Unified Expression of Rotational Hardening in Clay Plasticity

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Abstract: Rotational hardening is the constitutive ingredient of clay plasticity models that allows rotation of the yield surface in stress space to simulate, more realistically than isotropic models, the stress-strain response under different loading conditions. Based on well-known rotational hardening rules and extensive experimental results, a new unified expression of rotational hardening is proposed in this work, in which the parameters of the unified expression can be determined directly. The proposed rotational hardening expression was imported into a clay model and adopted to simulate the stress-strain behavior of different clays. The simulated results and the experimental results were compared to analyze the influence of the equilibrium values on the stress-strain behaviors. The comparisons show that the constitutive model with the proposed rotational hardening rule had good predictive capability for triaxial tests under different loading paths. DOI: 10.1061/(ASCE)GM.1943-5622.0000647. © 2016 American Society of Civil Engineers.

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Introduction

Anisotropy is a fundamental characteristic of natural soil. It is caused by the anisotropic arrangement of the soil particles and fabrication and manifests itself in various mechanical stress paths. The fabric orientation is altered after external loading. The macroscopic manifestation of such reorientation is the rotation of the yield surface, that is, rotational hardening. The rotation of the yield surface was first introduced by Sekiguchi and Ohta (1977), who depicted the rotation of the yield surface in stress space as a result of non-hydrostatic loading. Through experimental observations, Lade and Inel (1997) concluded that stress-reversal paths indicated the activation of another yield mechanism rotated in the direction of the previous monotonic history, rather than panning as in the metallic materials. Hueckel and Tutumluer (1994) adopted a combined isotropic kinematic hardening law to define the evolution of the yield surface, in which the rotation was governed by the deviatoric plastic strain in the generation of anisotropy. Gajo and Wood (2001) presented an approach that considered the rotational hardening of the bounding surface in the framework of kinematic hardening of the bounding surface plasticity, and concluded that the rotation was related to plastic volumetric strain and plastic shear strain. Whittle (1993) proposed a typical bounding MIT-E3 model in which limits were imposed on the principal directions of anisotropy, and found that the rotation of the yield surface was related only to the volumetric strain. Kobayashi et al. (2003) proposed a model in which the structure was enhanced with the enhancement of anisotropy, otherwise the structure decreases with the decreases in anisotropy. Wheeler et al. (2003) and Nakano et al. (2005) simulated anisotropy through an inclined elliptical based on the modified Cam clay (MCC) model and incorporated a rotational component of hardening to account for the influence of plastic anisotropy; the resulting rotational hardening law included dependence on the plastic shear increment and plastic volumetric strain increments, and this rotational hardening law has been experimentally validated.

Different rotational hardening laws are adopted for different constitutive models, and the rotational hardening parameters of these models thus differ accordingly. In general, these parameters can be estimated from the experimental data related to the original state and stress path. In the current study, a unified expression of the rotational component, based on the S-CLAY1 model (Wheeler et al. 2003), was developed and used to analyze the influence of plastic anisotropy in soft clays. This paper discusses the proposed new method for determining the absolute rotational rate parameter, as compared with the S-CLAY1 model. It then discusses the results of the experiments, in which three types of remolded and natural clays were chosen to analyze the influence of equilibrium values of plastic volumetric strain and plastic shear strain.

Current Expression of Rotational Hardening

Rotational hardening laws concern changes in the inclination of the yield curve caused by increments of plastic volumetric strain and plastic shear strain. For the rotational hardening law of anisotropic clays, a plethora of analytical expressions have been proposed, as summarized in Table 1. These expressions provide different equilibrium or bounding values of the rotational hardening law variable under various loading conditions, such as loading under fixed stress ratio or fixed strain-rate ratio, and subsequent drained or undrained loading up to critical state.

Classification of the Expression of Rotational Hardening

According to the relationship between the stress ratio η and its corresponding equilibrium value α, the rotational hardening expressions in Table 1 can be divided into three types.

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Table 1. Selected Approaches to Accounting for Rotational Hardening in Current Constitutive Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Rotational hardening law(^a)</th>
<th>Interpretation of parameter</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheeler et al. 2003</td>
<td>(da = \mu \left( \frac{3}{4} h - \alpha \right) \left( \frac{\beta}{3} - \alpha \right) \frac{de_v^p}{d\varepsilon} )</td>
<td>Parameter ( \mu ) controls the absolute rotational rate of the yield surface; parameter ( \beta ) controls the relative effectiveness of plastic shear strains and plastic volumetric strains in determining the overall current target value for ( \alpha ).</td>
<td>The influence of both plastic volumetric strain and plastic shear strain are included, but the parameter ( \mu ) cannot be determined in a direct way. At critical state, the equilibrium value of ( \alpha ) is ( \eta/3 ), which is not consistent with the critical state theory. Not applicable to triaxial extension loading paths.</td>
</tr>
<tr>
<td>Whittle 1993</td>
<td>(da = \frac{bp}{M_p} \left( M -</td>
<td>\alpha</td>
<td>\right) \left( \eta - \alpha \right) \frac{de_v^p}{d\varepsilon} )</td>
</tr>
<tr>
<td>Taiebat and Dafalias 2008</td>
<td>(da = \left( \frac{L C}{\lambda - \kappa} \right) \left( b \frac{p}{p_0} \right)^2 \left( \eta - \alpha \right) \left( \alpha - \alpha \right) \frac{de_v^p}{d\varepsilon} )</td>
<td>(&lt;L&gt;) is the plastic factor, ( a^p ) is the slope of the bounding line, and the term ( C(1 + \eta)^p((\lambda - \kappa)p_0)^3 ) controls the absolute rotational rate of the yield surface.</td>
<td>The influence of plastic shear strain on the rotational hardening law is not considered; back analysis is needed to determine the value of ( C ). There is a rotational limit line of the yield surface.</td>
</tr>
<tr>
<td>Zhang et al. 2007</td>
<td>(da = \sqrt{3}/2 \left( \eta - \alpha \right) \left( \alpha - \alpha \right) \frac{de_v^p}{d\varepsilon} )</td>
<td>Parameter ( b ) controls the absolute rotational rate of the yield surface, ( b_1 ) is the parameter controlling the rotational range, and the range is from 0.9 to 0.99.</td>
<td>Only the influence of the plastic shear strain is considered. The parameter ( b_1 ) can be determined in an indirect way.</td>
</tr>
<tr>
<td>Hashiguchi and Chen 1998</td>
<td>(da = b \left( \eta - \alpha \right) \left( \alpha - \alpha \right) \frac{de_v^p}{d\varepsilon} )</td>
<td>Parameter ( b ) controls the absolute rotational rate of the yield surface, and ( m_0 ) is the function of Lode angle and friction angle, which control the range of the rotation of the yield surface.</td>
<td>The influence of plastic volumetric strain is not considered. It is assumed that the ( K_0 ) line is also the ( NCL ) (normal consolidated line).</td>
</tr>
<tr>
<td>Hueckel and Tutumluer 1994</td>
<td>(da = b \left( \eta - \alpha \right) \left( \alpha - \alpha \right) \frac{de_v^p}{d\varepsilon} + r \frac{de_v^p}{d\varepsilon} )</td>
<td>Parameter ( b ) is a fitting parameter on the order of 102, and the parameter ( r ) controls the effectiveness of plastic volumetric strains on the rotation of the yield surface.</td>
<td>The influence of both plastic volumetric strain and plastic shear strain on the rotation of the yield surface has been considered. Parameter ( b ) is a fitting constant and lacks a physical basis, and cannot be derived directly.</td>
</tr>
<tr>
<td>Newson and Davies 1996</td>
<td>(da = \pm \left( \eta/M \right)^2 \frac{dp}{d\varepsilon} )</td>
<td>( \eta ) is the stress ratio at the start of the increment, ( \eta_a ) is the rotation of the ellipse at the start of the increment, ( dp ) is the stress increment, and ( \xi ) is a parameter controlling the rate of development of anisotropy.</td>
<td>The rotation of the yield surface is assumed to relate to the effective principal stress increment, but not shear stress. Because the nonassociated flow rule is adopted in this model, which makes the rotational hardening law complex, the absolute rotational rate parameter of the yield surface has not been determined.</td>
</tr>
<tr>
<td>Wang and Shen 2008</td>
<td>(da = b \left( \left( \beta_1 + \beta_2 \right) \eta - 2(\beta_1</td>
<td>\alpha</td>
<td>) \frac{de_v^p}{d\varepsilon} \right) )</td>
</tr>
</tbody>
</table>

\(^a\)\( \alpha \) is the inclination angle of the ellipse of the yield surface in the \( q-p' \) plane.

The first type assumes that the relationship between \( \eta/M \) (or \( M_x \) or \( M_z \)) and \( \alpha/M \) (or \( M_x \) or \( M_z \)) is linear and that there is no rotational limit line (e.g., Whittle 1993; Taiebat and Dafalias 2008; Zhang et al. 2007). The expression can be written as

\[
\alpha = x \eta
\]  

(1)

The second type assumes that the relationship between \( \eta/M \) (or \( M_x \) or \( M_z \)) and \( \alpha/M \) (or \( M_x \) or \( M_z \)) is linear and that a rotational limit line exists (e.g., Hashiguchi and Chen 1998; Wang and Shen 2008). The expression can be written as

\[
\alpha = \begin{cases} 
\eta & |\eta| \leq |M - |\alpha|| \\
0 & |\eta| > |M - |\alpha|| 
\end{cases}
\]  

(2)

The third type assumes that the relationship between \( \eta/M \) (or \( M_x \) or \( M_z \)) and \( \alpha/M \) (or \( M_x \) or \( M_z \)) is nonlinear (Wheeler et al. 2003). The expression can be written as
\[
\alpha = (a \eta - \alpha) (M^2 - \eta^2) + 2 \beta (b \eta - \alpha) (\eta - \alpha) = 0 \quad \text{for } \eta < M \\
a = b \eta \quad \text{for } \eta \geq M \\
(3)
\]

If \( a = 3/4 \) and \( b = 1/3 \), Eq. (3) recovers the expression of rotational hardening of the S-CLAY1 model (Wheeler et al. 2003), where parameter \( \beta \) controls the relative effect of plastic shear strain in rotating the yield and loading surfaces. The parameter \( \beta \) can be expressed as (Wheeler et al. 2003)

\[
\beta = \frac{3 (4M^2 - 4 \eta K_0^2 - 3 \eta K_0)}{8 (\eta K_0^2 - M^2 + 2 \eta K_0)} \\
(4)
\]

where \( \eta K_0 \) is the stress ratio under the \( K_0 \) consolidation condition.

To analyze the rationality of these three types of rotational hardening laws, this study used data collected on the stress ratio and its corresponding equilibrium value for 14 kinds of soils (e.g., Wheeler et al. 2003; Kevin 2006; Dan 2009), then normalized the data through division with \( M \). The influence of Lode angle on the value of \( M \) was considered in the study’s experiments, and the experimental results were used to validate the simulated results using the three types of rotational hardening laws summarized in Eqs. (1)–(3). The simulated results and experimental results are shown in Fig. 1.

From Fig. 1, it can be seen that the rotational hardening laws given by Eqs. (1)–(3) provided a reasonably good fit to the data for triaxial compression (positive \( \eta/M \), especially when the value of \( \eta/M \) was small. As \( \eta/M \) increased, the plastic shear strain was

**Fig. 1.** Equilibrium values of \( \alpha/M \) for radial stress paths at various \( \eta/M \); comparison of experimental data with simulated response using different rotational hardening laws: (a) Eq. (1); (b) Eq. (2); (c) Eq. (3)
Isotropic consolidation to a pressure of approximately 2–3 times the preconsolidation pressure makes the soil isotropic with respect to the strain response. The plastic volumetric strains can be expressed as

$$
\varepsilon_v^p = \frac{\lambda - k}{1 + \epsilon_0} \ln \left( \frac{p'_c}{p_0} \right) = \frac{\lambda - k}{1 + \epsilon_0} \ln(2 \sim 3) \approx \frac{\lambda - k}{1 + \epsilon_0} \quad (6)
$$

where $p'$ is the mean effective stress; $p_0$ defines the size of the initial yield surface; $\lambda$ and $\kappa$ are the slope of the compression line and swelling line, respectively; and $\epsilon_0$ is the initial void ratio. If the consolidation is isotropic, then $\eta = 0$, and the relation of plastic volumetric strain and plastic shear strain can be expressed as

$$
\frac{d\varepsilon_v^p}{d\varepsilon_v^p} = \frac{2(\eta - \alpha)}{M^2 - \eta^2} = -\frac{2\alpha}{M^2} \quad (7)
$$

When the inclination angle $\alpha$ reduces to 0.1$\alpha_0$, with $\alpha_0$ being the initial inclination angle, it is considered that isotropic and anisotropic samples behave in practically the same way. The expression of the control parameter $\mu$ can be obtained from Eqs. (5)–(7) as

$$
\mu = \frac{2\beta(1 + \epsilon_0)}{\lambda - \kappa} \ln \frac{10M^2 - 2\alpha_0\beta}{M^2 - 2\alpha_0}\quad (8)
$$

Based on the unified expression of rotational hardening and the experimental results shown in Fig. 1, the simulated results with $a = 1.0$ and $b = 0.0$ were closer to the experimental results than those with $a = 3/4$ and $b = 1/3$, so the rotational hardening expression proposed in this paper can be written as

$$
d\alpha = \mu[(\eta - \alpha)(d\varepsilon_v^p) + \beta(\eta - \alpha)d\varepsilon_v^p] \quad (9)
$$

where the parameter $\beta$ controls the relative effectiveness of plastic shear strains ($\varepsilon_v^p$) and plastic volumetric strains ($\varepsilon_v^p$) in determining the overall current target value for $\alpha$, which can be calculated by Eq. (4). The McCauley brackets imply that $d\varepsilon_v^p > 0$ for $d\varepsilon_v^p > 0$ or $d\varepsilon_v^p = 0$ for $d\varepsilon_v^p \leq 0$. Parameter $\mu$ controls the absolute rotational rate of the yield surface; the value is recommended by Wheeler et al. (2003) as $10\lambda - 15/\lambda$, where $\lambda$ is the slope of the compression curve in the $\eta$–$p'$ plane. In this section, a new method to determine the parameter $\mu$ is proposed.

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**Proposed Unified Expression of Rotational Hardening**

Based on the various expressions of rotational hardening shown in Table 1 and the rotational hardening law of the S-CLAY1 model (Wheeler et al. 2003), a unified expression is proposed in which the equilibrium values of $\alpha$ are $3/4$ and $1/3$, respectively. In this proposed expression, the two values $3/4$ and $1/3$ are substituted with $a$ and $b$, and the proposed unified expression for rotational hardening law can then be written as

$$
d\alpha = \mu[(a \eta - \alpha)(d\varepsilon_v^p) + \beta(\eta - \alpha)d\varepsilon_v^p] \quad (5)
$$

where the parameter $\beta$ controls the relative effectiveness of plastic shear strains ($\varepsilon_v^p$) and plastic volumetric strains ($\varepsilon_v^p$) in determining the overall current target value for $\alpha$, which can be calculated by Eq. (4). The McCauley brackets imply that $d\varepsilon_v^p > 0$ for $d\varepsilon_v^p > 0$ or $d\varepsilon_v^p = 0$ for $d\varepsilon_v^p \leq 0$. Parameter $\mu$ controls the absolute rotational rate of the yield surface; the value is recommended by Wheeler et al. (2003) as $10\lambda - 15/\lambda$, where $\lambda$ is the slope of the compression curve in the $\eta$–$p'$ plane. In this section, a new method to determine the parameter $\mu$ is proposed.
Table 2. Unified Expression of Rotational Hardening and Its Relations to Existing Rotational Hardening Laws

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mu$</th>
<th>$\beta$</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This paper</td>
<td>$\frac{2\beta(1 + \varepsilon_0)}{\kappa - \kappa}$</td>
<td>$\frac{3(4M^2 - 4\frac{\eta_{K_0}^2}{M} - 3\eta_{K_0})}{8(\eta_{K_0}^2 - M^2 + 2\eta_{K_0})}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Wheeler et al. 2003</td>
<td>$10/\lambda$ to $10/\lambda$</td>
<td>$\frac{3(4M^2 - 4\frac{\eta_{K_0}^2}{M} - 3\eta_{K_0})}{8(\eta_{K_0}^2 - M^2 + 2\eta_{K_0})}$</td>
<td>3/4</td>
<td>1/3</td>
</tr>
<tr>
<td>Whittle 1993</td>
<td>$b\frac{M_c}{M_p}(M -</td>
<td>\alpha</td>
<td>)$</td>
<td>0</td>
</tr>
<tr>
<td>Taiebat and Dafalias 2008</td>
<td>$C \left( \frac{1 + \varepsilon}{\lambda - \kappa} \right)^2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Zhang et al. 2007</td>
<td>$\sqrt{3/2} \frac{1 + \varepsilon}{\lambda - \kappa} b_M$</td>
<td>1</td>
<td>0</td>
<td>$b_1M$</td>
</tr>
<tr>
<td>Hashiguchi and Chen 1998</td>
<td>$b</td>
<td></td>
<td>\eta - \alpha</td>
<td></td>
</tr>
<tr>
<td>Hueckel and Tutumluer 1994</td>
<td>$b$</td>
<td>$r/b$</td>
<td>no equilibrium value</td>
<td>no equilibrium value</td>
</tr>
<tr>
<td>Newson and Davies 1996</td>
<td>$\exp(\eta - \eta_{sat})\xi$</td>
<td>0</td>
<td>$\pm M$</td>
<td>0</td>
</tr>
<tr>
<td>Wang and Shen 2008</td>
<td>$b$</td>
<td>1</td>
<td>0</td>
<td>1 or 0</td>
</tr>
</tbody>
</table>

Numerical Modeling under Different Loading Paths

Parameters $a$ and $b$ affect the relationship between the stress increment and the strain increment (Wheeler et al. 2003). In general, when these rotational hardening expressions with different combinations of $a$ and $b$ were applied to soil behavior, the simulated results gave reasonable predictions for various selected soils under specific loading paths. However, the influence of the combination of $a$ and $b$ on modeling the stress-strain behavior of clay under different loading conditions must also be evaluated to determine whether the simulated results are still reasonable when the stress path is changed. In this section, the influence of the combination of $a$ and $b$ on modeling the stress-strain behavior of clay under different loading conditions is analyzed.

It can be seen from Table 2 that to recover the listed existing rotational hardening laws, four combinations of parameters $a$ and $b$ were used: $(3/4, 1/3)$, $(0, 1)$, $(0, 0)$, and $(1, 0)$. The proposed rotational hardening law with the four combinations of $a$ and $b$ was imported into the S-CLAY1 model (Wheeler et al. 2003) to simulate the stress-strain behavior under different loading paths. Six parameters were used: $\varepsilon_0$, $\lambda$, $\kappa$, $M$, $\nu$, and $p_{sat}/kPa$. These parameters were determined directly from experimental data, following Wheeler et al. (2003).

The analysis is presented herein with reference to drained and undrained test results under different loading paths on various remolded and natural clays (e.g., Shanghai clay by Huang et al. 2011; Boston clay by Ladd and Varallyay 1965; Otaniemi clay by Karstunen and Koskinen 2008). The values of the input parameters for these three types of clays are listed in Table 3.

Drained Triaxial Tests

The triaxial test on Shanghai soft clay was carried out by Huang et al. (2011). The samples were consolidated to the in situ stress state along a path that retraced their normal consolidated stress histories. The specimens were then subjected to continuous drained probing tests radiating from the in situ stress state at a range of angles. The principal features of the yielding test are given in Fig. 2; the experimental results of SCD60 and SED56 were selected to compare with the simulated results.

Figs. 3 and 4 show the simulated drained test results with the proposed rotational hardening law. For the triaxial compression tests SCD60 and SED56, the simulation curves of $p^\prime - \varepsilon_v$ with different values of $a$ and $b$ were very close, with almost no differences among them, which shows that the influence of parameters $a$ and $b$ on the simulated results of $p^\prime - \varepsilon_v$ was relatively small. The differences in $q - \varepsilon_v$ was more noticeable among different combinations of $a$ and $b$ values, which shows that the influence of parameters $a$ and $b$...
On the simulated results of $q^*_s$ was larger than that of $p^*_v$. In general, the simulated results showed good agreement with the experimental results.

**Undrained Triaxial Tests**

The undrained triaxial compression tests on consolidated samples with different $K_0$ values for Boston blue clay (BBC) were performed by Ladd and Varallyay (1965). For these undrained tests on Boston clay, the samples were first isotropically consolidated up to an effective stress of 150 kPa, followed by isotropic unloading to different overconsolidation ratio (OCR) values. Subsequently, the samples were loaded by an increase of axial load up to failure in undrained conditions. The trends of shear strength and volume deformation with different OCR values were reproduced by this model. A comparison of experimental and simulated results for undrained triaxial compression tests on isotropically consolidated samples is shown in Fig. 5.

It can be seen that for the normally consolidated sample, the stress path was below the critical state line (CSL), whereas for the strongly overconsolidated specimen, the stress path was above the CSL when reaching the critical state. The numerical simulation using the proposed model captured the experimental trends well. In Fig. 5, the simulated stress-strain curves are located above the experimental curve. There were some discrepancies when simulating the stress-strain behavior with OCR = 1. In comparison, the model with values of $a = 1.0$ and $b = 0.0$ performed better than that with other values. For the heavily overconsolidated samples (OCR = 4, 8), the simulated results gave good predictions of the experiment results, but the results for the values $a = 0$ and $b = 0$ were less satisfactory. The influence of parameters $a$ and $b$ on the simulated results in the undrained tests was larger than that on the drained tests, which shows that the influence of plastic volumetric strains and plastic shear strains during undrained compression are more obvious than those during drained compression.

**Cyclic Loading Tests**

The cyclic loading tests of Otaniemi reconstituted clays were performed by Karstunen and Koskinen (2008). All samples of a given clay were first loaded along the same stress path, at a stress ratio of $\eta_0$, and to the same stress level $p_{0\text{max}}$. The value $\eta_0$ was chosen based on preliminary estimates of the normally consolidated values of $K_0$. Subsequently, after the initial consolidation stage, the samples were unloaded with the same ratio $\eta_0$ to a mean effective stress $p'_o$ of 6–12 kPa. After unloading, two loading–unloading cycles were carried out with constant stress ratios of $\eta_1$ and $\eta_2$. Details of the experiment are described in Karstunen and Koskinen (2008).

The test sample CAE3519R was chosen for comparison. Fig. 6 shows the simulated and experimental results for sample CAE3519R. It can be seen that the model again captures the volumetric stress-strain behavior very well. When the values of $a$ and $b$ are set as 0.0 and 0.0, which means no rotational hardening effect being taken into account, the discrepancy between
the simulation and experiments are the largest, especially in terms of shear stress-strain behavior, which demonstrates the importance of correctly considered the rotational hardening.

**Conclusion**

Based on the collected normalized stress ratio $\eta/M$ equilibrium value and $a/M$ behavior data of 14 kinds of soils, a unified expression of rotational component of hardening was proposed; the parameters of the unified expression can be determined in a new, direct way. The proposed rotational hardening expression was imported into the S-CLAY1 model. The model was then applied to simulate the stress-strain behavior of several clays under both monotonic and cyclic loading conditions, namely, undrained triaxial loading, drained triaxial compression, and cyclic loading conditions. The influence of model parameters $a$ and $b$ on the simulated material response under different stress paths was analyzed. In the analysis, four groups of parameters were chosen to simulate the results. The simulated results obtained from the proposed rotational

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**Fig. 5.** Comparison between experimental and numerical results for Boston blue clay with different OCRs: (a) and (b) OCR = 1; (c) and (d) OCR = 4; (e) and (f) OCR = 8
hardening model were compared with experimental results. The comparison shows the following:

1. For drained triaxial tests, the influence of rotational hardening and the newly introduced parameters was larger in the compression tests than in the extension tests.

2. For undrained triaxial tests on clays with different OCRs, the influence of rotational hardening was more obvious in the shear stress-strain response than in the volumetric stress-strain response.

3. For the cyclic loading tests, in general, the model was able to capture the observed stress-strain behavior, particularly well in both loading and unloading stages. The discrepancies in terms of the shear stress-strain response were larger.

Acknowledgments

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Reference


