Project A:
Active Vibration Suppression of Lumped-Parameters Systems using Piezoelectric Inertial Actuators [*]

A dynamic vibration absorber referred to as active resonator absorber (ARA) is considered here, while exploring its practical implementation using piezoelectric ceramic (PZT) inertial actuators. The ARA is a passive absorber with an additional dynamic feedback compensator within the PZT actuator (see Figure 1). Without any controller, it becomes a passive vibration absorber due to internal damping and elasticity properties of the piezoelectric materials, hence, it is inherently fail-safe. For active operation, the compensator parameters are designed such that a resonance condition is intentionally created within the absorber subsection to mimic the vibratory energy from the system of concern to which it is attached. The resonance condition can be created through the appropriate design of the compensator and implemented through adjusting the external electrical voltage applied to the PZT actuator. Although PZT materials inherently possess nonlinear characteristics (e.g., hysteresis), an equivalent linearized model could be developed for the actuator subsystem.

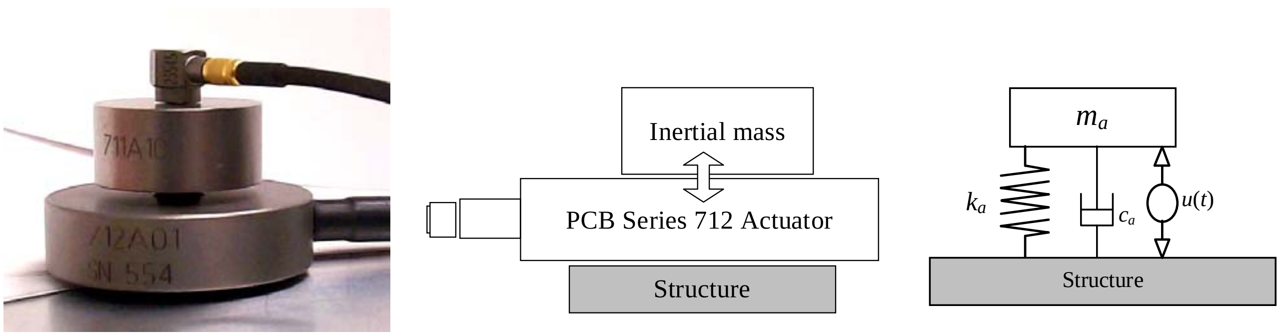

Figure 1 PCB[♦] series 712 PZT inertial actuator (left), schematic of operation (middle), and a simple SDOF mathematical model (right).

A very important component of any active vibration absorber is the actuator unit. Recent advances in smart materials have led to the development of advanced actuators using piezoelectric ceramics, shape memory alloys, and magnetostrictive materials. Over the past couple of decades, the piezoelectric ceramics have been utilized as potential replacements for conventional transducers. These materials are compounds of lead zirconate-titanate (PZT). The PZT properties can be optimized to suit specific applications by appropriate adjustment of the zirconate-titanate ratio. Specifically, a piezoelectric inertial actuator is an efficient and inexpensive solution for active structural vibration control. As shown in Figure 1, it applies a point force to the structure to which it is attached.

The underlying proposition in all dynamic vibration absorbers is to properly "sensitize" the absorber subsection such that it becomes absorbent of the vibratory energy (see Figure 2). In this project, an implementation of the tuned vibration absorbers referred to as active resonator absorber (ARA) is discussed. Using a simple position (or velocity or acceleration) feedback

[*] **Jalili, N.** and Knowles, D. W., "Structural Vibration Control using an Active Resonator Absorber: Modeling and Control Implementation", *Smart Materials and Structures,* **13** (5), 998-1005 (2004).

[♦] Active Vibration Control Instrumentation, A Division of *PCB Piezotronics, Inc.,* www.pcb.com.

control within the absorber subsystem, the ARA enforces the dominant characteristic roots of the absorber subsection to be on the imaginary axis, and hence leading to resonance (Figure 2-bottom). Once the ARA becomes resonant, it creates perfect vibration absorption at this frequency.

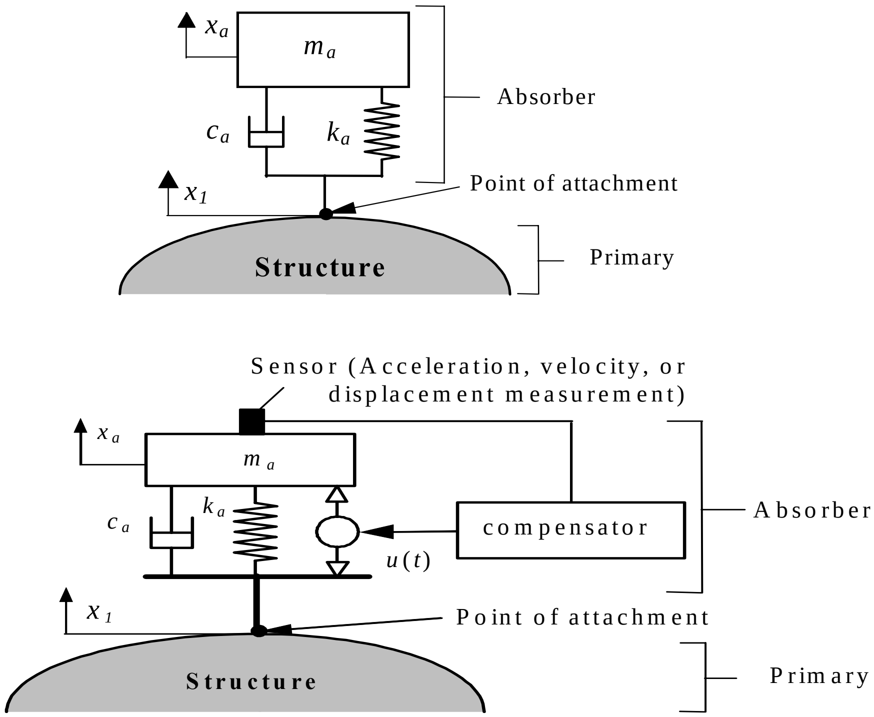

Figure 2 A general primary structure with passive (top) and active (bottom) absorber settings.

Overview of PZT Inertial Actuators: The PZT inertial actuators are most commonly made out of two parallel piezoelectric plates (see Figure 1). The resonance of such actuator can be adjusted by the size of the inertial mass (see Figure 1). Increasing the size of the inertial mass will lower the resonance frequency and decreasing the mass will increase it. The resonance frequency, f_r, can be expressed as

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k_a}{m_a}}\ , \tag{1}$$

where k_a is the effective stiffness of the actuator, m_a is defined as

$$m_a = m_{e_{PZT}} + m_{inertial} + m_{acc} \tag{2}$$

and $m_{e_{PZT}}$ is the PZT effective mass, $m_{inertial}$ is the inertial mass, and m_{acc} is the accelerometer mass. Using a simple SDOF system (see Figure 1), the parameters of the PZT inertial actuators can be experimentally determined. The model parameters consist of the effective mass, damping and stiffness of the absorber as well as the polynomial form of the non-linearity.

Active Resonator Absorber Concept: The concept of ARA is to design a compensator using a simple feedback (position or velocity, or acceleration) control within the absorber subsection for the mentioned "sensitization". Such arrangement places the dominant characteristic roots of the absorber subsection to be on the imaginary axis, and hence, creating resonance (see Figure 3).

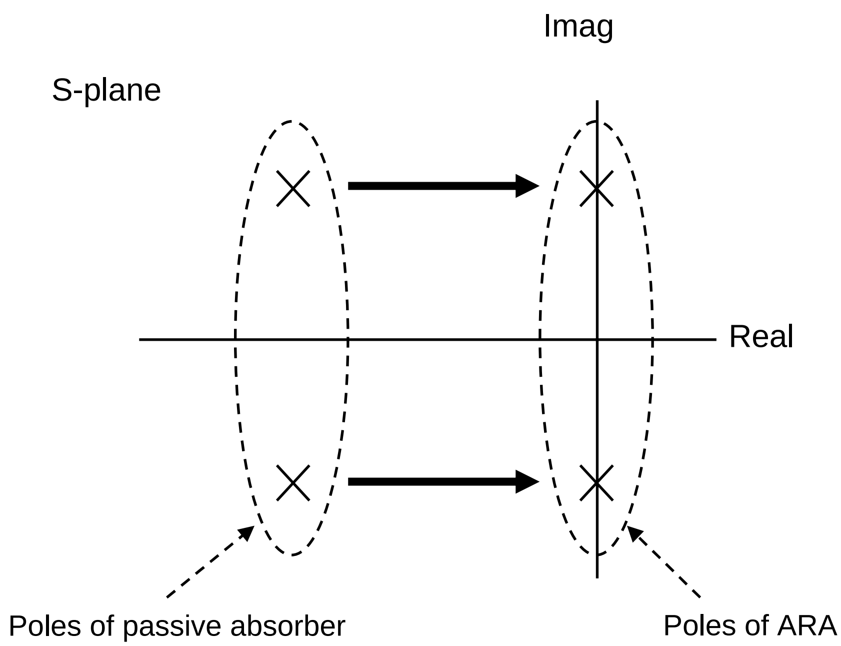

Figure 3 Schematic of the active resonator absorber concept through placing the poles of the characteristic equation on the imaginary axis.

The ARA requires only one signal from the absorber mass, absolute or relative to the point of attachment (see Figure 2-bottom). After the signal is processed through a compensator, an additional force is produced, for instance, by a PZT inertial actuator. By properly setting the compensator parameters, the absorber should behave as an ideal resonator at one or even more frequencies. As a result, the resonator will absorb vibratory energy from the primary mass at given frequencies. The frequency to be absorbed can be tuned in real-time. Moreover if the controller or the actuator fails, the ARA will still function as a passive absorber, thus it is inherently fail-safe.

For the case of linear assumption for the PZT actuator, the dynamics of the ARA (Figure 2-bottom) can be expressed as

$$m_a \ddot{x}_a(t) + c_a \dot{x}_a(t) + k_a x_a(t) - u(t) = c_a \dot{x}_1(t) + k_a x_1(t) \tag{3}$$

where $x_1(t)$ and $x_a(t)$ are the respective primary (at the absorber point of attachment) and absorber mass displacements. The mass m_a is given by equation (2), and control $u(t)$ is designed to produce designated resonance frequencies within the ARA.

The objective of the ARA is to keep the absorber subsystem marginally stable at particular frequencies in the determined frequency range. The objective of the feedback control, $u(t)$, is to convert the dissipative structure (Figure 2-top) into a conservative or marginally stable one (Figure 2-bottom) with a designated resonance frequency, ω_c. In other words, the control aims the placement of dominant poles at $\pm j\omega_c$ for the combined system, where $j=\sqrt{-1}$. As a result, the ARA becomes marginally stable at particular frequencies in the determined frequency range. Using a simple position (or velocity, or acceleration) feedback within the absorber subsection (i.e. $U(s)=\overline{U}(s)X_a(s)$), the corresponding dynamics of the ARA given by (3) in the Laplace domain become

$$(m_a s^2+c_a s+k_a)X_a(s)-\overline{U}(s)X_a(s)=C(s)\,X_a(s)=(c_a s+k_a)\,X_1(s) \tag{4}$$

The compensator transfer function $\overline{U}(s)$ is then selected such that the primary system displacement, at the absorber point of attachment, is forced to be zero, i.e.,

$$C(s)=(m_a s^2+c_a s+k_a)-\overline{U}(s)=0\,. \tag{5}$$

The parameters of the compensator are determined through introducing resonance conditions to the absorber characteristic equation, $C(s)$. That is, equations $\mathrm{Re}\{C(j\omega_i)\}=0$ and $\mathrm{Im}\{C(j\omega_i)\}=0$ are simultaneously solved, where $i=1,2,...,l$ and l is the number of frequencies to be absorbed. Using additional compensator parameters, the stable frequency range or other properties can be adjusted in real time.

Consider the case where $U(s)$ is taken as a proportional compensator with a single time constant based on the acceleration of the ARA given by

$$U(s)=\overline{U}(s)X_a(s)\quad\text{where}\quad \overline{U}(s)=\frac{gs^2}{1+Ts} \tag{6}$$

In the time domain, the control force, $u(t)$, is given as

$$u(t)=\frac{g}{T}\int_0^t e^{-(t-\tau)/T}\,\ddot{x}_a(\tau)\,d\tau \tag{7}$$

To achieve ideal resonator behavior, two dominant roots of equation (4) are placed on the imaginary axis at the desired crossing frequency ω_c. Substituting $s=\pm j\omega_c$ into equation (4) and solving for the control parameters, g_c and T_c, one can obtain

$$g_c=m_a\left(\frac{c_a{}^2}{m_a{}^2\left(\omega^2-k_a/m_a\right)}-\frac{k_a}{m_a\omega^2}+1\right),\quad T_c=\frac{c_a\sqrt{k_a/m_a}}{\sqrt{m_a k_a}\left(\omega^2-k_a/m_a\right)},\ \text{for}\ \omega=\omega_c \tag{8}$$

The control parameters, g_c and T_c, are based on the physical properties of the ARA (i.e. c_a, k_a, m_a) as well as the frequency of the disturbance ω, illustrating the ARA does not require any information from the primary system to which it is attached. However, when the physical properties of the ARA are not known within a high degree of certainty, a method to auto-tune the control parameters must be considered. The stability assurance of such auto-tuning proposition will bring primary system parameters into the derivations, and hence, the primary system can not be totally de-coupled.

Assignments:

In order to demonstrate the effectiveness of the proposed ARA, a simple single-degree-of-freedom (SDOF) primary system subjected to tonal force excitations is considered. As shown in Figure 4, two PZT inertial actuators are used for both the primary (model 712-A01) and the absorber (model 712-A02) subsections. Each system consists of passive elements (spring and damper of the PZT materials) and active compartment with the physical parameters listed in Table 1. The top actuator acts as the ARA with the controlled force $u(t)$, while the bottom one represents the primary system subjected to the force excitation f(t).

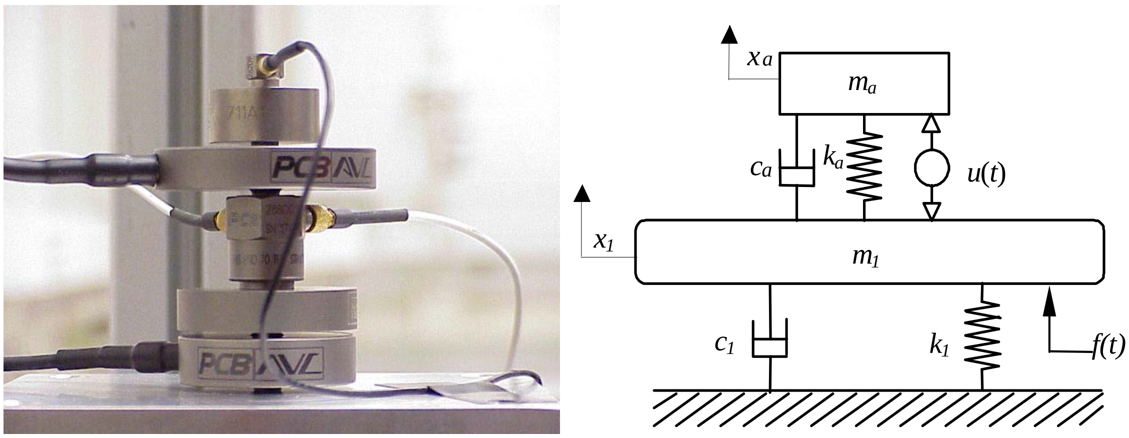

Figure 4 Implementation of ARA concept using two PZT actuators (left) and its mathematical model (right).

Table 1 Experimentally determined parameters of PCB Series 712 PZT inertial actuators.

PZT System Parameters	PCB Model 712-A01	PCB Model 712-A02
Effective Mass [gr], m_{ePZT}	7.199	12.14
Inertial Mass [gr], $m_{inertial}$	100	200
Stiffness [kN/m], k_a	3814.9	401.5
Damping [Ns/m], c_a	79.49	11.48

1. Derive the governing dynamics for the combined system. For the PZT actuator modeling, use the constitutive equations of piezoelectric materials. For this, the one-dimensional linear electromechanical constitutive relations between the strain, S_3, the stress, T_3, and the electric field, E_3, may be expressed as

$$S_3 = d_{33}\, E_3 + \frac{1}{Y_{33}} T_3 \tag{9}$$

where d_{33} is the linear electromechanical coupling coefficient and Y_{33} is the PZT material's elastic modulus measured at constant field. In most cases, PZT actuators produce displacement. If used in a restrainted configuration, they may generate forces. Force generation is always coupled with a reduction in displacement. To simplify the analysis, linear constitutive equations are often assumed and the PZT actuator is modeled as a spring/mass system. A system level actuator description with forces, in the presence of an external mechanical load F_0, is shown in Figure 5. The equation of motion can be written as

$$(M + m_a / 3)\ddot{x}_a + F_a + F_0 = 0 \tag{10}$$

where M is the external load mass, m_a is the actuator mass, and F_a is the force exerted by the actuator. Assume that the actuator has a n-layered stack configuration (see Figure 6) with each layer containing an area, A, and thickness, d. The resultant actuator stress (i.e., actuator force) may be expressed as

$$F_a = A\,Y_{33}\left(\frac{x_a}{n\,d} - \frac{d_{33}}{d}v_a\right) = \left(k_a x_a\right) - \left(\frac{A\,Y_{33}\,d_{33}}{d}v_a\right) = F_s - F_v \tag{11}$$

where $k_a = AY_{33}/nd$ is the PZT material equivalent stiffness.

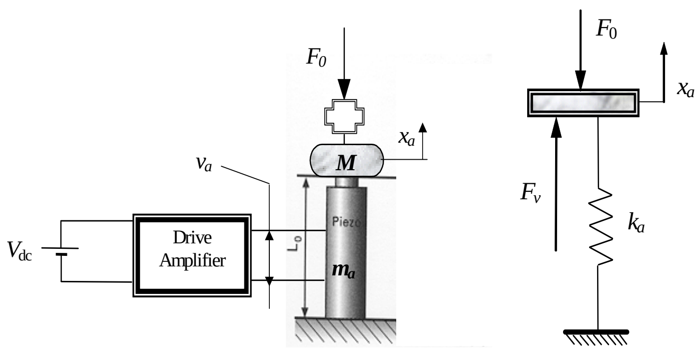

Figure 5 PZT actuator-structure.

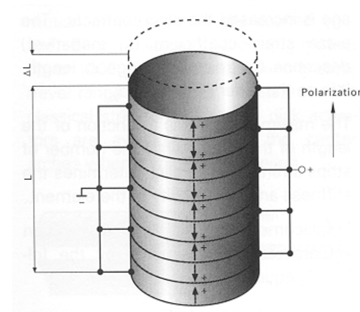

Figure 6 Linear (stacked design) piezoelectric actuator.

2. Use a single time constant, based on the acceleration of the ARA, and establish the sufficient and necessary condition for asymptotic stability as functions of compensator parameters, g and T, using Routh-Hurwitz method.

If the primary system is subjected to a harmonic excitation with unit amplitude and a frequency of 800 Hz with the ARA and primary system parameters taken as those given in Table 1 and d_{33}=10^{-7} C/N, numerically simulate the primary system and the absorber displacements for a set of stable compensator parameters.

3. The physical parameters are assumed to be known exactly. However, these parameters are not known exactly in practice or vary with time, so the case with estimated system parameters must be considered. To demonstrate the feasibility of the vibration control, the nominal system parameters (m_a, m_1, k_a, k_1, c_a, c_1) are fictitiously perturbed by 10% (i.e., representing the actual values) in the simulation. However, the nominal values of m_a, m_1, k_a, k_1, c_a, and c_1 are going to be used for calculation of the compensator parameters, g and T. Try to tune the system parameters accordingly such the vibration suppression is attained even for this perturbed system. Report your numerical results and discuss them.