. There were two reasons for this. First, practical three-dimensional vector problems require significantly more computation than two-dimensional or scalar problems. Second, spurious solutions known as *vector parasites* often result in unpredictable, erroneous results. However, recent developments in this field [4,5] appear to have solved the vector parasite problem. An increasing availability of computer resources coupled with a desire to model more complex electromagnetic problems has resulted in a wave of renewed interest in finite element methods for solving EM radiation problems.

The first step in finite-element analysis is to divide the configuration into a number of small homogeneous pieces or *elements*. An example of a finite-element model is shown in Figure 1. The model contains information about the device geometry, material constants, excitations and boundary constraints. The elements can be small where geometric details exist and much larger elsewhere. In each finite element, a simple (often linear) variation of the field quantity is assumed. The corners of the elements are called *nodes*. The goal of the finite-element analysis is to determine the field quantities at the nodes.

Most finite element methods are *variational* techniques. Variational methods work by minimizing or maximizing an expression that is known to be stationary about the true solution. Generally, finite-element analysis techniques solve for the unknown field quantities by minimizing an energy



Structure Geometry



Finite-Element Model

Figure 1: Finite-Element Modeling Example

functional. The energy functional is an expression describing all the energy associated with the configuration being analyzed. For 3-dimensional, time-harmonic problems this functional may be represented as,

$$F = \int_v \frac{\mu |\mathbf{H}|^2}{2} + \frac{\epsilon |\mathbf{E}|^2}{2} - \frac{\mathbf{J} \cdot \mathbf{E}}{2j\omega} dv \quad (1)$$

The first two terms in the integrand represent the energy stored in the magnetic and electric fields and the third term is the energy dissipated (or supplied) by conduction currents.

Expressing $\mathbf{H}$ in terms of $\mathbf{E}$ and setting the derivative of this functional with respect to $\mathbf{E}$ equal to zero, an equation of the form $f(\mathbf{J}, \mathbf{E}) = 0$ is obtained. A k^{th} -order approximation of the function f is then applied at each of the N nodes and boundary conditions are enforced, resulting in the system of equations,

$$\begin{bmatrix} J_1 \\ J_2 \\ \cdot \\ \cdot \\ J_n \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdot & \cdot \\ y_{21} & y_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & y_{nn} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ E_n \end{bmatrix} \quad (2)$$

The values of $\mathbf{J}$ on the left-hand side of this equation are referred to as the source terms. They represent the known excitations. The elements of the $\mathbf{Y}$ -matrix are functions of the problem geometry and boundary constraints. Since each element only interacts with elements in its own *neighborhood*, the $\mathbf{Y}$ -matrix is generally sparse. The terms of the vector on the right-hand side

represent the unknown electric field at each node. These values are obtained by solving the system of equations. Other parameters, such as the magnetic field, induced currents, and power loss can be obtained from the electric field values.

In order to obtain a unique solution, it is necessary to constrain the values of the field at all boundary nodes. For example, the metal box of the model in Figure 1 constrains the tangential electric field at all boundary nodes to be zero. A major weakness of the finite element method is that it is relatively difficult to model *open* configurations (i.e. configurations where the fields are not known at every point on a closed boundary). Various techniques such as *ballooning* and absorbing boundaries are used in practice to overcome this deficiency. These techniques work reasonably well for 2-dimensional problems, but so far they are not very effective for 3-dimensional electromagnetic radiation problems.

The major advantage that finite element methods have over other EM modeling techniques stems from the fact that the electrical and geometric properties of each element can be defined independently. This permits the problem to be set up with a large number of small elements in regions of complex geometry and fewer, larger elements in relatively open regions. Thus it is possible to model configurations that have complicated geometries and many arbitrarily shaped dielectric regions in a relatively efficient manner.

Commercial finite element codes [6,7] are available that have graphical user interfaces and can determine the optimum placement of node points for a given geometry automatically. These codes are used to model a wide variety of electromagnetic devices such as spark plugs, transformers, waveguides, and integrated circuits.

Specific implementations of three-dimensional electromagnetic finite element codes are described in Ph.D. dissertations by Maile [8] and Webb [9]. Silvester and Ferrari [10] have written an excellent text on this subject for electrical engineers.

Moment Methods

Like finite-element analysis, the *method of moments* (or moment method) is a technique for solving complex integral equations by reducing them to a system of simpler linear equations. In contrast to the variational approach of the finite element method however, moment methods employ a technique known as the *method of weighted residuals*. Actually, the terms method-of-moments

and method-of-weighted-residuals are synonymous. Harrington [11] was largely responsible for popularizing the term *method of moments* in the field of electrical engineering. His pioneering efforts first demonstrated the power and flexibility of this numerical technique for solving problems in electromagnetics.

All weighted residual techniques begin by establishing a set of trial solution functions with one or more variable parameters. The *residuals* are a measure of the difference between the trial solution and the true solution. The variable parameters are determined in a manner that guarantees a *best fit* of the trial functions based on a minimization of the residuals.

The equation solved by moment method techniques is generally a form of the *electric field integral equation* (EFIE) or the *magnetic field integral equation* (MFIE). Both of these equations can be derived from Maxwell's equations by considering the problem of a field scattered by a perfect conductor (or a lossless dielectric). These equations are of the form,

$$EFIE: \quad \mathbf{E} = \mathbf{f}_e (\mathbf{J}) \quad (3)$$

$$MFIE: \quad \mathbf{H} = \mathbf{f}_m (\mathbf{J}) \quad (4)$$

where the terms on the left-hand side of these equations are incident field quantities and $\mathbf{J}$ is the induced current.

The form of the integral equation used determines which types of problems a moment-method technique is best suited to solve. For example one form of the EFIE may be particularly well suited for modeling thin-wire structures, while another form is better suited for analyzing metal plates. Usually these equations are expressed in the frequency domain, however the method of moments can also be applied in the time domain.

The first step in the moment-method solution process is to expand $\mathbf{J}$ as a finite sum of basis (or expansion) functions,

$$\mathbf{J} = \sum_{i=1}^M J_i \mathbf{b}_i \quad (5)$$

where $\mathbf{b}_i$ is the i^{th} basis function and J_i is an unknown coefficient. Next, a set of M linearly independent weighting (or testing) functions, w_j , are defined. An inner product of each weighting function is formed with both sides of the equation being solved. In the case of the MFIE (Equation 4), this results in a set of M independent equations of the form,

$$\langle w_j, \mathbf{H} \rangle = \langle w_j, f_m(\mathbf{J}) \rangle \quad j = 1, 2, \dots, M \quad (6)$$

By expanding $\mathbf{J}$ using Equation (5), we obtain a set of M equations in M unknowns,

$$\langle w_j, \mathbf{H} \rangle = \sum_{i=1}^M \langle w_j, f_m(J_i, \mathbf{b}_i) \rangle \quad j = 1, 2, \dots, M \quad (7)$$

This can be written in matrix form as,

$$[\mathbf{H}] = [\mathbf{Z}] [\mathbf{J}] \quad (8)$$

where: $Z_{ij} = \langle w_j, f_m(b_i) \rangle$

$\mathbf{J}_i = J_i$

$\mathbf{H}_j = \langle w_j, \mathbf{H}_{inc} \rangle$

The vector $\mathbf{H}$ contains the known incident field quantities and the terms of the $\mathbf{Z}$ -matrix are functions of the geometry. The unknown coefficients of the induced current are the terms of the $\mathbf{J}$ vector. These values are obtained by solving the system of equations. Other parameters such as the scattered electric and magnetic fields can be calculated directly from the induced currents.

Depending on the form of the field integral equation used, moment methods can be applied to configurations of conductors only, homogeneous dielectrics only, or very specific conductor-dielectric geometries. Moment method techniques applied to integral equations are not very effective when applied to arbitrary configurations with complex geometries or inhomogeneous dielectrics. They also are not well-suited for analyzing the interior of conductive enclosures or thin plates with wire attachments on both sides [12].

Nevertheless, moment method techniques do an excellent job of analyzing a wide variety of important three-dimensional electromagnetic radiation problems. General purpose moment method

codes are particularly efficient at modeling wire antennas or wires attached to large conductive surfaces. They are widely used for antenna and electromagnetic scattering analysis. Several non-commercial general-purpose moment-method computer programs are available [13-17].

Finite Difference Time Domain Method

The Finite Difference Time Domain (FDTD) method is a direct solution of Maxwell's time dependent curl equations,

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad (9)$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t} \quad (10)$$

It uses simple central-difference approximations to evaluate the space and time derivatives.

The FDTD method is a time stepping procedure. Inputs are time-sampled analog signals. The region being modeled is represented by two interleaved grids of discrete points. One grid contains the points at which the magnetic field is evaluated. The second grid contains the points at which the electric field is evaluated.

A basic element of the FDTD space lattice is illustrated in Figure 2. Note that each magnetic field vector component is surrounded by four electric field components. A first-order central-difference approximation can be expressed as,

$$\frac{1}{A} \left[E_{z1}(t) + E_{y2}(t) - E_{z3}(t) - E_{y4}(t) \right] = -\frac{\mu_0}{2\Delta t} \left[H_{x_0}(t+\Delta t) - H_{x_0}(t-\Delta t) \right] \quad (11)$$

where A is the area of the near face of the cell in Figure 2. $H_{x_0}(t+\Delta t)$ is the only unknown in this equation, since all other quantities were found in a previous time step. In this way, the electric field values at time t are used to find the magnetic field values at time $t+\Delta t$. A similar central-difference approximation of Equation (10) can then be applied to find the electric field values at time $t+2\Delta t$ from the magnetic field values at time $t+\Delta t$. By alternately calculating the electric and magnetic fields at each time step, fields are propagated throughout the grid.

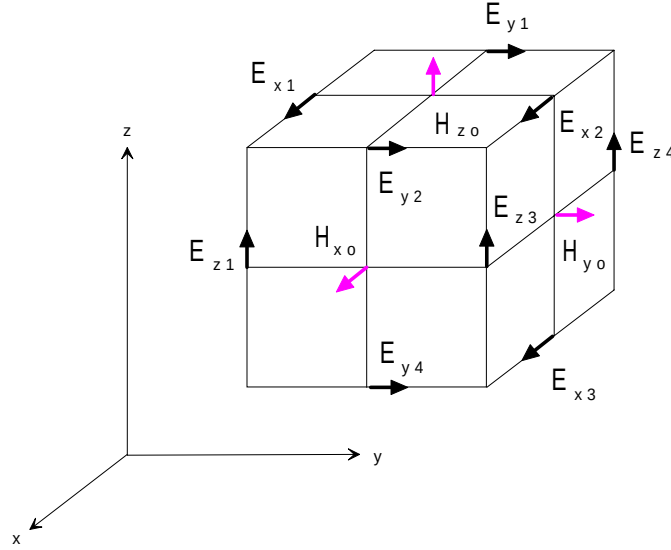


Figure 2: Basic Element of the FDTD Space Lattice

Time stepping is continued until a steady state solution or the desired response is obtained. At each time step, the equations used to update the field components are fully explicit. No system of linear equations must be solved. The required computer storage and running time is proportional to the electrical size of the volume being modeled and the grid resolution.

Figure 3 illustrates an arbitrary scatterer embedded in a FDTD space lattice. Special absorbing elements are used at the outer boundary of the lattice in order to prevent unwanted reflection of signals that reach this boundary. Values of μ , ϵ and σ assigned to each field component in each cell define the position and electrical properties of the scatterer. These parameters can have different values for different field orientations permitting anisotropic materials to be modeled. Their values can also be adjusted at each time-step depending on conditions making it easy to model nonlinear materials.

Because the basic elements are cubes, curved surfaces on a scatterer must be *staircased*. For many configurations this does not present a problem. However for configurations with sharp, acute edges, an adequately staircased approximation may require a very small grid size. This can significantly increase the computational size of the problem. Surface conforming FDTD techniques with non-rectangular elements have been introduced to combat this problem. One of the more promising of these techniques, which permits each element in the grid to have an arbitrary shape, is referred to as the Finite Volume Time Domain (FVTD) method [19].

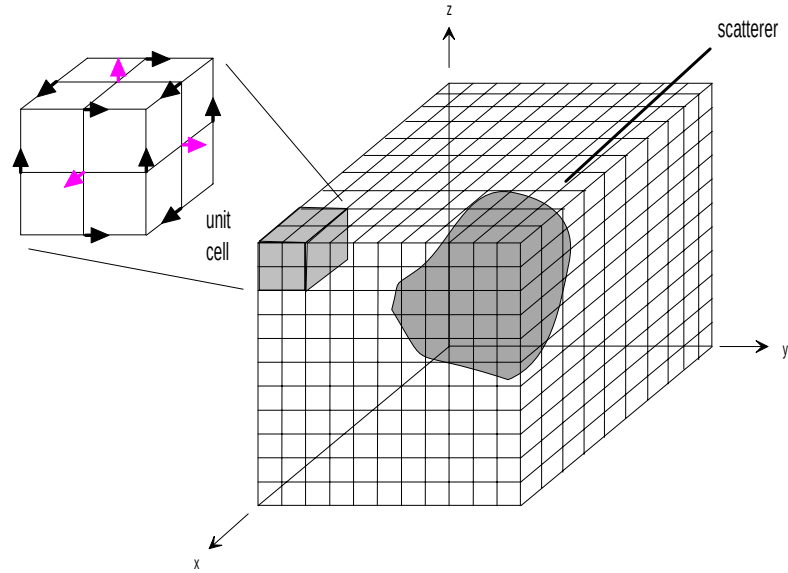


Figure 3: Scatterer in an FDTD Space Lattice

Frequency domain results can be obtained by applying a discrete Fourier transform to the time domain results. This requires additional computation, but a wide-band frequency-domain analysis can be obtained by transforming the system's impulse response.

The FDTD and FVTD methods are widely used for radar cross section analysis although they have been applied to a wide range of EM modeling problems. Their primary advantage is their great flexibility. Arbitrary signal waveforms can be modeled as they propagate through complex configurations of conductors, dielectrics, and lossy non-linear non-isotropic materials. Another advantage of these techniques is that they are readily implemented on massively parallel computers, particularly vector processors and SIMD (single-instruction-multiple-data) machines.

Wavetracer Inc. [19] sells a massively parallel computer and FDTD software for EM modeling. Lawrence Livermore [20] and Boston University [21] have run FDTD algorithms on a Connection Machine. These implementations have been used to provide animated graphical representations of EM waves as they propagate in and around a variety of interesting configurations.

The only significant disadvantage of this technique, is that the problem size can easily get out of hand for some configurations. The *fineness* of the grid is generally determined by the dimensions of the smallest features that need to be modeled. The volume of the grid must be great enough to encompass the entire object and most of the *near field*. Large objects with regions that contain

small, complex geometries may require large, dense grids. When this is the case, other numerical techniques may be much more efficient than the FDTD or FVTD methods.

Finite Difference Frequency Domain Method

Although conceptually the Finite Difference Frequency Domain (FDFD) method is similar to the Finite Difference Time Domain (FDTD) method, from a practical standpoint it is more closely related to the finite element method. Like FDTD, this technique results from a finite difference approximation of Maxwell's curl equations. However, in this case the time-harmonic versions of these equations are employed,

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \quad (12)$$

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} \quad (13)$$

Since, there is no time stepping it is not necessary to keep the mesh spacing uniform. Therefore optimal FDFD meshes generally resemble optimal finite element meshes. Like the moment-method and finite-element techniques, the FDFD technique generates a system of linear equations. The corresponding matrix is sparse like that of the finite element method.

Although it is conceptually much simpler than the finite element method, very little attention has been devoted to this technique in the literature. Perhaps this is due to the *head start* that finite element techniques achieved in the field of structural mechanics.

There are apparently very few codes available that utilize this technique. A notable exception is the FDFD module that is included in the GEMACS software marketed by Advanced Electromagnetics [22].

Transmission Line Matrix Method

The Transmission Line Matrix (TLM) method is similar to the FDTD method in terms of its capabilities, but its approach is unique. Like FDTD, analysis is performed in the time domain and the entire region of the analysis is gridded. Instead of interleaving E-field and H-field grids however, a single grid is established and the *nodes* of this grid are interconnected by virtual transmission lines. Excitations at the source nodes propagate to adjacent nodes through these transmission lines at each time step.

The symmetrical condensed node formulation introduced by Johns [23] has become the standard for three-dimensional TLM analysis. The basic structure of the symmetrical condensed node is illustrated in Figure 4. Each node is connected to its neighboring nodes by a pair of orthogonally polarized transmission lines. Generally, dielectric loading is accomplished by loading nodes with reactive stubs. These stubs are usually half the length of the mesh spacing and have a characteristic impedance appropriate for the amount of loading desired. Lossy media can be modeled by introducing loss into the transmission line equations or by loading the nodes with lossy stubs. Absorbing boundaries are easily constructed in TLM meshes by terminating each boundary node transmission line with its characteristic impedance.

The advantages of using the TLM method are similar to those of the FDTD method. Complex, nonlinear materials are readily modeled. Impulse responses and the time-domain behavior of systems are determined explicitly. And, like FDTD, this technique is suitable for implementation on massively parallel machines.

The disadvantages of the FDTD method are also shared by this technique. The primary disadvantage being that voluminous problems that must use a fine grid require excessive amounts of computation.

Nevertheless, both the TLM and FDTD techniques are very powerful and widely used. For many types of EM problems they represent the only practical methods of analysis. Deciding whether to utilize a TLM or FDTD technique is a largely personal decision. Many engineers find the transmission line analogies of the TLM method to be more intuitive and easier to work with. On the other hand, others prefer the FDTD method because of its simple, direct approach to the

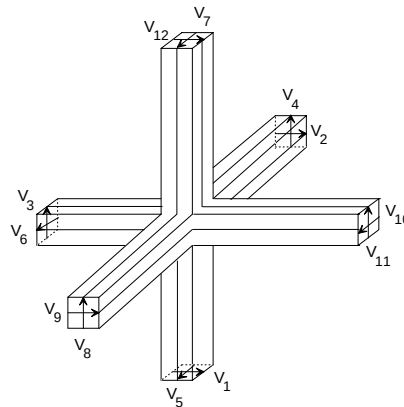


Figure 4: The Symmetrical Condensed Node

solution of Maxwell's field equations. The TLM method requires significantly more computer memory per node, but it generally does a better job of modeling complex boundary geometries. This is because both $\mathbf{E}$ and $\mathbf{H}$ are calculated at every boundary node.

A listing for a general purpose TLM code written in FORTRAN can be found in a Ph.D. dissertation by S. Akhtarzad [24]. This program can be adapted to a variety of applications. A general overview of the TLM method and a two-dimensional TLM code is provided in a book by Hoefer [25].

Generalized Multipole Technique

The Generalized Multipole Technique (GMT) is a relatively new method for analyzing EM problems. It is a frequency domain technique that (like the method of moments) is based on the method of weighted residuals. However, this method is unique in that the *expansion* functions are analytic solutions of the fields generated by sources located some distance away from the surface where the boundary condition is being enforced.

Moment methods generally employ expansion functions representing quantities such as charge or current that exist on a boundary surface. The expansion functions of the Generalized Multipole Technique are spherical wave field solutions corresponding to *multipole* sources. By locating these sources away from the boundary, the field solutions form a smooth set of expansion functions on the boundary and singularities on the boundary are avoided.

Like the method of moments, a system of linear equations is developed and then solved to determine the coefficients of the expansion functions that yield the best solution. Since the expansion functions are already field solutions, it is not necessary to do any further computation to determine the fields. Conventional moment methods determine the currents and/or charges on the surface first and then must integrate these quantities over the entire surface to determine the fields. This integration is not necessary at any stage of the GMT solution.

There is little difference in the way dielectric and conducting boundaries are treated by the GMT. The same multipole expansion functions are used. For this reason, a general purpose implementation of the GMT models configurations with multiple dielectrics and conductors much more readily than a general purpose moment-method technique. On the other hand, moment method techniques,

which employ expansion functions that are optimized for a particular type of configuration (e.g. thin wires), are generally much more efficient at modeling that specific type of problem.

Over the last ten years, the GMT has been applied to a variety of EM configurations including dielectric bodies [26,27], obstacles in waveguides [28], and scattering from perfect conductors [29,30]. Work in this young field is continuing and new developments are regularly announced. Recent significant developments include the addition of a thin-wire modeling capability [31,32] and a “ringpole” expansion function for modeling symmetric structures [33].

A commercial GMT code has been developed at the Swiss Federal Institute of Technology. This code is called the MMP (Multiple MultiPole) code. A two-dimensional PC version is available through Artech House Publishers [34]. A comprehensive text describing the GMT technique and the MMP code is also available [35].

Conjugate Gradient Method

The conjugate gradient method is another technique based on the method of weighted residuals. It is very similar conceptually to conventional moment method techniques. Nevertheless, there are two features that generally distinguish this technique from other moment methods. The first has to do with the way in which the weighting functions are utilized. The second involves the method of solving the system of linear equations.

Conventional moment methods define the inner product of the weighting functions, w_j , with another function g as,

$$\langle w_j, g \rangle = \int_s (w_j \cdot g) \, ds \quad (14)$$

This is referred to as the *symmetric product*. The conjugate gradient method uses a different form of the inner product called the *Hilbert inner product*. This is defined as,

$$\langle w_j, g \rangle = \int_s (w_j \cdot g^*) \, ds \quad (15)$$

where the $*$ denotes complex conjugation. If both functions are real, these two definitions are equivalent. However, when complex weighting functions are utilized, the symmetric product is a

complex quantity and therefore not a valid *norm*. In this case, the Hilbert inner product is preferred [36].

The other major difference between conventional moment methods and the conjugate gradient method involves the technique used to solve the large system of equations these methods generate. Conventional moment method techniques generally employ a Gauss-Jordan method or another direct solution procedure. Direct solution techniques solve the system of equations with a given number of calculations (generally $\mathbf{O}[N^3]$, where N is the order of the matrix).

Conjugate gradient methods utilize an iterative solution procedure. This procedure, called the *method of conjugate gradients*, can be applied to the system of equations or it can be applied directly to the operator equation [37]. Iterative solution procedures such as the method of conjugate gradients are most advantageous when applied to large, sparse matrices.

Boundary Element Method

The Boundary Element Method (BEM) is a weighted residual technique. It is essentially a moment-method technique whose expansion and weighting functions are defined only on a boundary surface. Most general purpose moment-method EM modeling codes employ a boundary element method [13-16].

Like the finite element method, its origins are in the field of structural mechanics. Electrical engineers are likely to use the more general term *moment method* to describe an implementation of this technique. Outside of electrical engineering however, the terms *boundary element method* or *boundary integral element method* are commonly used.

Uniform Theory of Diffraction

The Uniform Theory of Diffraction (UTD) is an extension of the Geometrical Theory of Diffraction (GTD). Both of these techniques are high-frequency methods. They are only accurate when the dimensions of objects being analyzed are large relative to the wavelength of the field. In general, as the wavelengths of an electromagnetic excitation approach zero, the fields can be determined using geometric optics. UTD and GTD are extensions of geometric optics that include the effects of diffraction.

Diffraction is a local phenomena at high frequencies. Therefore, the behavior of the diffracted wave at edges, corners, and surfaces can be determined from an asymptotic form of the exact solution for simpler canonical problems. For example, the diffraction around a sharp edge is found by considering the asymptotic form of the solution for an infinite wedge. GTD and UTD methods add diffracted rays to geometric optical rays to obtain an improved estimate of the exact field solution.

The Basic Scattering Code (BSC) is a popular implementation of UTD. It is available from the ElectroScience Laboratory of the Ohio State University [38].

Hybrid Techniques

It is apparent from the previous sections that none of the techniques described is well-suited to all (or even most) electromagnetic modeling problems. Most moment method codes won't model inhomogeneous, nonlinear dielectrics. Finite element codes can't efficiently model large radiation problems. GMT and UTD codes are not appropriate for small, complex geometries or problems that require accurate determination of the surface and wire currents. Unfortunately, most practical printed circuit card radiation models have all of these features and therefore cannot be analyzed by any of these techniques.

One solution, which has been employed by a number of researchers, is to combine two or more techniques into a single code. Each technique is applied to the region of the problem for which it is best suited. The appropriate boundary conditions are enforced at the interfaces between these regions. Normally a surface integral technique such as the boundary element method will be combined with a finite method such as the finite element, FDTD, or TLM method. Several successful implementations of hybrid techniques are described in the literature [39-48].

So far, none of the available hybrid techniques model the radiation from printed circuit cards very well. This is due to the fact that most of these methods were developed to predict radar cross section (RCS) values or for other scattering problems where the source is remote from the configuration being modeled. Work in this area is continuing however. Several researchers are involved in efforts to develop hybrid techniques that can be applied to a variety of presently intractable problems.

Advances in the development and implementation of codes based on a single technique continue to be important. However, there will always be problems that defy analysis by any one technique.

Hybrid methods permit numerical modeling techniques to be applied to a whole new class of configurations.

Conclusions

Several numerical modeling techniques have been described. A fundamental description of each technique and an overview of the types of problems they are best suited to analyze have been presented. References have been provided that direct the reader to more detailed information and sources of computer codes.

The state-of-the-art in numerical modeling is progressing rapidly. Each year new types of problems can be analyzed. Implementations of these techniques are getting more accurate and powerful. Many practical EMC problems are already being solved using numerical computer models. Before long, numerical modeling techniques are likely to become as indispensable to the EMC engineer as they already are to the antenna and microwave engineer.

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