Clemson University Department of Mechanical Engineering

ME 821 Final Term Project	**Due: Mon. 8 Dec. 2008**
Fall 2008	**Dr. Nader Jalili**

Self-erected Inverted Pendulum

Problem Statement:

The aim of this final term project is to collectively combine all your controls knowledge, gained in this course, to design an effective controller to bring a rigid pendulum attached to a moving cart (see Figure 1a) into an upright position (see Figure 1b), what is referred to as "self-erected inverted pendulum".

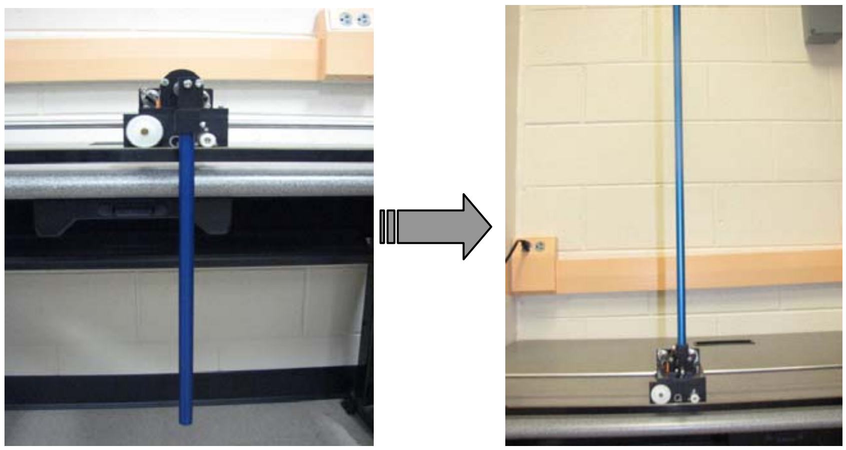

Figure 1. (a) Pendulum in gantry position, and (b) in upright position.

The schematic of the inverted pendulum is shown in Figure 2, where it is pivoted onto a shaft attached to the linear cart with mass M_c. The cart is equipped with a DC motor which is connected to the cart gear through a shaft. The gear ratio between the motor shaft and the cart gear is K_g. When the motor runs, the shaft drives the cart gear resulting in the cart movement inside the rack. As the cart moves, it rotates the encoder wheel from which the position of the cart can be obtained. Another encoder is located in the place where the pendulum is mounted to give the information about the angular position of the pendulum.

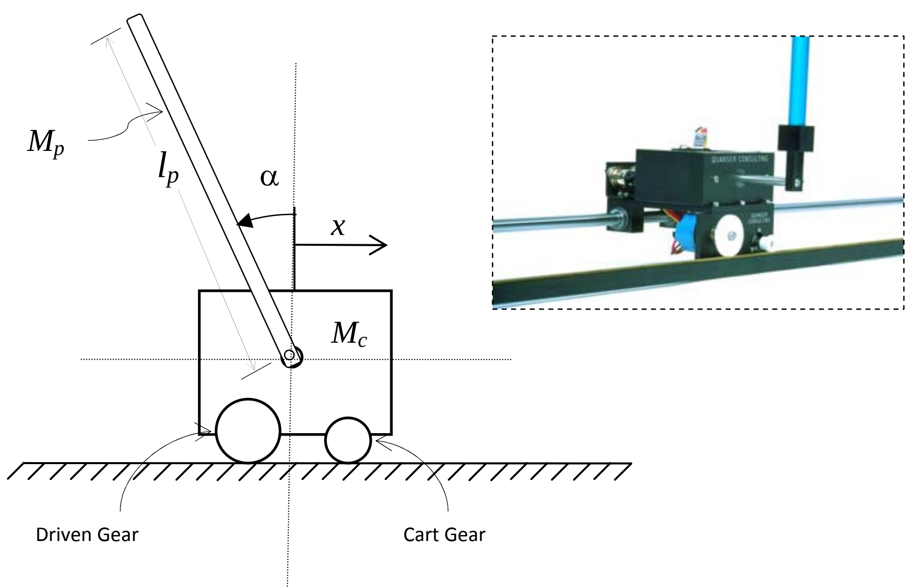

Figure 2. Schematic of inverted pendulum.

For your convenience, the details of the system modeling along with a list of parameters are provided in Appendix A. Both the original and linearized representations are given. You are, however, required to verify these models and modify them if necessary to include other ever-present unmodeled dynamics.

Assignments:

The objective here is to design the input voltage to the DC motor, V_{in} in Appendix A, such that the pendulum initial position ($\alpha(0) = 180^0$), which is a stable equilibrium point, is moved to around the unstable equilibrium position (i.e., $\alpha(t_f) \approx 0^0$) in a finite time t_f and stay in this position despite ever-present disturbances and uncertainties. This is an open-ended problem to be completed as a group project, which should include i) thorough mathematical derivations of the controller(s), ii) complete stability analysis, iii) extensive numerical analysis and results, and iv) experimental implementation and verifications. Your final report must adhere to the reporting requirements and formatting given in Appendix B.

Submission Procedure:

The final report of your project including original Word files of your report, all Matlab/Simulink, Maple files and any other related documents need to be put on a CD with the root directory named as "ME821_FA08_Proj_GroupX", where X is your group letter assigned to you or the last name of your members. This CD along with the hard copy of your report must be turned in on **Monday December 8, 2008, 4:00 PM** in my office (205 Fluor Daniel EIB). This is a **HARD** deadline and cannot be extended. Please turn in ONE report per team.

Appendix A: Detailed Dynamic Modeling of the Inverted Pendulum

The equations of motion of the inverted pendulum shown in Figure 2 can be obtained by Lagrangian approach. For this, the potential energy of the system is first obtained as:

$$V = M_p g\left(\frac{l_p}{2}\right)\cos\alpha(t) \tag{1}$$

where M_p is the mass of the pendulum and l_p is the length of the pendulum. The kinetic energy of the system can be divided into cart and pendulum as given respectively by:

$$K_c = \frac{1}{2}M_c\dot{x}^2(t), \quad K_p = \frac{1}{2}\left[I_p\dot{\alpha}^2(t) + M_p\left(\frac{l_p}{2}\right)^2\dot{\alpha}^2(t) + M_p\dot{x}^2(t) - M_p l_p\dot{x}(t)\dot{\alpha}(t)\cos\alpha(t)\right] \tag{2}$$

where M_c is the mass of the cart and I_p is the inertia of the pendulum about its center.

Hence, the total kinetic energy can be given by $T = K_c + K_p$. Consequently, the Lagrangian $L = T - V$ can be readily formed to be used in the standard Lagrangian equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) + \left(\frac{\partial B}{\partial \dot{q}_i}\right) = Q_i, \quad i = 1,2 \tag{3}$$

where $q_1 = x$ and $q_2 = \alpha$ are the generalized coordinates, and Q_1 and Q_2 are the non-conservative forces acting on cart and pendulum, respectively. The equations of motion can now be obtained for both linear cart position x and pendulum angular displacement α as:

$$(M_c + M_p)\ddot{x}(t) + B_c\dot{x}(t) - M_p\left(\frac{l_p}{2}\right)\ddot{\alpha}(t)\cos\alpha(t) + M_p\left(\frac{l_p}{2}\right)\dot{\alpha}^2(t)\sin\alpha(t) = \frac{K_g K_t}{r_g}i(t) \tag{4}$$

$$(I_p + M_p\left(\frac{l_p}{2}\right)^2)\ddot{\alpha}(t) + B_p\dot{\alpha}(t) - M_p\left(\frac{l_p}{2}\right)\ddot{x}(t)\cos\alpha(t) - M_p g\left(\frac{l_p}{2}\right)\sin\alpha(t) = 0 \tag{5}$$

Current $i(t)$ in Eq. (4) can be related to input voltage as:

$$L\frac{di(t)}{dt} + Ri(t) + \frac{K_m K_g\dot{x}(t)}{r_g} = V_{in}(t) \tag{6}$$

where L is the motor armature inductance, R is the resistance, $i(t)$ is the current flowing through the motor circuit, K_m is the back electromotive force (emf) constant, K_g is the gear ratio between the cart gear and motor gear, r_g is the radius of the cart gear, and $V_{in}(t)$ is the input voltage to the DC motor.

Special Case: If the inductance of the motor armature can be assumed negligible, then Eq. (6) reduces to:

$$Ri(t) + \frac{K_m K_g\dot{x}(t)}{r_g} = V_{in}(t) \tag{7}$$

Hence, Eq. (4) can be simplified as:

$$(M_c + M_p)\ddot{x}(t) + \left(B_c + \frac{K_m K_t K_g^{\,2}}{Rr_g^{\,2}}\right)\dot{x}(t) - M_p\left(\frac{l_p}{2}\right)\ddot{\alpha}(t)\cos\alpha(t) + M_p\left(\frac{l_p}{2}\right)\dot{\alpha}^2(t)\sin\alpha(t) = \frac{K_g K_t}{Rr_g}V_{in}(t) \tag{8}$$

Linearized equations of motion: If α can be assumed to be very small (i.e., $\alpha(t) \le 6^0$), then Eqs. (8) and (5) can reduce to:

$$(M_c + M_p)\ddot{x}(t) + \left(B_c + \frac{K_m K_t K_g^{\,2}}{R r_g^{\,2}} \right)\dot{x}(t) - M_p\left(\frac{l_p}{2}\right)\ddot{\alpha}(t) = \frac{K_g K_t}{R r_g} V_{in}(t) \tag{9}$$

$$(I_p + M_p\left(\frac{l_p}{2}\right)^2)\ddot{\alpha}(t) + B_p\dot{\alpha}(t) - M_p g\left(\frac{l_p}{2}\right)\alpha(t) - M_p\left(\frac{l_p}{2}\right)\ddot{x}(t) = 0 \tag{10}$$

Table A.1. System parameters

Parameter	Symbol	Values	Units
Mass of the Cart	M_C	0.360	Kg
Mass of the Pendulum	M_P	0.230	Kg
Length of the Pendulum	l_P	0.6413	m
Radius of the cart gear	r_g	0.00635	m
Maximum cart travel length	L_t	0.814	m
Internal gear ratio	K_g	3.7:1	-
Motor torque constant	K_t	0.00767	Nm/Amp
Back emf constant	K_m	0.00767	V.sec/rad
Motor input voltage range	V_{in}	±5	Volts
Motor armature resistance	R	2.6	Ω
Motor armature inductance	L	0.18	mH
Cart encoder resolution	K_{EC}	2.275×10^{-05}	m/count
Pendulum encoder resolution	K_{EP}	0.0015	rad/count
Gravitational constant of earth	g	9.81	m/sec^2
Viscous damping coefficient at the pendulum axis	B_p	0.05	N.sec/m.rad
Viscous friction of motor shaft and cart	B_c	0.2	N.sec/m