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LPV gain-scheduled control of SCR aftertreatment systems

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Hydrocarbons, carbon monoxide and some of other polluting emissions produced by diesel engines are usually lower than those produced by gasoline engines. While great strides have been made in the exhaust aftertreatment of vehicular pollutants, the elimination of nitrogen oxide (NO_x) from diesel vehicles is still a challenge. The primary reason is that diesel combustion is a fuel-lean process, and hence there is significant unreacted oxygen in the exhaust. Selective catalytic reduction (SCR) is a well-developed technology for power plants and has been recently employed for reducing NO_x emissions from automotive sources and in particular, heavy-duty diesel engines. In this article, we develop a linear parameter-varying (LPV) feedforward/feedback control design method for the SCR aftertreatment system to decrease NO_x emissions while keeping ammonia slippage to a desired low level downstream the catalyst. The performance of the closed-loop system obtained from the interconnection of the SCR system and the output feedback LPV control strategy is then compared with other control design methods including sliding mode, and observer-based static state-feedback parameter-varying control. To reduce the computational complexity involved in the control design process, the number of LPV parameters in the developed quasi-LPV (qLPV) model is reduced by applying the principal component analysis technique. An LPV feedback/feedforward controller is then designed for the qLPV model with reduced number of scheduling parameters. The designed full-order controller is further simplified to a first-order transfer function with a parameter-varying gain and pole. Finally, simulation results using both a low-order model and a high-fidelity and high-order model of SCR reactions in GT-POWER interfaced with MATLAB/SIMULINK illustrate the high NO_x conversion efficiency of the closed-loop SCR system using the proposed parameter-varying control law.

Keywords: selective catalytic reduction; gain-scheduling control; principal component analysis

1. Introduction

The lean burn conditions of combustion in automotive diesel engines that yield improved efficiency produce an exhaust containing an excess of oxygen. While this net-oxidising exhaust enables the comparatively straightforward oxidation of hydrocarbons and carbon monoxide (CO) on precious metal catalysts, it complicates the chemical reduction of nitrogen oxide (NO_x) to N_2. This scenario has led to the vigorous development of technologies for NO_x reduction to meet the stringent NO_x exhaust limits mandated by US and European agencies. The NO_x emissions are one of the main air pollutants that are responsible for ozone depletion and photochemical smog formation causing severe respiratory problems to humans. Selective catalytic reduction (SCR) is a well-proven technology used in power generation for more than 30 years. The use of SCR technology has been also favoured by automotive industry in recent years due to its lower cost compared to other NO_x reduction aftertreatment systems. SCR technology has been used in Europe since 2008 and introduced in the US in 2010 (Johnson 2010). It is important to note that stationary power generation involves very slow variation of operating conditions allowing simple open-loop controllers to efficiently tackle the SCR control design problem. However, automobile engines work in a broad envelope of fast varying conditions necessitating the use of advanced closed-loop SCR control techniques.

The basic operating principle of the closed-loop SCR system is as follows: a urea injector, driven by a command signal from the controller, pumps the aqueous urea solution into the exhaust stream through a nozzle. Ammonia (NH_3) and carbon dioxide (CO_2) are formed as a result of urea decomposition and HNCO hydrolysis upstream the SCR catalyst (Koebel, Elsener, and Kleemann 2000). The mixture

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of ammonia, CO_2, remaining urea, and the exhaust then enters the SCR catalyst, where NH_3 reacts with NO_x from the exhaust producing nitrogen and water. As shown in Figure 1, a sensor placed at the catalyst outlet measures concentration of the unreacted NO_x and supplies this information to the controller, thereby closing the loop. The control problem consists of achieving the appropriate regulation of the urea injection to minimise NO_x emissions without significant ammonia slip.

The key requirement for an SCR catalyst is to selectively reduce NO_x to N_2 in the presence of ammonia. SCR washcoats contain base metals such as copper (Cu), iron (Fe) and zeolites (Ze) that store ammonia to enable NO_x reduction. The overall SCR chemistry is generally well-understood and qualitatively similar for Cu–Ze and Fe–Ze catalysts. Ammonia is obtained by the thermal decomposition and hydrolysis of externally supplied aqueous urea. The three key reactions involved in the SCR NO_x reduction process are the following (Tronconi and Forzatti 1992):

$$4NH_3 + 4NO + O_2 \longrightarrow 4N_2 + 6H_2O,$$
$$4NH_3 + 3NO_2 \longrightarrow 7/2N_2 + 6H_2O,$$
$$4NH_3 + 2NO + 2NO_2 \longrightarrow 4N_2 + 6H_2O.$$

The first and second reactions are referred to as the 'standard SCR reaction' and the 'slow SCR reaction', respectively. The third reaction involves both NO and NO_2 at equimolar amounts, and is much faster than the first reaction with only NO and is referred to as the 'fast SCR reaction'. It is also important to note that several side reactions can occur along with the standard and fast SCR reactions, the most important of which is the ammonia oxidation:

$$4NH_3 + 3O_2 \longrightarrow 2N_2 + 6H_2O,$$
$$2NH_3 + 5/2O_2 \longrightarrow 2NO + 3H_2O.$$

These reactions are not desired since they involve the consumption of the ammonia reductant (Nova, Ciardelli, Tronconi, Chatterjee, and Bandl-Konrad 2006; Metkar, Salazar, Muncrief, Balakotaiah, and Harold 2011). The extent of byproduct formation varies with the operating conditions, such as temperature and feed composition. In fact, the NO_x conversion is improved by the presence of NO_2 except at high temperatures where ammonia oxidation limits NO_x conversion.

There have been some efforts in lumped parameter modelling of the SCR reactions in the literature. These simplified models are appropriate for model-based control, since they reduce the complexity of the design. Tronconi and Forzatti (1992) developed one- and two-dimensional steady-state isothermal models of SCR for different geometries of the catalyst. Upahhyay and van Nieuwstadt (2002) derived a lumped parameter model of SCR by first assuming that the catalyst behaves as an isothermal continuously stirred tank reactor and neglecting mass transfer, and subsequently using the method of weighted residuals. The reaction mechanism involves the $DeNO_x$ reaction, adsorption/desorption of NH_3 and NH_3 oxidation. This model considers the reduction of only NO by ammonia. Since, in Fe–Ze catalyst NO_2-based reactions are highly favoured, and NO_2 is more toxic compared to NO, Devarakonda, Parker, Johnson, Strots, and Santhanam (2008) presented a set of ordinary differential equations (ODEs) to model the SCR reactions considering both the fast SCR reaction involving NO_2 and the standard SCR reaction.

Model-based control and optimisation of the SCR system have been the focus of few recent publications. Upahhyay and van Nieuwstadt (2006) presented a model-based control strategy using a sliding mode control approach. A nonlinear observer is designed using the measured NO_x concentration downstream the SCR catalyst for the estimation of the surface coverage fraction and ammonia slip concentration. A similar control design method which incorporates both the NO and NO_2 conversion efficiency in addition to ammonia slip is proposed in Devarakonda, Parker, Johnson, Strots, and Santhanam (2008). Schar, Onder, and Geering (2006) used a similar model as in Upahhyay and van Nieuwstadt (2002) and designed a model-based feedforward controller to limit the ammonia slip and a PI feedback controller for disturbance rejection. Chi and Dacosta (2005) presented a more advanced SCR model where the catalyst

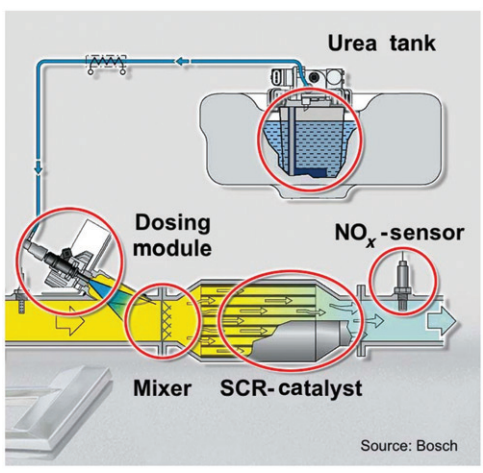

Figure 1. SCR operating principle (source: http://www.bosch.com).

channel is discretised axially and radially. A simplified first-order model of the system is then used in Chi and Dacosta (2005) for control design purposes, where the parameters are estimated in real-time using a model reference adaptive controller. In Hsieh and Wang (2009), a PID-nonlinear model predictive controller is designed and experimentally validated for a urea-SCR system.

The lumped parameter model of the SCR reactions we use in this article for control design purposes is a nonlinear model corresponding to the *ammonia adsorption and desorption*, *standard SCR reduction*, as well as *ammonia oxidation to* NO. It is noted that the general design approaches presented in this article can be extended to the *fast SCR reaction*, where ammonia reacts with both NO and NO_2, considering the 4-state lumped parameter model developed in Devarakonda et al. (2008). Traditional gain-scheduling control methods for nonlinear systems are based on interpolated, linear controllers that are scheduled based on a function of varying parameters. These methods are inherently *ad hoc* and the resulting controllers provide no stability or performance guarantees for rapid changes in the scheduling variables (Rugh and Shamma 2000). These issues are the main motivation for recent research efforts on multi-variable robust gain-scheduled control techniques in the context of linear parameter-varying (LPV) systems (Apkarian and Gahinet 1995; Apkarian and Adams 1998; Rugh and Shamma 2000). LPV systems are linear systems whose state-space description is a known function of time-varying parameters. The time variation of the LPV parameters is not known *a priori*, but is assumed to be measurable in real-time. For such systems, the controller is designed to be an LPV system, whose state-space entries depend causally on the parameters' history. Stabilisation and more realistic problems involving closed-loop performance objectives while exploiting known bounds on the LPV parameters' variation rate can also be posed in the LPV design framework. The LPV control design problem can be efficiently solved by formulating the problem into either: (i) a finite-dimensional, convex feasibility problem which can be solved exactly, or (ii) an infinite-dimensional convex feasibility problem which can be solved approximately. Quasi-LPV (qLPV) systems represent a special class of LPV systems where the scheduling variables contain system states (Shamma and Cloutier 1993).

The main contribution of this work is threefold: (i) this is the first attempt to apply LPV control methodology to cope with the nonlinearities and operating condition variability (as seen later, temperature and catalyst surface coverage fraction) in the SCR system dynamics; (ii) in some cases, the LPV modelling process will be shown to present challenges specifically due to the number of LPV parameters, which will be addressed by making use of the principal component analysis (PCA) method; (iii) finally, a closed-form solution to the design of a parameter-varying feedforward controller will be proposed to ensure a limit on the ammonia slip that would not otherwise be possible using only feedback control. The proposed control design methods of this article will be validated on a low-order SCR reaction model, as well as, a high-fidelity and high-order model of SCR reactions developed in GT-POWER.

The qLPV model we develop for the SCR system in this article includes several scheduling parameters and may not be practical in its initial LPV representation for control implementation purposes. A remedy to reduce the number of scheduling parameters is the use of the PCA method, as an effective method to reduce the dimensionality of a data set consisting of a large number of interrelated variables, while retaining as much as possible of the variation present (Jolliffe 2002). This technique has been recently extended to LPV models in Kwiatkowski and Werner (2008) to reduce the number of scheduling parameters. We take advantage of this method for our SCR model, where the catalyst temperature is varying. An additional benefit of using PCA for reducing the number of LPV scheduling parameters is that the LPV system with the new set of LPV parameters is in affine form. Systems with affine dependency on the LPV parameters are computationally tractable in terms of the solution to the optimisation problem corresponding to the control design procedure. It is noted that in order to simplify the presentation and the design process, this first attempt of LPV control design for the SCR process does not include exhaust flow as scheduling parameter. Some of the variability of exhaust flow is indirectly taken into account in the variability of the surface coverage fraction estimate. Future efforts will include exhaust flow as an additional scheduling parameter using the the PCA-based LPV parameter reduction method described in this article.

In this article, we use a third-order nonlinear model developed in Upahhyay and van Nieuwstadt (2002) and design a reduced order gain-scheduled LPV controller to improve the NO conversion efficiency while the ammonia slip is kept around a desired value. The simulation results of this article illustrate a comparison between the performance of the developed output feedback parameter-varying controller with those of an observer-based static state-feedback parameter-varying controller and a sliding mode observer-based controller.

2. SCR system lumped parameter model and its qLPV representations

In this article, our objective is to utilise a previously developed zero-dimensional (i.e. lumped parameter) model of the SCR aftertreatment system in Upahhyay and van Nieuwstadt (2002) to design a self-scheduled LPV controller. The lumped parameter model developed in Upahhyay and van Nieuwstadt (2002) includes a set of three ODEs where the NO and NH_3 concentrations, as well as ammonia surface coverage fraction are the three states of the resulting dynamic equations. The ammonia coverage fraction is defined as the ratio of the number of stored sites filled with NH_3 to the total number of storage sites in the catalyst. The model below is associated with the first reaction, i.e. standard SCR, described above. The complete model is as follows:

$$\begin{bmatrix} \dot{C}_{NO} \\ \dot{\theta} \\ \dot{C}_{NH_3} \end{bmatrix} = \begin{bmatrix} f_1(C_{\mathrm{NO}}, \theta, C_{\mathrm{NH_3}}) \\ f_2(C_{\mathrm{NO}}, \theta, C_{\mathrm{NH_3}}) \\ f_3(C_{\mathrm{NO}}, \theta, C_{\mathrm{NH_3}}) \end{bmatrix} + \begin{bmatrix} \frac{F}{V} \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ \frac{F}{V} \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{\mathrm{NO}} \\ \theta \\ C_{\mathrm{NH_3}} \end{bmatrix}$$

$$f_1 = -C_{\mathrm{NO}}\left(\Theta_{\mathrm{SC}} k_{red}\theta + \frac{F}{V}\right) + k_{ox}\Theta_{\mathrm{SC}}\theta,$$

$$f_2 = -\theta(k_{ads}C_{\mathrm{NH_3}} + k_{des} + k_{red}C_{\mathrm{NO}} + k_{ox}) + k_{ads}C_{\mathrm{NH_3}},$$

$$f_3 = -C_{\mathrm{NH_3}}\left(\Theta_{\mathrm{SC}} k_{ads}(1-\theta) + \frac{F}{V}\right) + \Theta_{\mathrm{SC}} k_{des}\theta, \tag{1}$$

where C_{NO} and $C_{\mathrm{NH_3}}$ are concentrations of NO and NH_3 downstream the catalyst in mol/m^3, respectively. The reaction rate constant for reaction i is represented by $k_i = A_i \exp(\frac{-E_i}{RT})$, $i = ads, des, red, ox$ where E_i is the activation energy and A_i is the pre-exponential term for the corresponding reaction. The parameter Θ_{SC} is the maximum ammonia storage capacity and θ is the surface coverage fraction. The parameters F, V, R and T represent the flow rate through catalyst, catalyst volume, universal gas constant and catalyst surface temperature, respectively. The input u is the concentration of ammonia entering the catalyst and is the only controllable variable. The input d is the NO concentration upstream the catalyst treated as an external disturbance. The only available measurement for control design purposes is the concentration of NO downstream the catalyst denoted by y. Observability and controllability of the system (1) are discussed in Upahhyay and van Nieuwstadt (2006). The numerical values of the reaction rate constants parameters (A_i and E_i) corresponding to the model described above that we used in this article are listed in Table 1.

It should be noted that the lumped parameter model described above has a number of limitations, some of which have been discussed in Upahhyay and van Nieuwstadt (2002). Some of these assumptions are well-justified, others are important to keep the complexity of the model low but may limit the ability of the model to capture some of the catalyst behaviours. The assumptions made to develop the model are as follows:

- The dynamics of NO_x sensor is not considered mainly due to the fact that it is faster compared to the SCR reactions.
- Breakthrough time is ignored. This is the time that it takes for the ammonia to be adsorbed into the catalyst surface leading to a time delay between when the ammonia is injected and when ammonia slip is observed. The breakthrough time can be calculated from the shock velocity of the NO_x adsorption wave estimated by the Rhee equation (Sharma, Harold, and Balakotaiah 2005).
- Dependence of the storage capacity Θ_{SC} on the temperature is ignored, and a constant value of $\Theta_{\mathrm{SC}} = 36.36\,\mathrm{mol/m^3}$ is considered throughout this article.

2.1 *LPV model for constant temperature case*

The dynamic model of the SCR system introduced above can be transformed to a qLPV form as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\Theta_{\mathrm{SC}} k_{red} x_2 - \frac{F}{V} & k_{ox}\Theta_{\mathrm{SC}} & 0 \\ -k_{red}x_2 & -(k_{ox} + k_{des}) & k_{ads}(1-x_2) \\ 0 & \Theta_{\mathrm{SC}} k_{des} & -\Theta_{\mathrm{SC}} k_{ads}(1-x_2) - \frac{F}{V} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{F}{V} \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ \frac{F}{V} \end{bmatrix} u. \tag{2}$$

where $x_1 = C_{\mathrm{NO}}, x_2 = \theta, x_3 = C_{\mathrm{NH_3}}$ are the three states corresponding to the above state-space representation. As observed, for the case with constant temperature, and hence invariant rate constants, the system matrix A is affinely dependent on the second state, i.e. surface coverage fraction θ, and therefore θ

Table 1. Numerical values corresponding to the system (1).

Reaction	A	E (K J/mol)
Standard SCR	3.2E+09 m^3/mol s	85
NH_3 oxidation	4.3E+06 1/s	112
NH_3 adsorption	2.1E+07 m^3/mol s	42
NH_3 desorption	3.3E+11 1/s	113

is considered as the scheduling parameter in the above qLPV model provided that it is known in real-time. In the state-space representation above, the system matrix A can be represented as $A(\theta) = A_0 + \theta A_1$ and the rest of the system matrices are parameter-independent.

2.2 *LPV model for temperature-varying case*

In the previous section, the catalyst temperature was assumed to remain constant. In this section, we modify the control-oriented LPV model of the SCR system to account for the variability of the exhaust temperature as a result of different engine speeds and loads. It has been reported that there is a small difference between the exhaust temperature and the catalyst downstream temperature (Devarakonda et al. 2008), and the catalyst operating temperature can be considered as the average of these two temperatures.

When trying to develop a qLPV model for the temperature-varying case, a challenge is the large number of LPV parameters that appear in the model since the reaction rate constants are now dependent on the catalyst temperature. In this article, we use the PCA method to reduce the number of LPV parameters to achieve a reasonably complex model useful for the synthesis of LPV gain-scheduling controllers. This method enables a systematic trade-off between the number of reduced parameters and the desired model accuracy (Kwiatkowski and Werner 2008). In this section, we briefly describe the process involved and present the results of applying the PCA method to the SCR system model discussed earlier where the scheduling parameters $\hat{\theta}$ and T are estimated and measured, respectively (i.e. $\rho = \begin{bmatrix} \hat{\theta} & T \end{bmatrix}^T$). For the temperature-varying case, the system matrices can be parameterised as a function of a time-varying parameter vector $\delta(t)$ that depends on the scheduling parameter vector $\rho(t)$ as follows:

$$\delta_1 = k_{red}\hat{\theta}, \quad \delta_2 = k_{ox}, \quad \delta_3 = k_{des}, \quad \delta_4 = k_{ads}, \quad \delta_5 = k_{ads}\hat{\theta}. \tag{3}$$

Using this parameter vector selection, the system matrix A becomes

$$A = \begin{bmatrix} -\frac{F}{V} - \Theta_{sc}\delta_1 & \Theta_{sc}\delta_2 & 0 \\ -\delta_1 & -\delta_3 - \delta_2 & \delta_4 - \delta_5 \\ 0 & \Theta_{sc}\delta_3 & -\Theta_{sc}\delta_4 + \Theta_{sc}\delta_5 - \frac{F}{V} \end{bmatrix},$$

which is now affine in terms of δ_i's. Assume that $\delta = \begin{bmatrix} \delta_1 & \cdots & \delta_p \end{bmatrix}^T$, where $p = 5$ in our case. We generate typical data for the scheduling signals and construct the data matrix

$$E = \begin{bmatrix} \delta(0), \ldots, \delta((N-1)T_s) \end{bmatrix},$$

where T_s is the sampling time and N is the number of samples. This data matrix is then normalised to achieve scaled zero-mean values denoted by E^n, i.e. $E^n = \mathcal{N}(E)$ where $\mathcal{N}$ is a row-wise scaling. Next, a singular value decomposition (SVD) is performed on E^n as follows:

$$E^n = \begin{bmatrix} U_m & U_k \end{bmatrix} \begin{bmatrix} \Sigma_m & 0 & 0 \\ 0 & \Sigma_k & 0 \end{bmatrix} \begin{bmatrix} V_m^T \\ V_k^T \end{bmatrix}.$$

The basic idea behind the PCA is that if the data are correlated, some of the singular values are smaller compared to others (Jolliffe 2002). Therefore, the k smaller singular values can be neglected and the reduced parameter vector becomes

$$\phi(t) = q(\rho(t)) = U_m^T \mathcal{N}(\delta(t)) \in \mathbb{R}^m, \tag{4}$$

where $m < p$ and $\phi(t)$ can be reconstructed from the data only corresponding to the m largest singular values as

$$\hat{E}^n = U_m \Sigma_m V_m^T \approx E^n,$$

where $\hat{E}^n$ is an approximation of the original data. For quantifying this approximation accuracy the fraction of total variation is defined as

$$\upsilon = \frac{\sum_{i=1}^{m} \sigma_i^2}{\sum_{i=1}^{p} \sigma_i^2},$$

where υ close to 1 shows a better accuracy. Note that there is a trade-off between the accuracy and model complexity. Keeping a higher number of singular values leads to a better accuracy but not necessarily an appropriate parameter reduction. The bounds on $\phi(t)$ are determined as follows:

$$\underline{\phi}_i = \min_j q_i(\rho(jT_s)), \quad \bar{\phi}_i = \max_j q_i(\rho(jT_s)). \tag{5}$$

The approximation of the parameter vector $\delta(t)$ in terms of the reduced parameter vector at any given

time is then computed by

$$\hat{\delta}(t) = \mathcal{N}^{-1}(U_m\phi(t)). \tag{6}$$

Finally, the approximation of the parameter-dependent matrices of the LPV model in the control design procedure are reconstructed using the above $\hat{\delta}(t)$. As mentioned earlier the only parameter-dependent matrix of the SCR qLPV model is the matrix A which is affine in terms of δ. Therefore, it suffices to solve the corresponding LMIs for the LPV control design only at the vertices in (5).

3. LPV control design approach for SCR system

In this section, we present a SCR control design method based on the qLPV model described in Section 2. For control design purposes, we use the H_∞-norm as the performance measure. The rationale to use the H_∞-norm as the performance measure is its ability in attenuating the effect of disturbance inputs with bounded energy. The H_∞-norm of a single-input single-output LTI system $G(s)$ is defined as the peak value of its frequency response amplitude $|G(j\omega)|$. For an LPV system represented by T_{zd}, the operator that maps the energy-bounded input $d(t)$ to the output $z(t)$, the H_∞-norm is defined as (Apkarian and Adams 1998)

$$\|T_{zd}\|_\infty = \sup_{\rho\in\Omega} \sup_{d\in L_2-\{0\}} \frac{\|z\|_2}{\|d\|_2},$$

where $\|\cdot\|_2$ is the vector's two norm and Ω is the set of LPV parameters allowable variation set.

The control design objective is to minimise the H_∞-norm from the disturbance input (NO concentration upstream the SCR catalyst) to an appropriately defined control output, which is a weighted combination of NO concentration downstream the catalyst and ammonia slip as

$$z = a_1x_1 + a_3x_3, \tag{7}$$

where a_1 and a_3 are the positive scalar weights. The proposed control design method combines a parameter-dependent feedforward control law with a parameter-dependent feedback action (Figure 2). Two parameter-dependent feedback control methods are examined where the parameter-varying feedback control gains are determined as: (i) a dynamic output feedback control law which will be further simplified to result in a reduced-order controller, or (ii) a static state-feedback control law augmented with a state estimator.

In a typical SCR system, the only measurement available is the NO_x concentration downstream the catalyst. In this article, we also assume that

$$y = C_{\mathrm{NO}}. \tag{8}$$

There have been recent efforts on the development and use of ammonia sensors for SCR closed-loop control (Herman, Wu, Cabush, and Shost 2009), which are not wide-spread due to their cost and will not be pursued in this article. Our control design objective is to increase the NO conversion efficiency, decrease the amount of injected ammonia and keep the ammonia slip at a desired level. For design purposes, we consider ammonia concentration as the control input; however, it is noted that in an actual SCR system, the concentration of urea is the control variable.

3.1 *LPV output feedback control design approach*

We first consider the system dynamics (2), (7) and (8) is represented in state-space form by the following equations:

$$\begin{aligned} \dot{x} &= Ax + B_1d + B_2u, \\ z &= C_1x, \\ y &= C_2x, \end{aligned} \tag{9}$$

where A is dependent on θ and the system matrices A, B_1, B_2, C_1 and C_2 are appropriately defined. The objective is to design a gain-scheduling output

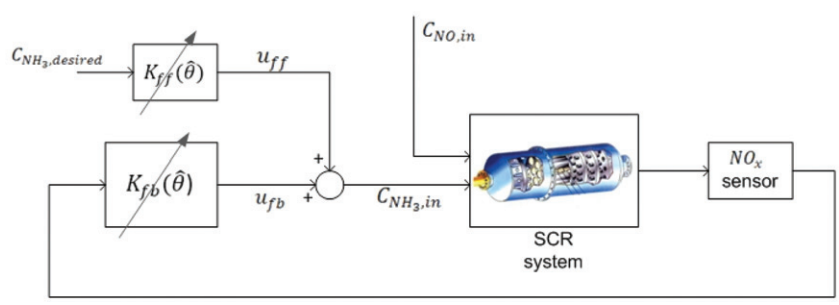

Figure 2. Schematic of the LPV control strategy for ammonia-SCR system.

feedback controller represented with the following state-space formulation

$$\begin{aligned} \dot{x}_k &= A_k(\theta)x_k + B_k(\theta)\,y, \\ u &= C_k(\theta)x_k + D_k(\theta)\,y, \end{aligned} \tag{10}$$

where x_k represents the state vector corresponding to the feedback controller. As discussed earlier, the surface coverage fraction θ is assumed to be the scheduling parameter. However, the value of θ is not measurable in practice and must be estimated in real-time. Here, we estimate θ using the method described below to prevent the use of a state observer. We demonstrate that the proposed approximation is good enough for our control design purposes. Considering the reaction rates and activation energies corresponding to the adsorption and reduction in Table 1, it can be verified that the NO concentration dynamics is slower than the ammonia concentration dynamics at lower surface coverage fraction. For the latter statement to be true, the following condition should hold true:

$$\Theta_{\mathrm{SC}}k_{red}\theta + \frac{F}{V} \ll \Theta_{\mathrm{SC}}k_{ads}(1-\theta) + \frac{F}{V}.$$

This is determined from (1) and results in the condition

$$\theta \ll \frac{k_{ads}}{k_{ads}+k_{red}}. \tag{11}$$

Based on the values given in Table 1, the right-hand side of the inequality (11) varies in $[0.840, 0.997]$ for temperature range of $[200^\circ\mathrm{C}, 500^\circ\mathrm{C}]$ and hence the inequality (11) holds true due to the low surface coverage fraction. Finally, since the condition (11) is met, the following holds:

$$-C_{\mathrm{NO}}\left(\Theta_{\mathrm{SC}}k_{red}\theta + \frac{F}{V}\right) + k_{ox}\Theta_{\mathrm{SC}}\theta + \frac{F}{V}d = 0$$

and hence, θ can be estimated by

$$\hat{\theta} = \frac{\frac{F}{V}(C_{\mathrm{NO}} - d)}{-C_{\mathrm{NO}}\Theta_{\mathrm{SC}}k_{red} + k_{ox}\Theta_{\mathrm{SC}}}, \tag{12}$$

which implies that the knowledge of inlet and outlet NO concentration is sufficient to estimate θ at each time instant. It is important to note that the disturbance term d (NO concentration upstream the catalyst) is assumed to be measurable to allow the calculation of the approximation in (12). There are alternative but more complicated methods to estimate the catalyst ammonia coverage ratio such as with the use of a Kalman filter as in Hsieh and Wang (2010) or a state estimator as described in Section 3.2.

We use the basic characterisation of the gain-scheduling control design with guaranteed stability and H_∞-performance as described in Apkarian and Adams (1998) for the qLPV model represented by (2). Since the dependence on the scheduling parameter θ is affine the synthesis LMIs need to be solved only at the two corners (i.e. maximum and minimum allowable values for θ). The range of variation for θ is between 0 and 1. For simplicity we consider constant basis functions for the Lyapunov functions $R(\cdot)$ and $S(\cdot)$ and the auxiliary controller matrices $\hat{A}_k$, $\hat{B}_k$, $\hat{C}_k$ and D_k in the corresponding synthesis LMIs (Apkarian and Adams 1998). Therefore, we obtain a finite number of decision variables to optimise. Taking into account the above structure for the decision variables, the only parameter-dependent controller matrix is A_k characterised by

$$A_k(\hat{\theta}) = A_{k_0} + \hat{\theta}A_{k_1}. \tag{13}$$

The resulting controller is a full-order one; however, it was observed that the dominant pole of this system is constant for a fixed temperature and over the full range of variation of the LPV parameter $\hat{\theta}$. Therefore, we calculated a reduced order controller that has the same DC gain as the original full-order controller. We determined that the reduced order controller is a first-order transfer function represented by

$$C(s) = \frac{\alpha(\hat{\theta})}{s+p}, \tag{14}$$

where $-p$ is the dominant pole of the obtained controller. In order to have equal DC gains for both full-order and reduced order controllers, the following should hold

$$\frac{\alpha(\hat{\theta})}{p} = -C_k A_k^{-1}(\hat{\theta})B_k, \tag{15}$$

which indicates that the knowledge of p provides the controller gain parameter α scheduled based on $\hat{\theta}$. Interestingly, it was observed that in the temperature range of SCR operation (200°–500° C) a straight line well-represents the dependence of the DC gain α on the scheduling parameter $\hat{\theta}$. This will be demonstrated in the following section.

The use of combined feedforward and feedback control can significantly improve the closed-loop performance over simple feedback control whenever there is a major disturbance that can be measured before it affects the system output. In the ideal situation, feedforward control can completely eliminate the effect of the measured disturbance on the system output. Even when there are modelling errors, feedforward control can often reduce the effect of the measured disturbance on the output better than that

achieved by feedback control by itself (Skogestad and Postlethwaite 2005). Our objective of adding a feedforward control as illustrated in Figure 2 is to deal with a known disturbance, i.e. NO concentration upstream the catalyst, rather than modelling errors. The feedforward component of the control input is used here to ensure that the ammonia slip reaches a desired level at steady state. This is done using the known disturbance input in calculating the estimate of the coverage surface fraction $\hat{\theta}$ as in (12). The corresponding feedforward control to achieve the desired objective is calculated using the following law:

$$u_{ff} = \frac{V}{F}\left[C_{\mathrm{NH_3,desired}}\left(\Theta_{\mathrm{SC}}k_{ads}(1-\hat{\theta}) + \frac{F}{V}\right) - \Theta_{\mathrm{SC}}k_{des}\hat{\theta}\right], \tag{16}$$

where $C_{\mathrm{NH_3,desired}}$ is the desired ammonia slip and $\hat{\theta}$ is the real-time estimate of the LPV parameter using (12).

3.1.1 *Temperature effect on reduced order control design*

As discussed earlier, the dominant pole of the full-order LPV output feedback controller (at a fixed operating point) was found out to be a function of temperature over the range of variation of the parameter $\hat{\theta}$. Therefore, the reduced order controller will now be considered in the following form:

$$C(s) = \frac{\alpha(T,\hat{\theta})}{s + p(T)} \tag{17}$$

and the DC gain of the controller is calculated as

$$\frac{\alpha(T,\hat{\theta})}{p(T)} = -C_k A_k^{-1}(\hat{\theta})B_k. \tag{18}$$

As mentioned earlier the term $-C_k A_k^{-1}(\hat{\theta})B_k$ has an approximate affine dependency on the scheduling parameter $\hat{\theta}$. Therefore, we have

$$-C_k A_k^{-1}(\hat{\theta})B_k = g_1(T)\hat{\theta} + g_2(T) \tag{19}$$

in which the linear characteristic changes with respect to the operating temperature. The parameters $p(T)$, $g_1(T)$ and $g_2(T)$ are mappings that are determined offline. Using the data generated from the simulation model, we found out that these parameters can be estimated by polynomials of second and third order. The algorithm uses the exhaust gas temperature to calculate the controller dominant pole. Further simulation studies revealed that this dependence is logarithmic, i.e.

$$\log p(T) = \gamma_0 + \gamma_1 T + \gamma_2 T^2 \tag{20}$$

as illustrated in Figure 3 as a function of temperature. The parameters $g_1(T)$ and $g_2(T)$ and their estimates are also shown in Figure 4.

The SCR feedback control method proposed in this article relies on the consistent and precise measurement of NO concentration which sometimes is not a valid assumption as NO_x sensors have significant cross-sensitivity to ammonia (Schar et al. 2006; Willems et al. 2007). The remedy for this issue can be: (i) utilising a factory provided cross-sensitivity correction to compensate for the error in the NO_x sensor reading, or (ii) the use of an estimation approach to approximate the actual NO_x concentration and ammonia cross-sensitivity factor if a post-catalyst ammonia sensor is available (Hsieh and Wang 2010). This issue of cross-sensitivity is beyond the scope of this article and will not be pursued here.

3.2 *LPV state-feedback control design approach*

For comparison purposes, in this section we propose to design an LPV state-feedback controller combined with an LPV observer to minimise the H_∞-norm of the closed-loop system as the performance measure. A benefit of this approach is the reduced design complexity compared to the output feedback method of Section 3.1. Considering the LPV system model described by (9), the following state-space representation for the LPV observer is considered:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B_1 d + B_2 u + L(y - \hat{y}), \\ \hat{y} &= C_2\hat{x}, \\ \hat{z} &= C_1\hat{x}, \end{aligned} \tag{21}$$

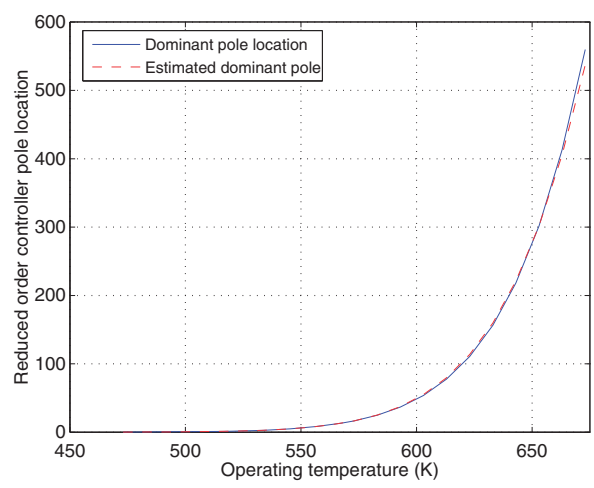

Figure 3. The absolute value of the temperature-varying controller pole $p(T)$: solid line is the actual pole and dotted line is the approximation using (20).

where L is the observer gain designed such that the error system is stable and satisfies the specified performance criterion. Substituting $u = K(\hat{\theta})\hat{x}$ and $e = x - \hat{x}$ in the above equations result in

$$\begin{aligned} \dot{\hat{x}} &= (A + B_2K)\hat{x} + B_1 d + LC_2 e, \\ \dot{e} &= (A - LC_2)e. \end{aligned} \tag{22}$$

The augmented system of the plant and the observer then becomes

$$\begin{aligned} \dot{\zeta} &= \mathcal{A}\zeta + \mathcal{B}d, \\ \hat{z} &= \mathcal{C}\zeta, \end{aligned} \tag{23}$$

where

$$\mathcal{A} = \begin{bmatrix} A + B_2K & LC_2 \\ 0 & A - LC_2 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},$$
$$\mathcal{C} = \begin{bmatrix} C_1 & 0 \end{bmatrix}, \quad \zeta = \begin{bmatrix} \hat{x} \\ e \end{bmatrix}.$$

It is noted that since the disturbance term d is measurable as discussed earlier, it has been included in the observer state-space representation (21). The following lemma presents a synthesis condition to design the observer and a parameter-dependent state-feedback controller to guarantee the closed-loop system stability and H_∞-performance.

Lemma 1: *For the LPV system represented by* (9) *with* $|\dot{\theta}| \le \nu$ *and a positive scalar* β, *there exist an LPV observer* (21) *and a state-feedback controller such that the closed-loop system is stable and a prescribed level of* H_∞*-performance* γ *is guaranteed if parameter-dependent matrices* $R(\theta) > 0$, $P_1(\theta) > 0$, $P_2(\theta) > 0$, $S(\theta)$ *and* $Q(\theta)$ *exist that satisfy the following set of LMIs*:

$$A^T R + RA - C_1^T S^T - SC_1 + 2\beta R < 0, \tag{24}$$

$$\begin{bmatrix} \left\{ \begin{matrix} AP_1 + B_2Q \\ +(*) - \dot{P}_1 \end{matrix} \right\} & LC_2P_2 & B_1 & P_1C_1^T \\ (\star) & \left\{ \begin{matrix} AP_2 - LC_2P_2 \\ +(*) - \dot{P}_2 \end{matrix} \right\} & 0 & 0 \\ (\star) & (\star) & -\gamma I & 0 \\ (\star) & (\star) & (\star) & -\gamma I \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0. \tag{26}$$

Then, the LPV observer gain and the feedback controller gain are determined by

$$L(\theta) = R^{-1}(\theta)S(\theta), \tag{27a}$$

$$K(\theta) = Q(\theta)P_1^{-1}(\theta). \tag{27b}$$

In the above formulation $(*)$ *denotes the transpose of the terms it proceeds and* $(\star)$ *is used to denote the sub-matrices lying under the main diagonal.*

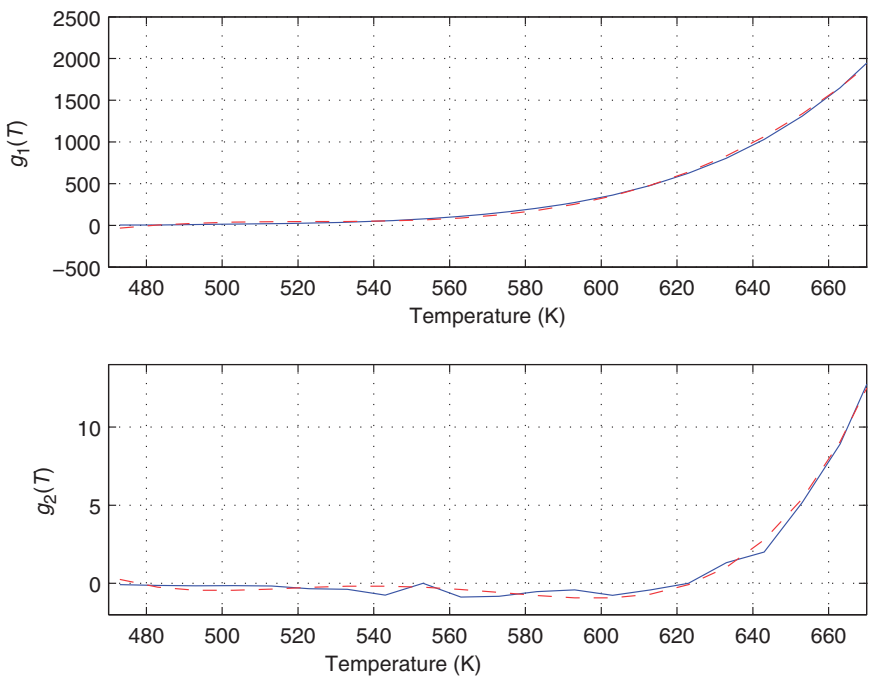

Figure 4. The temperature-dependent functions g_1 and g_2 in (19): solid line is the actual value and dashed line is the one approximated using a polynomial function.

Proof of the result is based on the bounded real lemma formulation corresponding to the H_∞-norm of the closed-loop system (23). To avoid the bilinear terms in the matrix inequality associated with the closed-loop system, a block-diagonal structure is enforced on the Lyapunov matrix as in (26). The algebra led to the LMIs (24) and (25) is omitted for brevity. It is noted that due to the cross-product of the observer gain L and the Lyapunov matrix in (25), the LMI (24) is solved first independently to determine the observer gain.

Remark 1: The derivative terms $\dot{P}_1$ and $\dot{P}_2$ in (25) can be substituted by $\dot{P}_j = \pm(\nu \frac{\partial P_j}{\partial \theta})$ (for $j = 1, 2$). The notation $\pm(\cdot)$ is used to indicate that both $+$ and $-$ should be included in the LMI condition since due to the dependency of the LMI on $\dot{\theta}$ affinely, it needs to be solved only at the vertices of $\dot{\theta}$.

Remark 2: It is noted that the term $2\beta R$ in (24) is added for placing the poles of the error system dynamics at the left hand side of the vertical line $s = -\beta$.

4. Simulation results

To validate our proposed control design, in this section, we present closed-loop simulation using the proposed LPV feedback/feedforward controllers. Simulation and validation studies are pursued using (i) the low-order lumped parameter model of the SCR system (Sections 4.1 and 4.2), and (ii) a high-fidelity numerical model of the reactions developed in GT-POWER (Section 4.3).

4.1 *Simulation results for the fixed temperature case*

In order to satisfy the dual goals of maximising NO conversion efficiency and keeping the ammonia slip at a desired level, we chose $z = a_1 x_1 + a_3 x_3$ as the controlled weighted output in the state-space representation (9) where α_1 and α_3 are the weighting scalers on the NO and NH_3 concentrations, respectively. The profile of NO concentration upstream the catalyst is shown in Figure 5 with the temperature fixed at 573 K. For simulation purposes, measurement noise was added to the NO measurement downstream the catalyst. The feedforward part of the controller is designed to keep the ammonia slip around 10 ppm.

First, we demonstrate the results of using the designed reduced order LPV output feedback controller. After solving the synthesis LMIs, the dominant pole is determined to be at -16.5. Figure 6 shows the dependence of the controller's DC gain versus $\hat{\theta}$ as in (15). It is interesting to note that the gain appears to be a linear function of $\hat{\theta}$, and hence $\alpha(\hat{\theta})$ in (14) can be adapted in real time to changes of $\hat{\theta}$. The determined reduced order controller in (14) is applied to the SCR system using the design configuration of Figure 2. Shown in Figure 7 is the closed-loop simulation results comparing the state-feedback, output-feedback and sliding mode control design methods. The three plots shown in Figure 7 correspond to NO concentration downstream the catalyst, the released ammonia and the injected ammonia, and illustrate the differences in transient responses corresponding to the three controllers. As observed from this figure, using LPV output feedback, the ammonia slip of around 10 ppm is quickly achieved due to the use of the feedforward term resulting in the improved steady state performance. The reduced order controller (14) is scheduled based on the parameter $\hat{\theta}$ as in (12). In Figure 8, the approximation of θ using (12) is shown illustrating

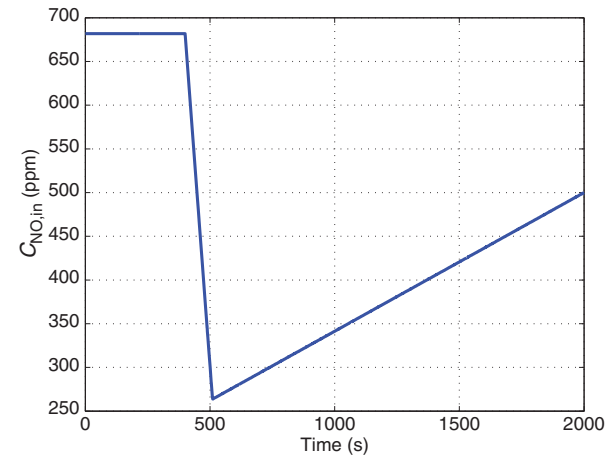

Figure 5. NO concentration in SCR inlet.

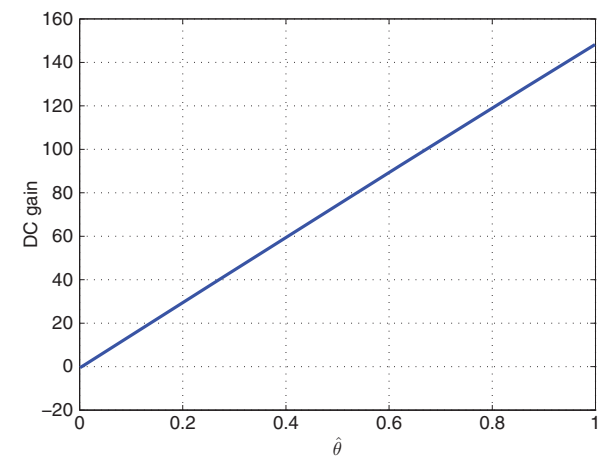

Figure 6. Controller DC gain as a function of $\hat{\theta}$.

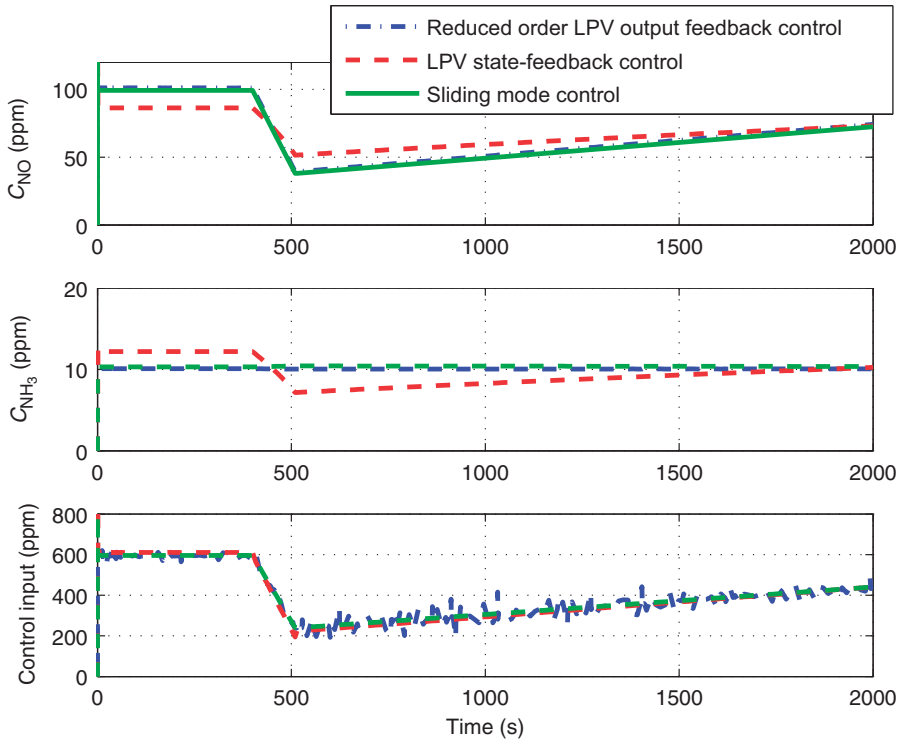

Figure 7. Comparison between different controllers for the fixed temperature case.

its accuracy for the estimation of the coverage ratio to be used for control gain adaptation. Figure 7 also includes the result of the state-feedback controller integrated with the observer designed using Lemma 1.

Finally, we compare the proposed LPV H_∞-controllers of this article with a sliding mode-based controller described in Khalil (2002). To this end, a nonlinear observer is designed first to estimate the surface coverage fraction and the ammonia slip concentration as

$$\dot{\hat{x}} = h(\hat{x}, u, d) + L(\hat{C}_{NO} - C_{NO}),$$

where $\hat{x} = \left[\, \hat{C}_{NO} \quad \hat{\theta} \quad \hat{C}_{NH_3} \,\right]^T$ is the observer state, h is the nonlinear model in (1), L is the observer gain vector and C_{NO} is the measured NO concentration downstream the catalyst. In Figure 7, the result of closing the feedback loop using sliding mode control following the approach in Upahhyay and van Nieuwstadt (2006) and Devarakonda et al. (2008) is shown.

Next, we apply the proposed reduced order output feedback control strategy of this article for different exhaust temperatures. In Figure 9, we evaluate the closed-loop system performance in terms of the NO conversion efficiency for temperatures fixed at 573, 673, and 773 K. As expected, the SCR catalyst performs better at higher temperatures. This trend, however, changes at very high temperatures where ammonia oxidation limits the NO_x conversion.

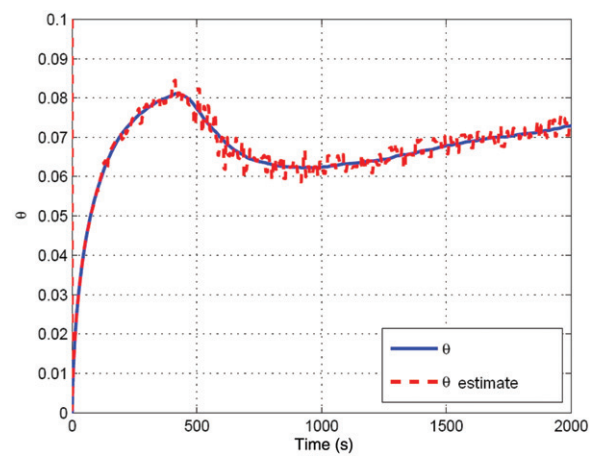

Figure 8. Surface coverage fraction estimate.

Finally, we compare the performance (in terms of NO conversion efficiency) of the closed-loop systems using the sliding mode controller, the LPV state-feedback controller, and the reduced order LPV feedback/feedforward controller. In the design of the feedforward controller, the ammonia slip is kept around 10 ppm and the NO conversion efficiency for the three control design methods are compared in Table 2. The following metric is used to evaluate the

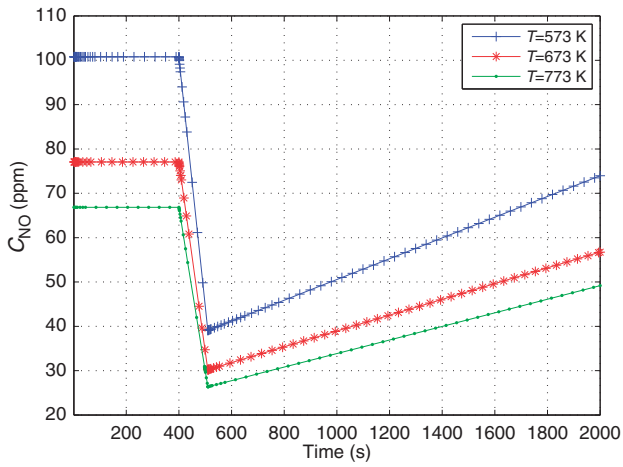

Figure 9. NO concentration downstream the catalyst corresponding to different exhaust temperatures.

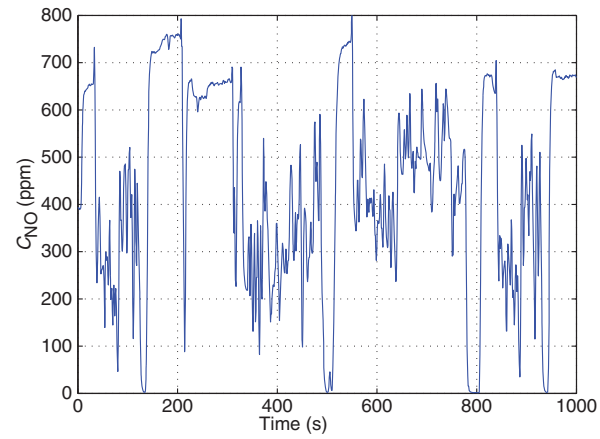

Figure 10. The NO concentration from a UDDS transient cycle.

Table 2. Comparisons. between different control methods.

Control strategy	$DeNO_x$ (%)	Ammonia slip RMS	Injected ammonia RMS	Computation time (s)
LPV state feedback	85.67	10.23	550	0.2
Sliding mode control	85.66	10.37	406	0.2
LPV output feedback	85.16	10.03	358	1.5

conversion efficiency:

$$\mathrm{DeNO}_x = \frac{\sum_i C_{\mathrm{NO,in}}(i) - \sum_i C_{\mathrm{NO,out}}(i)}{\sum_i C_{\mathrm{NO,in}}(i)}. \tag{28}$$

The NO concentration profile (upstream the catalyst) used to evaluate the NO conversion efficiency is shown in Figure 10. This emission profile corresponds to an EPA Urban Dynamometer Driving Schedule (UDDS) which has been developed for chassis dynamometer testing of heavy-duty vehicles and . The basic parameters of the cycle are: duration of 1060 s, distance of 5.55 miles, average speed of 18.86 mile/h and maximum speed of 58 mile/h. To perform a comparative study between the transient behaviour of the three control design methods discussed in this article, we have shown in Figure 11 the profiles of NO concentration downstream the catalyst, the amount of ammonia slip and the injected ammonia for the different control designs. The controllers are designed to satisfy the dual objective of keeping ammonia slip around 10 ppm and maximising the conversion efficiency, which in this case was calculated to be close to 85% as shown in Table 2. As indicated in Table 2, the computation time corresponding to the LPV output feedback controller is considerably higher compared to other two. This is justified due to the real-time adaptation of both feedback and feedforward control laws with respect to the LPV parameters.

Next, we investigate the robustness of the proposed control design methods through a comparative study. The described feedback control methods in this article do not take the exhaust flow variation into account during the design process. We will study using simulations the effect of varying flow on different transient performance criteria including NO_x conversion, injected ammonia and ammonia slip. To this purpose, we design the controllers at a particular exhaust flow and validate it on the plant model with exhaust flow varying in a range from one-tenth of the nominal flow to as much as four times the nominal flow, for which the controllers were designed. The comparison is depicted in Figure 12. Figure 12(a) and (b) illustrates the comparison between the amount of ammonia slip and the ammonia injected, respectively. Figure 12(c) shows the NO downstream the catalyst for the three controllers and the cases with fixed flow and varying flows. It is noted that the sliding mode controller gains vary with respect to the changes in the surface coverage fraction θ, whose estimate is provided by the nonlinear observer. Hence, the sliding mode control gains are indirectly scheduled based on the exhaust flow, while the output feedback LPV controller gains are scheduled in real time based on the estimate of θ from (12) that involves the exhaust flow. This implies that even though the variation of the exhaust flow is not considered directly in the design process, it does alter the controller via the controller

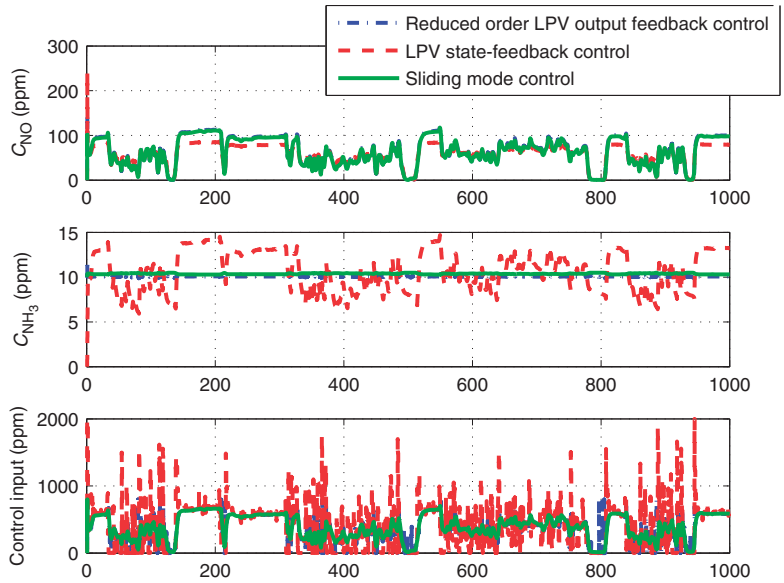

Figure 11. The NO concentration downstream the catalyst, ammonia slip and injected ammonia for the three control design methods using the NO profile of Figure 10.

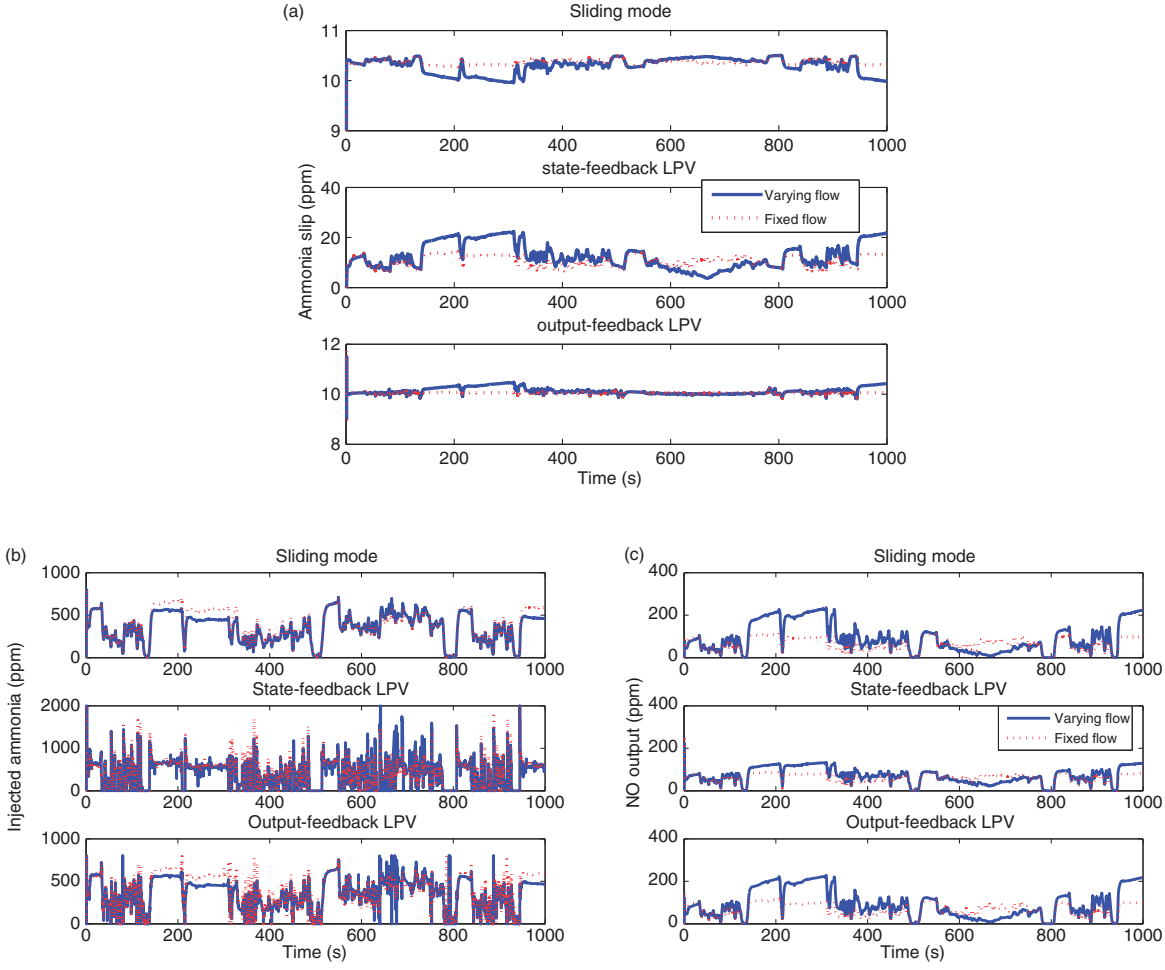

Figure 12. Comparison between the three controllers for the varying exhaust flow: (a) ammonia slip, (b) injected ammonia and (c) output NO.

gain adaptation in real time. This fact explains the comparative behaviour of the output feedback LPV and sliding mode controllers. As observed from the plots, the varying flow has little impact on the injected ammonia for the state-feedback case, and this leads to a change in the slip corresponding to this controller due to the variation in exhaust flow. On the other hand, both the output feedback LPV and sliding mode controllers keep the slip at the desired level and this explains the change in the injected ammonia for these two controllers to adapt to the varying flow. It is worth mentioning that compared to the other two controllers, the LPV output feedback shows less variation in the ammonia slip for the case of a varying exhaust flow.

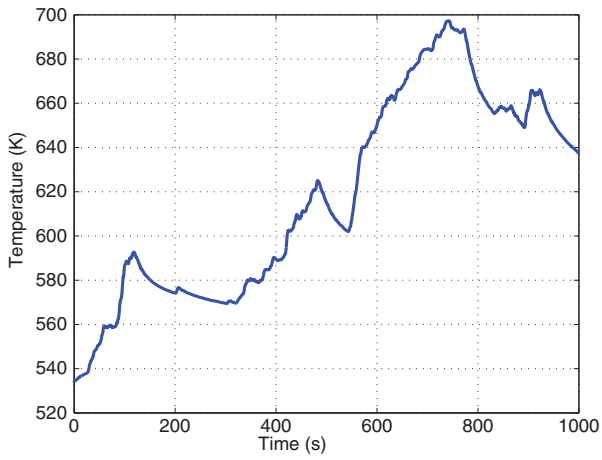

Figure 13. Temperature profile used for the simulations.

The changes in the NO downstream the catalyst can be similarly justified. Additional simulations were also performed by changing other model parameters including the catalyst storage capacity, and robustness of LPV output feedback controller compared to other two was observed. The results are not reported here due to the space limitation.

4.2 *Simulation results for the temperature-varying case*

To show the effectiveness of the proposed design method for the temperature-varying case, we compare the performance of the two reduced order LPV controllers discussed in Section 3.1 for the example considered earlier and with a temperature-varying profile shown in Figure 13. The first controller we design is scheduled only based on $\hat{\theta}$ and the second one is scheduled based on both $\hat{\theta}$ and the exhaust temperature T. The results are shown in Figures 14 and 15. The NO conversion efficiencies for the first and second cases are calculated to be 61.39% and 80.52%, respectively. Also, using the feedforward term similar to (16) but temperature-dependent we are able to keep the ammonia slippage around our desired value of 10 ppm by using a gain-scheduling control method. The control inputs corresponding to the two control methods are shown in the last subplots of Figures 14 and 15. As observed, for the temperature-varying case the LPV controller scheduled based on only $\hat{\theta}$ results in

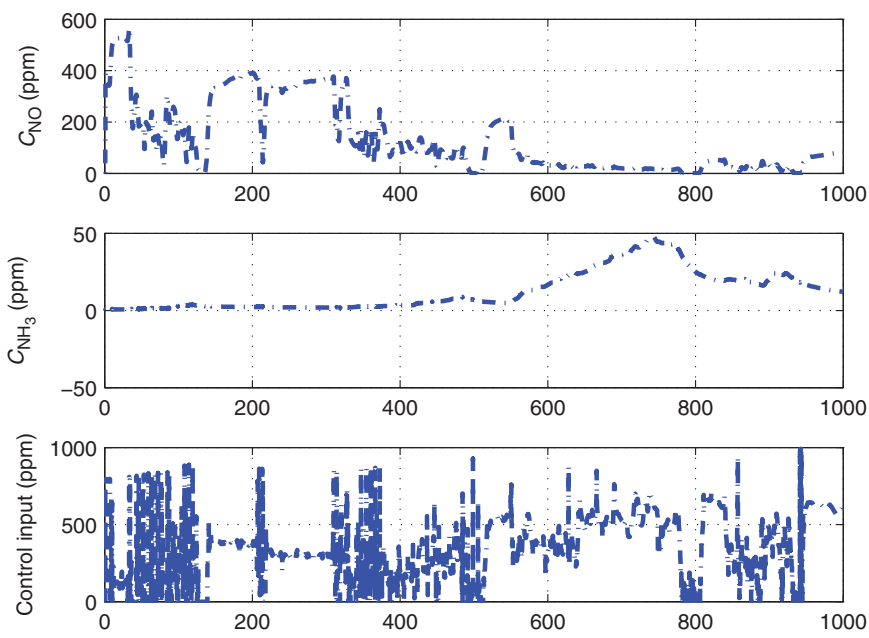

Figure 14. NO concentration downstream the catalyst, ammonia slip and injected ammonia for LPV control scheduled only on $\hat{\theta}$.

a control action (injected ammonia) with a large number of switchings between low ammonia and high ammonia, while the LPV controller scheduled on both $\hat{\theta}$ and T leads to a smoother control action due to the consideration of varying temperature in the reduced order controller dynamics.

For the SCR qLPV model with varying temperature, there are five scheduling parameters. Using the PCA technique described in Section 2.2, we are able to reduce the number of parameters to only one or two parameters. Figure 16 shows the number of required principal components versus the fraction of total variation. It is observed that the first principal component captures 63% and the first two principal components capture 93% of the model dynamics. Due to the relatively good accuracy and lower complexity, the model with one parameter is used for controller design purposes. Hence, the nonlinear system is controlled by an LPV H_∞-output feedback controller that is scheduled on the reduced parameter vector $\phi(t)$. Since the model is affine in terms of ϕ, by taking advantage of the multi-convexity property, the controller synthesis LMIs hold true for all $\phi_i \in [\underline{\phi}_i, \bar{\phi}_i]$ if they hold only at the vertices (Gahinet, Apkarian, and Chilali 1996). The parameter reduction makes the computations very tractable in the LPV design process since the number of LMIs to be solved depends exponentially on the number of parameters. In addition, the PCA method results in a model that is affine in the reduced parameter space.

Simulation results demonstrate that the designed full-order controller can be reduced to a first-order parameter-varying one without significantly sacrificing performance. The simulation results are shown in Figure 17. The performance of the reduced order controller gain-scheduled based on $\hat{\theta}$ and T is compared to that of the open-loop case in which constant amount of ammonia (320 ppm) is injected into the catalyst. By making use of a gain-scheduled feedforward control law, we are able to keep the NH_3 slip around 10 ppm as before. However, this is not possible using open-loop control. Therefore, the constant amount of injected ammonia to the open-loop SCR system is adjusted so that the mean value of NH_3 slip remains around 10 ppm. Figure 18 shows the contribution of feedback and feedforward in the overall control strategy, respectively. The NO conversion efficiency for the closed-loop and open-loop systems was calculated to be 82.5% and 73.4%, respectively.

4.3 *GT-POWER simulation results*

In this section, we validate the designed LPV output feedback/feedforward control scheme depicted in Figure 2 on a high-order computational model of the SCR reactions using the GT-SUITE simulation software tool. GT-SUITE (and in particular GT-POWER, which is part of GT-SUITE) is an engine/powertrain computational modelling platform used for integrated simulations of an engine and a vehicle (Gamma Technologies 2009). In this platform, the SCR aftertreatment system can be modelled using its corresponding reaction kinetics. This tool is used

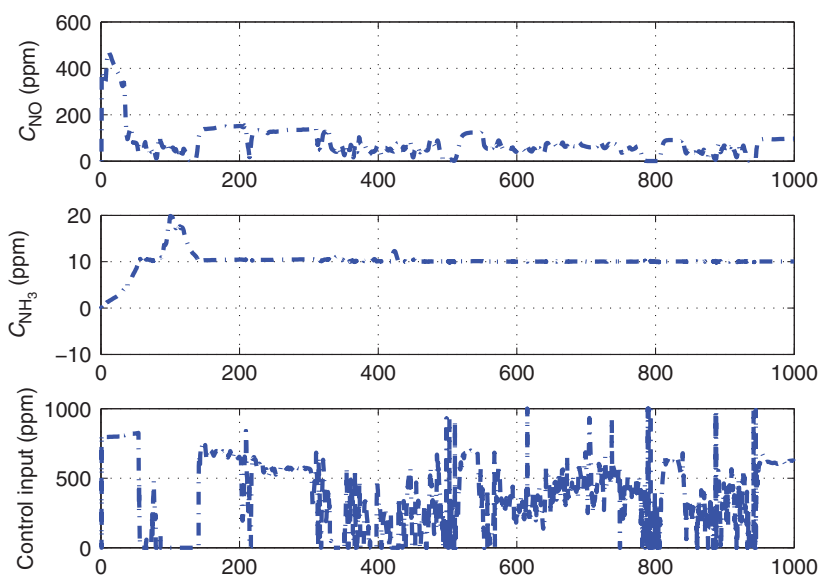

Figure 15. NO concentration downstream the catalyst, ammonia slip and injected ammonia for LPV control scheduled on both $\hat{\theta}$ and T.

in the simulations described next to obtain a better understanding of the SCR model described in Section 2, as well as to evaluate the performance of the designed LPV controller. Subsequently, the GT model is coupled with MATLAB/SIMULINK, in which the control algorithm is implemented. Figure 19 shows a schematic of the model built in GT-POWER and used for this validation. Basically, the simulation model consists of the following components:

- flow inlet to introduce exhaust gases and flow outlet,
- connecting pipes,
- NO and ammonia injectors which are controlled from Simulink,
- catalyst brick which defines the catalyst geometry and cells configuration,
- surface reaction mechanisms that occur in the SCR and the corresponding kinetics,
- NO sensors upstream and downstream the catalyst,
- link to Matlab/Simulink,
- monitors to display signals.

General chemistry system equations for exhaust after-treatment pipe gas flow are described by a set of convection–reaction equations, and more specifically, continuity equation for different species, energy equation, momentum equation and diffusion equations. The quasi-steady solver is used for the flow solver with the time step of 1 s. Chemistry solver is set to backward differentiation formulae (BDF), which is a robust differential algebraic equation solver. This chemistry solver is the recommended solver for most catalysts and is appropriate in handling stiff kinetics. For this model, the catalyst is coated with vanadium and the same parameters are used as in Table 1.

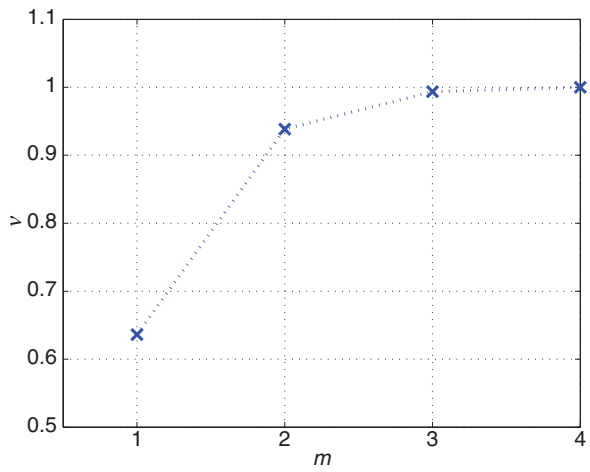

Figure 16. Accuracy of the low-order model vs. the number of singular values considered.

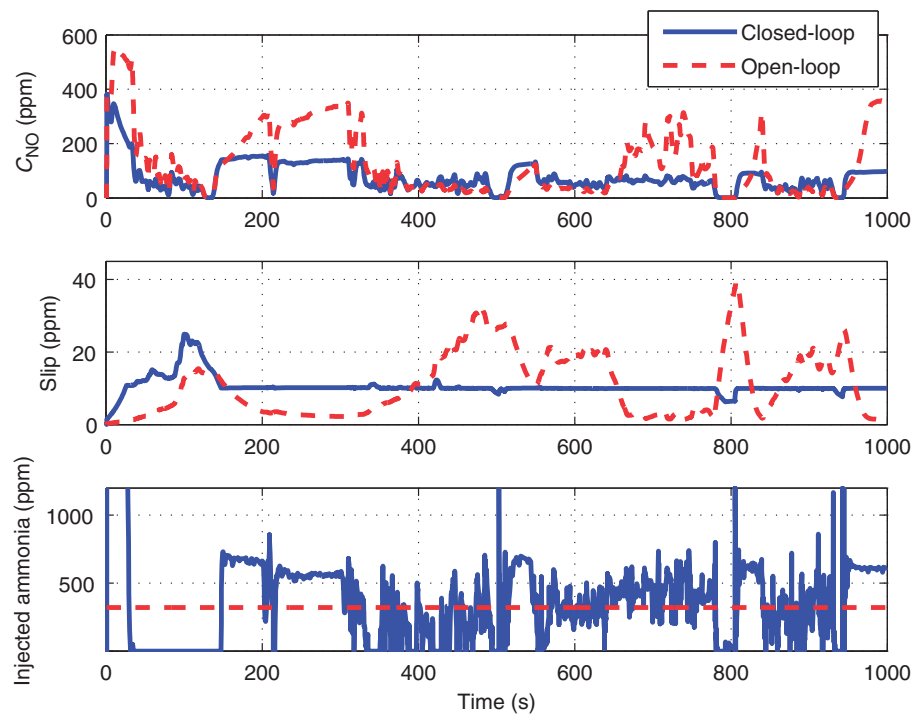

Figure 17. NO concentration downstream the catalyst, ammonia slip and injected ammonia for open-loop and closed-loop controllers designed based on PCA.

The results of three simulations are reported here. In the first set of simulations, we consider a constant NO concentration and a fixed temperature of 500 ppm and 573 K, respectively. Using the reduced order LPV output feedback control described in the previous section, Figure 20 shows the transient NO conversion rate, ammonia slip, and injected ammonia. It is noted that the feedforward control is designed to achieve the ammonia slip of 10 ppm.

The second set of simulations is performed for the fixed temperature of 573 K and a transient NO profile as shown in Figure 10. The feedforward control is designed so that the ammonia slip is around 25 ppm at the steady sate. Figure 21 shows the transient NO conversion, ammonia slip and the injected ammonia. In the last set of simulations, we consider a transient NO profile along with the varying exhaust temperature as shown before in Figure 13. To simplify the control design, we use the PCA method described in Section 2.2 for LPV parameter reduction purposes. Figure 22 depicts the NO conversion, ammonia slip, and injected ammonia for a desired slip of 25 ppm. It is noted that unlike the first case that led to the desired ammonia slip, the second and third cases lead to slips that are fluctuating around the desired value of 25 ppm. A justification is that the lumped parameter model used for control design purposes can capture the low-frequency dynamics of the SCR system and hence it is more appropriate for slowly varying perturbations about a steady-state operation. Both feedback control gains and feedforward controls are scheduled based on the estimate of θ given in (12) that is dependent on both NO downstream the catalyst and exhaust temperature. Therefore, the discrepancy between the high-order GT-POWER model and lumped parameter model (1) would lead to an error in the ammonia slip. It is noted that the use of an estimator (e.g. a Kalman filter) that takes into account the dynamics of the surface coverage fraction can help provide a better estimate of this parameter leading to improvement in the performance in terms of the desired ammonia slip.

5. Concluding remarks

In this article, we propose an LPV control design method for ammonia SCR aftertreatment system in automotive applications. The objective is to minimise NO_x emission while keeping ammonia slippage low. Both output feedback and state-feedback LPV controllers were examined and compared. The latter method required an observer for state estimation, but the former method only used the concentration of NO upstream and downstream the catalyst. Eliminating the fast modes in the full-order LPV output feedback controller resulted in a simple first-order transfer function gain scheduled as a function of the LPV parameter (the surface coverage fraction). Two other control design methods (namely sliding mode and LPV state-feedback controllers) were also examined,

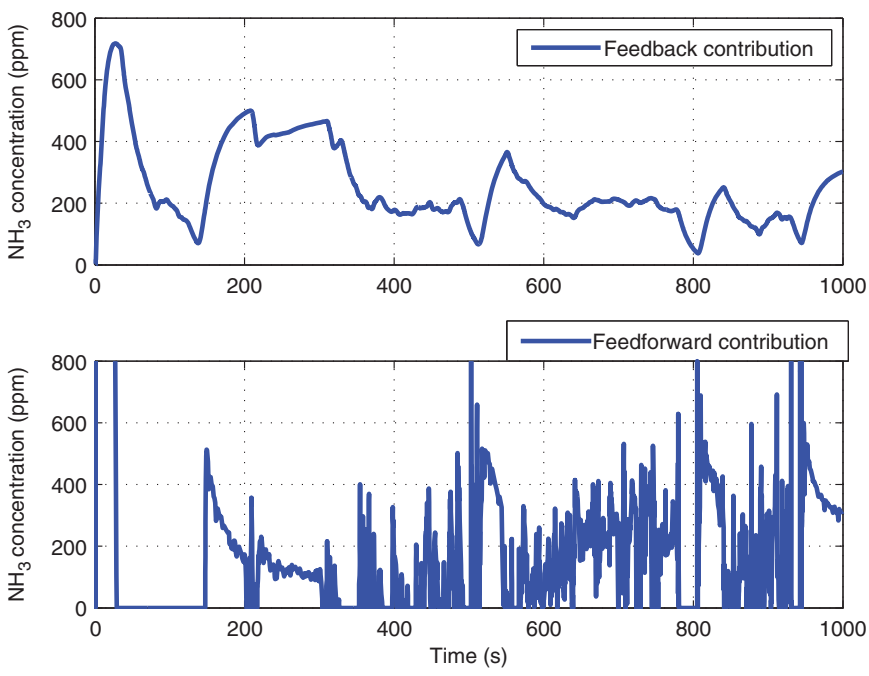

Figure 18. Feedback and feedforward control contribution.

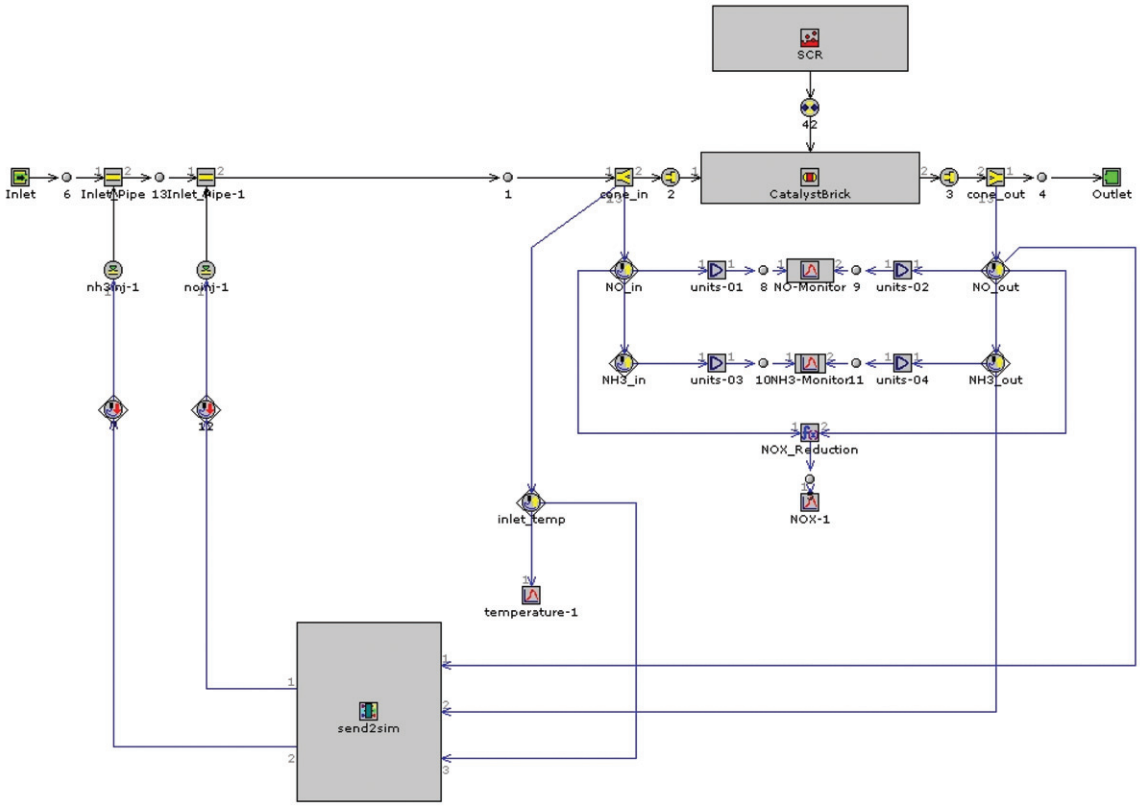

Figure 19. Model of the GT-POWER simulation.

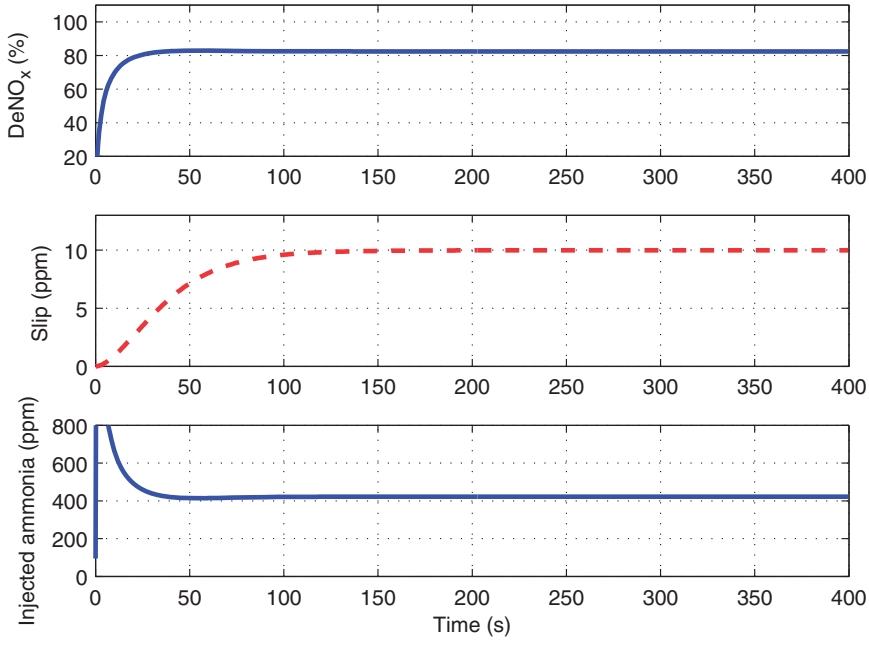

Figure 20. GT-POWER simulations with constant temperature and NO concentration.

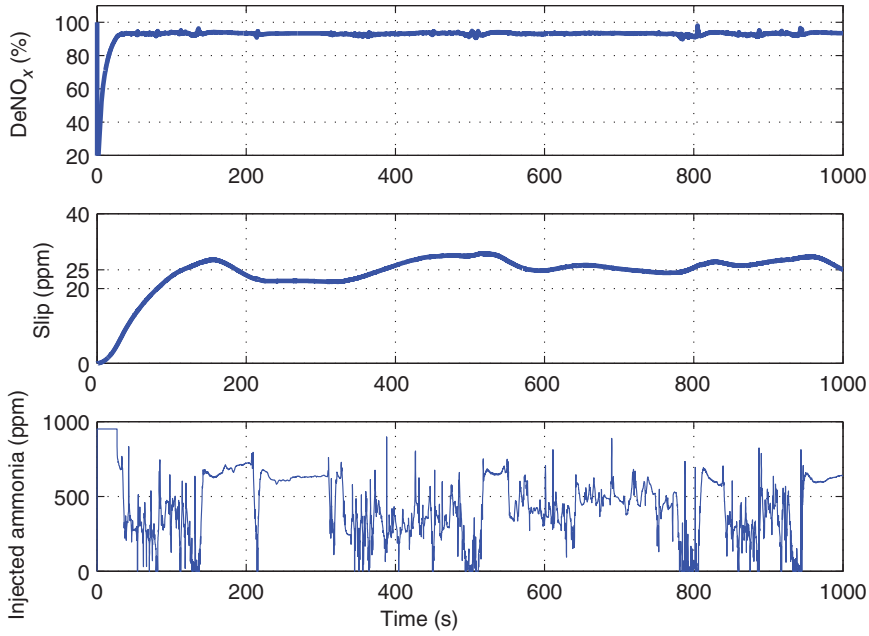

Figure 21. GT-POWER simulations with fixed temperature and transient NO.

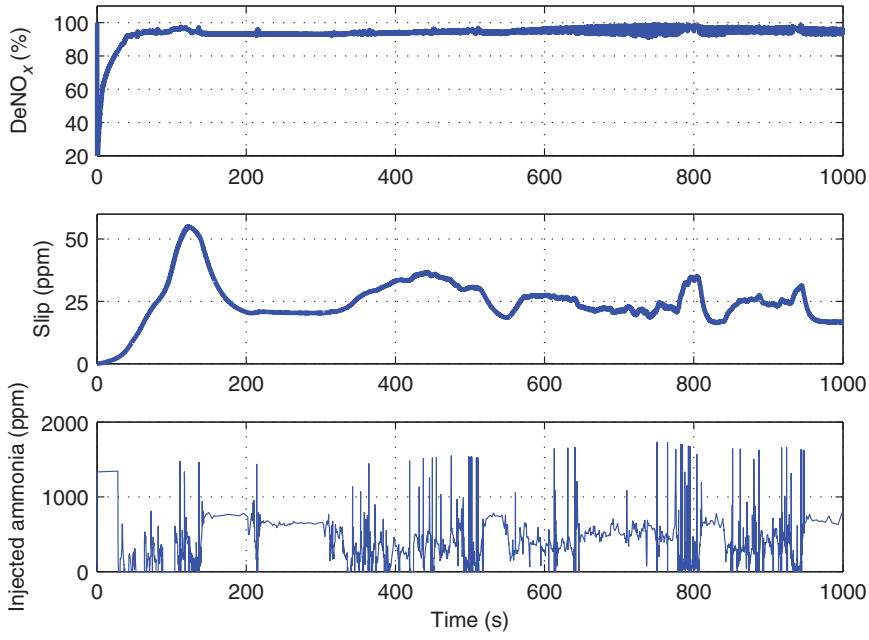

Figure 22. GT-POWER simulations with varying temperature and transient NO.

simulated and compared. It was observed that the proposed reduced order controller LPV feedback/feedforward scheme had the ability to better handle the dual objectives of regulating the ammonia slip and improving the NO conversion efficiency. We also investigated the impact of exhaust temperature on the structure of the reduced order controller as an additional LPV parameter. To address the complexity of the qLPV model due to the large number of gain-scheduling parameters, a PCA method was used as a parameter reduction tool based on the SVD of the collected data matrix. The use of PCA not only led to a lower number of LPV parameters but also provided an affine model (in terms of the reduced LPV parameters) that significantly simplified the control design process. Finally, a high-fidelity model of the SCR in the GT-POWER simulation was used to investigate the performance of the designed controllers.

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