

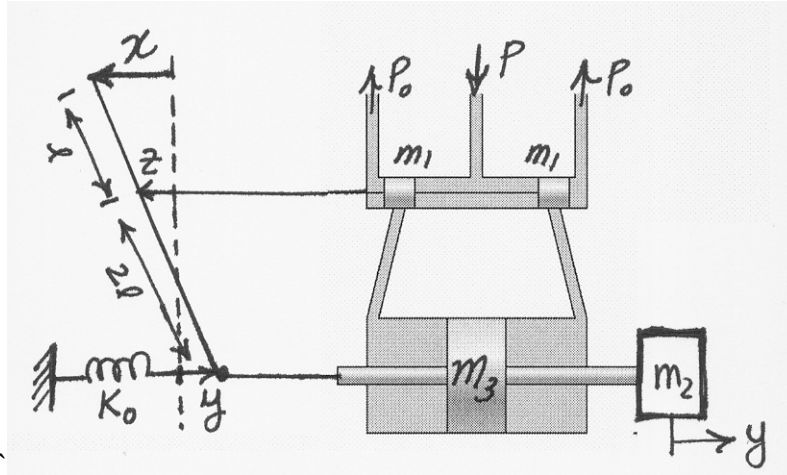


Figure 2. Hardware closed-loop implementation.

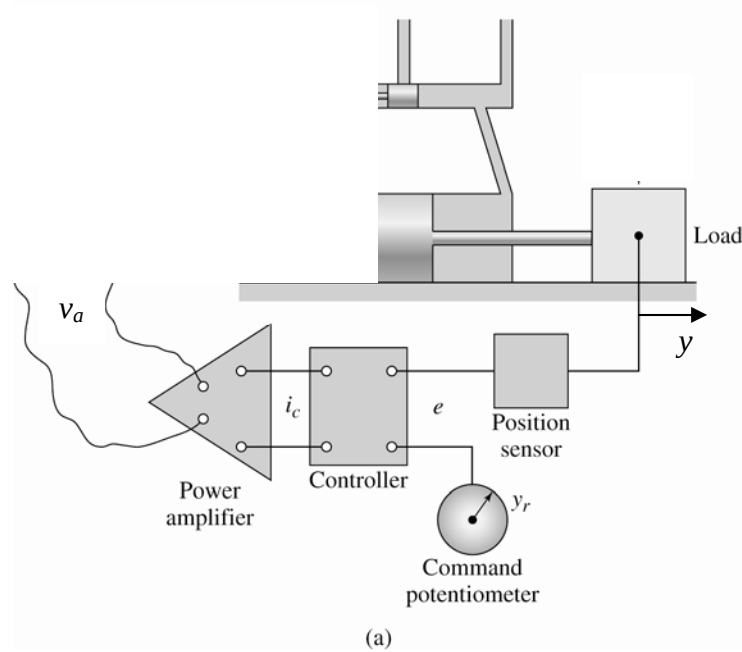


Figure 3. Software level controller.

The controller is taken to be in the form of :

$$v_a(t) = K_p(y_r(t) - y(t)) - K_d\dot{y}(t) \quad (1)$$

where $y_r(t)$ is the desired position (a time-varying variable at this stage) of the load (mass m_2), and K_p and K_d are the respective proportional and derivative feedback gains to be designed.

The proposed controller can be schematically shown in Figure 4, where both K_p and K_d have the respective “volt/m” and “volt.sec/m” dimensions and error $e(t)$ is defined as

$e(t) = y_r(t) - y(t)$. All the constant values including mechanical and electrical parameters remain unchanged as given in Table 1 of *Project 1*.

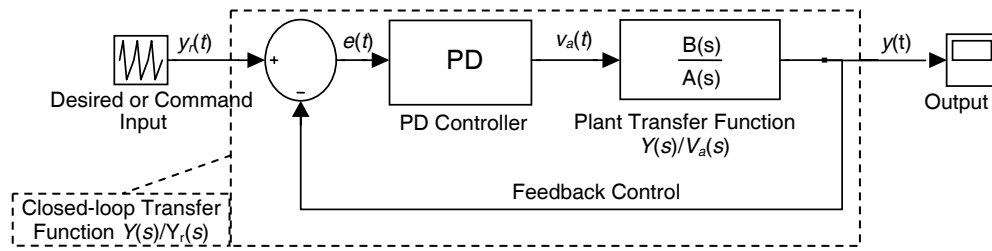


Figure 4. Feedback control of the linear system shown in Figure 1.

Analytical Questions:

1. Set the equations governing the dynamics of this system (this is the same as in *Project 1* except that in the hydraulic part you need to take into account the new geometry introduced in Figure 2 – see a clarification page at the end of this document). Now, replace the input voltage, v_a , with the expression given in equation (1) and classify the new input $y_r(t)$ (i.e., the desired load position) and load position, $y(t)$, as the input and output variables, respectively.
2. Assuming zero initial conditions, transfer the governing equations developed in Part 1 into Laplace domain and find the transfer function of the system, $TF(s) = Y(s)/Y_r(s)$. This will form the closed-loop transfer function of the system from $y_r(t)$ to $y(t)$ as shown in Figure 4 (inside dashed lines).
3. Extract the characteristic equation from Part 2 and determine whether the system is stable or not (using characteristic roots). What is the order of the system? How does this compare with the characteristic equations developed in *Project 1*?
4. What is the position of the load as $t \rightarrow \infty$ (steady-state error) in response to a unit DC input voltage (i.e., $y_r(t) = u(t)$ or $Y_r(s) = 1/s$)? Does this agree with the stability results of Part 3? Compare this with the open-loop case of *Project 1* and draw your conclusion.

Open-loop Control:

5. With $K_p = 1$, $K_d = 0$ and $y_r(t) = 0.4$ mm, implement the open-loop block diagram of Figure 4 (with **NO** feedback line) in Simulink. Does the load follow the command properly? Draw your conclusion.

Closed-loop Control:

6. Implement the closed loop controller of Figure 4 in Simulink. It is now desired to vary K_p and K_d to obtain the three cases of under-damped, critically-damped and over-damped systems. Try to numerically obtain K_p and K_d . One approach is to numerically calculate the corresponding roots of the characteristic equation (using “fzeros” in Matlab, for instance) and determine the corresponding damping ration, ξ , of the dominant root (using “sgrid” in Matlab, for instance).

7. Implement each set of your selection of K_p and K_d and plot the system response ($y(t)$). Determine the damping ratio, settling time and overshoot and collect the results in a table for the three cases of under-damped, critically-damped and over-damped systems.

Case	K_p	K_d	ξ	t_s	M_p
Under Damped					
Critically Damped			N/A		N/A
Over Damped			N/A		N/A

8. Now, try to see the effect of each controller gains (K_p and K_d) by varying them in some ranges. Decide as to what procedure you will take. For instance, keep one of them unchanged and vary the other one and so on. Observe and take notes from the responses.
9. Find out each gain's effect on the performance of the closed loop controller. From the exercises above, find out the best (to the extent possible) combination of these gains and report. Justify your choice of "best" gains. Explain the procedure used.
10. Calculate the peak overshoot, settling time, rise time, and peak time for your best controller, and plot the corresponding system output ($y(t)$).

Table 1. System parameters.

PARAMETER	SYMBOL	VALUE	UNITS
Motor Torque Constant, Back Emf Constant	K_t, K_e, K_b	0.00767	Nm/amp V/(rad/sec)
Armature Resistance	R_a	2.6	Ohms
Armature Inductance	L_a	0.18	mHenry
Maximum Input Voltage	V_{max}	10.0	Volts
Motor Inertia	I	$3.87e^{-7}$	Kgm ²
Motor Gear (Pinion) Radius	r	0.635	cm
Spool mass	m_1	0.050	Kg
Piston Mass	m_2	0.250	Kg
Load Mass	m_3	1.5	Kg
Spool Valve Spring	k_1	25	N/m
Spool Valve Damper	c_1	0.5	Kg/sec
Load Spring	k_2	200	N/m
Load Damper	c_2	12.5	Kg/sec
Motor Shaft Equivalent Spring	k	450	N/m
Motor Shaft Equivalent Viscous Friction	c	0.05	Kg.m/sec
Pump Supply Pressure	P_s	500	MPa
Spool Valve Constant	B_1	24	Kg/sec.m
Spool Valve Constant	B_2	10	Kg/sec.m
Cylinder Effective Diameter	D	40	cm
Feedback mechanism spring	K_0	50	N/m