Clemson University Department of Mechanical Engineering

ME 403/416/616 Project	**Due: Friday December 5, 2008**
Fall 2008	**Dr. Nader Jalili**

GANTRY SYSTEM

It is desired to control the position of a spring driven cart, while carrying a pendulum. This system, known as gantry system, is used in many industrial applications. We are assuming small angle of rotation for the pendulum, so only linear vibrations are considered. The pendulum is also assumed to be uniform.

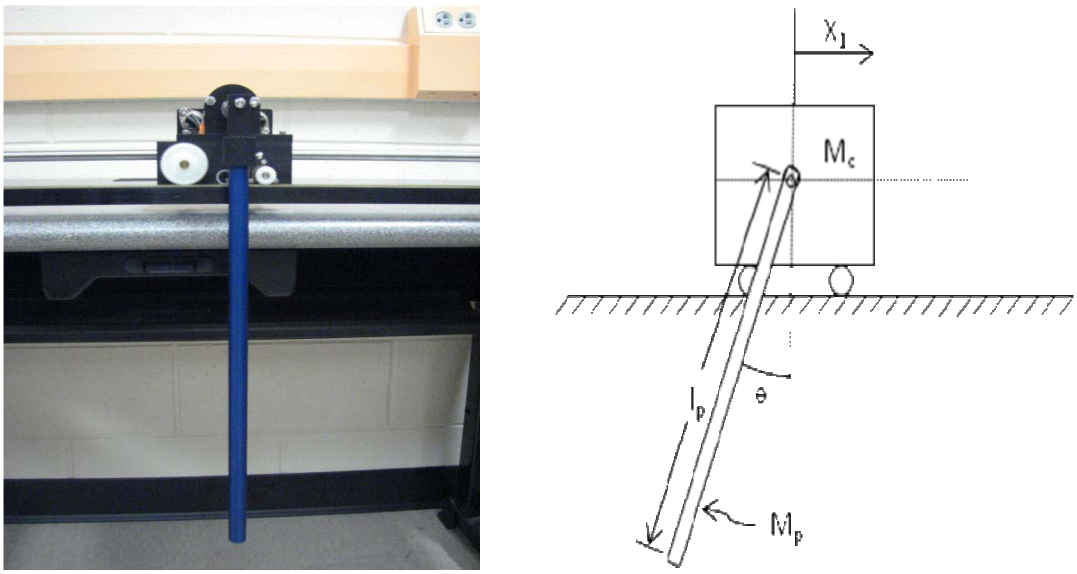
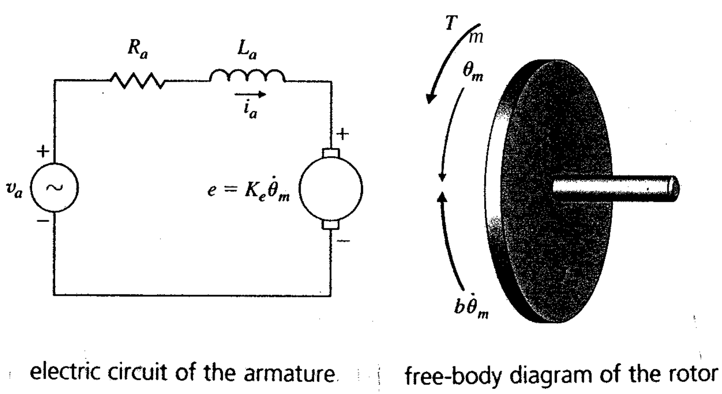


Figure 1. (left) Gantry system, and (right) its free-body-diagram.

The cart (M_c) is equipped with a DC motor. As modeled in Figure 2, this motor has an electrical constant K_e, a torque constant K_t, an armature inductance L_a, and a resistance R_a. The motor shaft is connected to a gear with radius r as shown in Figure 3. The motor shaft has an equivalent inertia of J_m and an equivalent viscous friction b. The gear ratio from motor shaft, θ_m, to cart gear, θ_1, is K_g. Notice, the inertia of the gears involved are neglected, and the cart gear contact with the rack is assumed to be a pure rolling contact (i.e., $x_1 = r\,\theta_1$ in Figure 3). The viscous damping coefficient as seen at the pendulum axis is B_p.



$$\boxed{T_m = K_t\, i_a}$$

Figure 2. Model of the DC motor.

When the motor runs, the torque created at the output shaft is translated to a linear force which results in the cart's motion (see Figure 3). When the cart moves, the encoder shaft turns and the voltage measured from the potentiometer can be calibrated to obtain the track position of driven cart, x_2.

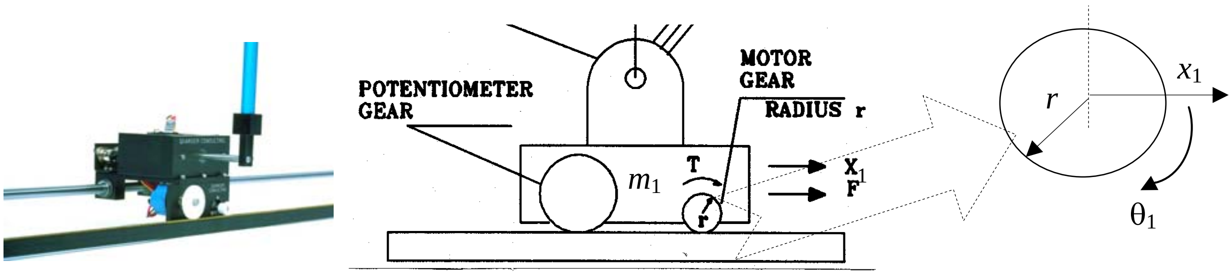


Figure 3. Model of the force translation on the driver cart.

Dymamic Modelign and Analysis:

1. Set the equations governing the dynamics of this system. Classify these equations and identify the input voltage, v_a, and output cart position, x_1, as the input and output variables, respectively.
2. Represent the dynamics in the state-variable form (a set of 1st order differential equations) and identify your states. Identify the system coefficient and input force matrices.
3. Assuming zero initial conditions, transfer the governing equations developed in part 1 into Laplace domain and find the transfer function of the system, $TF(s) = X_1(s)/V_a(s)$.
4. Extract the characteristic equation from part 2 or 3, and determine whether the system is stable or not (using characteristic roots). What is the order of system?
5. What is the position of the driven cart as $t \rightarrow \infty$ (steady-state error) in response to a unit DC input voltage? Does this agree with the stability results of Part 4?
6. Find the simplified expressions for the TF of part 3, the order of the resulting system, and steady-state error for the following cases:
   a. Neglecting the effect of armature inductance L_a.
   b. Neglecting the effects of both armature inductance L_a and viscous friction c.
   c. Neglecting the effects of armature inductance L_a, viscous friction c, and motor inertia J_m.
7. Determine the stability characteristics of each simplified system resulted in part 6 (a-c). Will any of these assumptions change the stability characteristics of the system?

Numerical Questions:

8. Using either Simulink or Matlab, solve the complete system developed in part 1 with the numerical values listed in Table 1 for both:
   a. Constant input voltage (at maximum voltage, for instance), i.e., $v_a(t) = V_{\max}$.
   b. Sinusoidal input voltage at maximum amplitude and frequencies 0.5, 1 and 5 Hz (i.e., $v_a(t) = V_{\max} \sin(\pi\, t)$, $v_a(t) = V_{\max} \sin(2\pi\, t)$, and $v_a(t) = V_{\max} \sin(10\pi\, t)$.

   Report the system responses ($x_1(t)$) using graphs or tables. (<u>Hint:</u> You can develop a block diagram in Simulink to solve the model developed in part 1 or use Matlab to solve for the set of 1st order ODEs developed in part 2).

9. Use Simulink to numerically solve this system via the transfer function developed in part 3. Verify your results with the findings in part 8.
10. Numerically solve part 6 (a-c) and verify with the analytical results developed in part 6.

Control System Design: Clearly, the open loop position response of the system is not acceptable or at least desirable. A "proportional plus derivative – PD" controller can now be used to improve the system response. The controller is taken to be in the form of:

$$v_a(t) = K_p(r(t) - x_1(t)) - K_d \dot{x}_1(t) \qquad (1)$$

where $r(t)$ is the desired cart position (a time-varying variable at this stage) of the cart, and K_p and K_d are the respective proportional and derivative feedback gains to be designed. The proposed controller can be schematically shown in Figure 4, where both K_p and K_d have the respective "volt/m" and "volt.sec/m" dimensions and error $e(t)$ is defined as $e(t) = r(t) - x_1(t)$. All the constant values including mechanical and electrical parameters remain unchanged as given in Table 1.

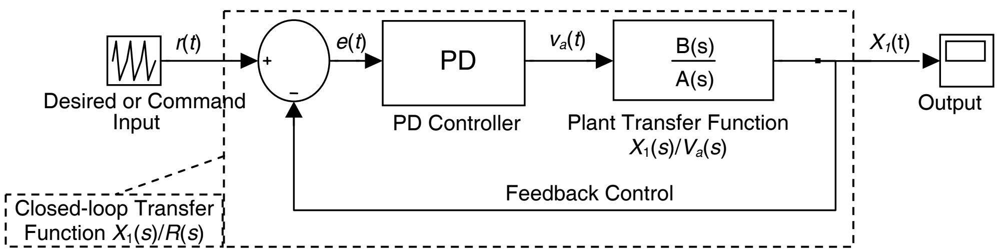


Figure 4. Feedback control of the linear system shown in Figure 1.

11. Set the equations governing the dynamics of this system (this is the same as in Part 1). Replace the input voltage, v_a, with the expression given in equation (1) and classify the new input $r(t)$ (i.e., the desired cart position) and output driven cart position, $x_1(t)$, as the input and output variables, respectively.
12. Assuming zero initial conditions, transfer the governing equations developed in Part 1 into Laplace domain and find the transfer function of the system, $TF(s) = X_1(s)/R(s)$. This will form the closed-loop transfer function of the system from $r(t)$ to $x_1(t)$ as shown in Figure 4 (inside dashed lines).
13. Extract the characteristic equation from Part 12 and determine whether the system is stable or not (using characteristic roots). What is the order of system? How does this compare with the characteristic equations developed in Part 4?
14. What is the position of the driven cart as $t \rightarrow \infty$ (steady-state error) in response to a unit DC input voltage (i.e., $r(t) = u(t)$ or $R(s) = 1/s$)? Does this agree with the stability results of Part 13? Compare this with the open-loop case (Part 5) and draw your conclusion.

Open-loop Control:

15. With $K_p = 1$, $K_d = 0$ and $r(t) = 10$ cm, implement the open-loop block diagram of Figure 4 (with **NO** feedback line) in Simulink. Does the cart follow the command properly? Draw your conclusion.

Closed-loop Control:

16. Implement the closed loop controller of Figure 4 in Simulink. It is now desired to vary K_p and K_d to obtain the three cases of under-damped, critically-damped and over-damped systems. Try to numerically obtain K_p and K_d. One approach is to numerically calculate the corresponding roots of the characteristic equation (using “fzeros” in Matlab, for instance) and determine the corresponding damping ration, ξ, of the dominant root (using “sgrid” in Matlab, for instance).
17. Implement each set of your K_p and K_d and plot the system response ($x_1(t)$). Determine the damping ratio, settling time and overshoot and collect the results in a table for the three cases of under-damped, critically-damped and over-damped systems.

Case	Kp	Kd	ξ	ts	Mp
Under Damped					
Critically Damped			N/A		N/A
Over Damped			N/A		N/A

18. Now, try to see the effect of each controller gains (K_p and K_d) by varying them in some ranges. Decide as to what procedure you will take. For instance, keep one of them unchanged and vary the other one and so on. Observe and take notes from the responses.
19. Using Routh-Hurwitz, Root-Locus or Nyquist techniques, find out each gain’s effect on the performance of the closed loop controller. From the exercises above, find out the best (to the extent possible) combination of these gains and report. Justify your choice of “best” gains. Explain the procedure used.
20. Now, monitor pendulum angle as you change controller gains and determine the best gain selection for minimum angle variation of the pendulum. Do you see any contradiction between best gains selection for cart position and pendulum minimum vibration?

Reporting Format:

Present your report in the order below:

- Analytical model, transfer functions, dynamic representation, 40 pts.
- Control development, numerical results, Matlab, Simulink, … 40 pts.
- Discussion on the results and conclusions 10 pts

The remaining 10 pts. is reserved for the presentation style. Please return **ONE** report per group by the due date.

Table 1. Linear Position Servo System Parameters.

PARAMETER	SYMBOL	VALUE	UNITS
Motor Torque Constant, Back Emf Constant	K_t, K_e, K_b	0.00767	Nm/amp V/(rad/sec)
Armature Resistance	R_a	2.6	Ohms
Armature Inductance	L_a	0.18	mHenry
Maximum Input Voltage	V_{max}	6.0	Volts
Internal Gear Ratio	K_g	3.7:1	-
Armature Inertia	J_m	$3.87e^{-7}$	Kgm^2
Motor Gear Radius	r	0.635	cm
Cart Mass	M_c	0.360	Kg
Motor Shaft Equivalent Viscous Friction	b	0.02	Kg.m/sec
Mass of the Pendulum	M_p	0.230	Kg
Length of the Pendulum	L_p	0.6413	m
Viscous Damping coefficient, as seen at the pendulum axis	B_p	0.0024	N.m.s/rad
Gravitational Constant of Earth	g	9.81	m/s^2