Without Notice! Evacuation and Humanitarian Aid

Educational Concepts

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This game introduces students to a variety of concepts, including humanitarian aid, probability, and reliability.

1. Humanitarian Aid

In adverse conditions, such as natural or man-made disasters, supplies and logistical assistance are often needed. Humanitarian aid serves these needs with the goals of saving lives and alleviating suffering. Providing such services can be dangerous, depending on the natural, political, and social conditions of the area being served. The International Red Cross is an example of a humanitarian organization.

In the game, students pretend that they work for a humanitarian aid organization with the goals of moving refugees and delivering food and supplies. Through the game, students start to gain an appreciation of the danger that may be encountered during the process of administering aid (through Chance cards) and the unreliability of the road system that has been affected by the disaster, etc.

Depending on the strategies employed by the students, they may also gain some experience with collaboration among humanitarian agencies. If students choose to swap cargoes or buy/sell supplies from/to other players, they can see the benefits of working together on an ad-hoc basis.

2. Probability

At each roll of the die, students are generating a random number from 0 to 9. In a fair, 10 sided die, each value has an equal probability of turning up on top (1/10).

2.1. One Move – The Bernoulli Distribution

On one move, students compare their roll to the value shown on a link (line) on the board. For example, if the link has a 0.8, students must roll an 8 or higher. They could evaluate their probability of successfully moving along this link prior to their roll by counting the possible outcomes (number on top of the die) that satisfy the minimum value they are trying to achieve. In this example, there are two ways to be successful, i.e., rolling an 8 or a 9; thus, the probability of success is 0.2 (2/10) and the probability of failure is 0.8 (8/10).

For slightly older or more advanced students, the distribution associated with an individual move is called the Bernoulli distribution. This distribution dictates a single trial (roll of the die) with two possible outcomes (success and failure). The probability of success is often denoted p and the probability of failure may be denoted q. In our above example, p = 0.2, q = 0.8. The outcome (success or failure) can be represented mathematically as in equation (1).

$$\boldsymbol{X} = \begin{cases} \mathbf{1} \ (\boldsymbol{success}) \\ \mathbf{0} \ (\boldsymbol{failure}) \end{cases} \quad \textbf{(1)}$$

With this representation, the expected value *E(X)* and variance *Var(X)* can be calculated. Expected value (mean or probability weighted average) is calculated by taking each possible value of X and multiplying it by the probability of that value, as shown in equation (2).

$$\boldsymbol{E}(\boldsymbol{X}) = \mathbf{1} \times \boldsymbol{p} + \mathbf{0} \times \boldsymbol{q} = \boldsymbol{p} \quad \textbf{(2)}$$

The variance of random variable X represents the spread of the distribution and is calculated as the difference between the expected value of X^2 and the square of the expected value of X.

$$\boldsymbol{Var}(\boldsymbol{X}) = \boldsymbol{E}[\boldsymbol{X}^2] - (\boldsymbol{E}[\boldsymbol{X}])^2 = \boldsymbol{p} - \boldsymbol{p}^2(\boldsymbol{E}[\boldsymbol{X}]) \quad \textbf{(3)}$$

The Bernoulli distribution serves as the basis for many other discrete probability distributions.

2.2. Multiple Moves – Independence

With a fair die, each roll is independent of the previous one, meaning that the probability of a particular outcome (number on top) remains the same, regardless of the previous rolls. If two successive links on the board have the values 0.8 and 0.5, students can calculate the probability of success on the first roll (p_1 = 0.2) and on the second roll (p_2 = 0.5). Since the rolls (events) are independent, the probability of success on both rolls together is the product of the individual probabilities of success, as shown in equation (4).

$$\boldsymbol{P}(\boldsymbol{X}_1 = \mathbf{1}, \boldsymbol{X}_2 = \mathbf{1}) = \boldsymbol{p}_{\boldsymbol{X}_1}(\mathbf{1})\boldsymbol{p}_{\boldsymbol{X}_2}(\mathbf{1}) = \boldsymbol{p}_1\boldsymbol{p}_2 = \mathbf{0.2} \times \mathbf{0.5} = \mathbf{0.1} \quad \textbf{(4)}$$

Note that such calculations in the game do not account for the Chance cards, which do not have quantifiable probabilities.

3. Reliability

Connectivity (or terminal) reliability refers to the probability that two locations (nodes) are connected. In the context of the game, this can be considered the probability that the student can reach his/her destination from his/her current or starting position on the board (omitting the effect of the Chance cards).

3.1. Network Terminology

Networks consist of a set of nodes (vertices) and links (arcs, edges). *Nodes (vertices)* are at either end of a link and can represent intersections, decision points, origins, destinations, etc. *Links (arcs, edges)* connect the nodes and can represent segments of roadways, the rail track between consecutive train stations, etc. To travel from an origin to a destination, one follows a *path*, which can be represented by a list of consecutive links or nodes.

3.2. Reliability Calculations

Each link can be thought of as having two states: operational and failed. The state can be represented similar to the Bernoulli notation above and shown in equation (5).

$$\boldsymbol{x_i = \begin{cases} 1 & if\ link\ i\ is\ operational \\ 0 & if\ link\ i\ fails \end{cases}} \qquad \textbf{(5)}$$

The network state – in our case, whether the origin-destination pair is connected – can similarly be represented as in equation (6).

$$\boldsymbol{\phi(X) = \begin{cases} 1 & if\ the\ network\ functions \\ 0 & if\ the\ network\ fails \end{cases}} \qquad \textbf{(6)}$$

The $\phi(\boldsymbol{X})$ is called the structure function and is one of the more difficult things to determine in reliability calculations for large networks. This function tells us something about how the network is organized or structured. Two of the basic structures are the organization of links (1) in series or (2) in parallel. When links are in series (such as those in a path) as shown below, the structure function is similar to the independent probability calculation above.

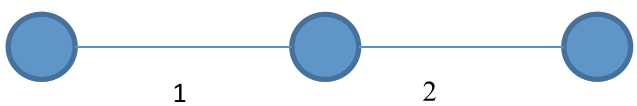

Figure 1 Links in Series

$$\boldsymbol{\phi(X) = \prod_i x_i} \qquad \textbf{(7)}$$

In the example in Figure 1, equation (7), becomes $x_1 \times x_2$. Only if both links 1 and 2 are in the functional state, with value 1, will the state of the network be functional, with value 1. If either or both of the links fail, the network has failed. The probability of the network in Figure 1 being in state 1 is $p_1 \times p_2$.

The case where links are organized in parallel is illustrated in Figure 2. The structure function for the parallel case is shown in equation (8).

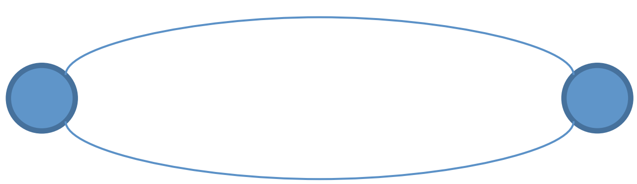

Figure 2 Links in Parallel

$$\boldsymbol{\phi(X) = 1 - \prod_i (1 - x_i)} \qquad \textbf{(8)}$$

For the example shown in Figure 2, the structure function will take the value 1 if either link 1 or link 2 function or if both do. Equation (8) becomes $1-(1-x_1)(1-x_2)$. The reliability of the network (probability of being in state 1) is then $1-(1-p_1)(1-p_2)$.

Larger networks pose challenges in identifying a structure function that can be written in terms of series and parallel structures. If the network can be represented as a system of parallel paths, equation (9) can be used; however, the links cannot be present in multiple paths.

$$\boldsymbol{\phi}(X) = \mathbf{1} - \prod_{s=1}^{S}\left(\mathbf{1} - \prod_{i\in P(s)} x_i\right) \qquad \textbf{(9)}$$

Where

i indicates a link on path number s

P(s) is the set of links on path *s*

S is the set of paths, $s \in S$

x_i is the state of link *i*.

Most networks do not consist solely of parallel paths and the same link is part of multiple paths, in this case, exact approaches to finding the network reliability require the use of Boolean algebra, which is outside the scope of this document. Approximate approaches also exist, but the analysis required for the network shown in the game would require time exceeding many younger students' attention spans or the use of computers and are thus not shown here. Interested readers may refer to chapter 8 in Bell and Iida (1997).

Bell, M. G. H. and Y. Iida (1997). Transportation Network Analysis. New York, John Wiley & Sons.