Final Project of Vibration of Continuous Media

Report Due: Friday May 2, 2008

This document contains a brief description of your final-term project that must be fully developed, experimented and completed along with a complete report and discussions (pls. refer to the "formal report style" document already provided to you during group project assignment.

Piezoelectrically-actuated Cantilever Beams:

(The Next-generation Platform for Cantilever-based Sensors and Actuators)

Project Description: The ultimate goal of this project is to develop a comprehensive model for a piezoelectrically-actuated cantilever beam and perform a detailed modal and vibration analyses as well as experimental verification. The numerical and experimental results are to be compared.

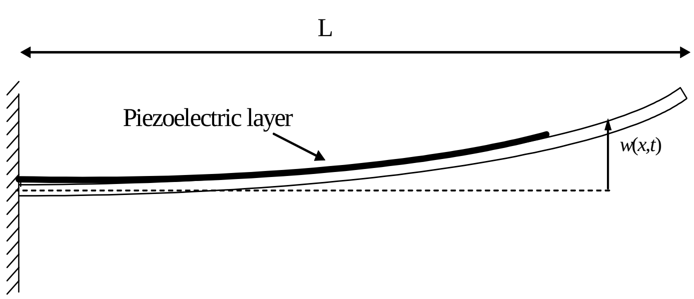
Background and Mathematical Modeling: Consider a uniform flexible cantilever beam with a piezoelectric actuator bonded on its top surface. As shown in Figure 1, one end of the beam is free and the other end is vertically clamped into a fixed base. The beam has total thickness t_b, width w_b and length L, while the piezoelectric film possesses thickness t_p. The piezoelectric actuator is perfectly bonded on the beam. Using the Euler-Bernoulli beam theory for the beam, one can extend the stress/strain relationship within the beam layer

$$\sigma_x^b = E^b \varepsilon_x \tag{1}$$

to the following relationship within the piezoelectric layer as

$$\sigma_x^p = E^p \varepsilon_x - E^p d_{31} \frac{v(t)}{h_p} \tag{2}$$

where E^b is the beam Young's modulus of elasticity, E^p is the piezoelectric Young's modulus of elasticity, d_{31} is the piezoelectric constant, and $v(t)$ is the applied voltage to the piezoelectric layer.



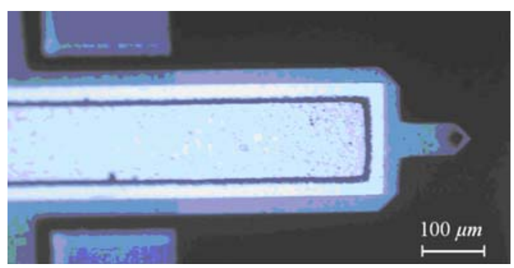


Figure 1. (left) Schematic of the flexible beam covered with piezoelectric patch actuator, and (right) actual image of a piezoelectrically-driven microbeam under microscope covered with ZnO actuator layer.

Assignments:

1) Using extended Hamilton's Principle, derive the governing dynamics for the system and the associated boundary conditions. Use Figure 2 for the geometry of the beam. Notice, due to discontinuous geometry as the one shown in Figure 2, equations (1) and (2) are modified as:

$$\sigma_x^b = E^b \varepsilon_x = -E^b y \frac{\partial^2 w(x,t)}{\partial x^2} \tag{3}$$

$$\sigma_x^p = E^p(\varepsilon_x - d_{31}\frac{v(t)}{t_p}) = E^p\left(-(y-y_n)\frac{\partial^2 w(x,t)}{\partial x^2} - d_{31}\frac{v(t)}{t_p}\right) \tag{4}$$

where y_n is the neutral surface in the composite (beam/piezoelectric layer) portion of the cantilever given by

$$y_n = \frac{E^p t_p w_p (t_p + t_b)}{2(E^p t_p w_p + E^b t_b w_{b1})} \tag{5}$$

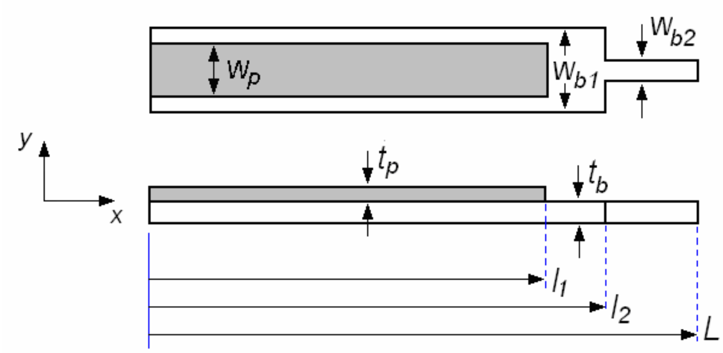


Figure 2. Schematic representation of microcantilever beam with an attached piezoelectric layer on its top surface.

2) Although the beam is clearly non-uniform, represent the eigenfunction problem and obtain the associated eigenfrequencies assuming uniform cross-section. Numerically and using Table 1, obtain the first 3 eigenfrequencies of this system.

3) Solve the forced vibration problem in which $v(t)$ is considered to be a unit-step input and obtain the cantilever tip deflection, $w(l,t)$. Plot your responses in both time-domain and frequency-domain and verify your findings in part (2) above using your frequency response obtained here.

4) Assuming plate-like structure for this system, repeat part (2) above and compare your results with the ones obtained in part (2), especially on the first few eigenfrequencies.

5) Experimentally, obtain the first few eigenfrequencies and compare the results with both parts (2) and (4). You will be provided help and instructions to perform this task.

Table 1. Physical and numerical parameters.

Parameters	**Values**
$L\,(\mu m)$	486
$L_1\,(\mu m)$	325
$L_2\,(\mu m)$	360
$E^b\,(Gpa)$	105
$E^p\,(Gpa)$	104
$\rho_b\,(kg/m^3)$	2330
$\rho_p\,(kg/m^3)$	6390
$w_{b1}\,(\mu m)$	250
$w_{b2}\,(\mu m)$	55
$w_p\,(\mu m)$	130
$t_b\,(\mu m)$	4
$t_p\,(\mu m)$	4
d_{31} (pC/N)	11