THE ROLE OF TRAFFIC FLOW PREVIEW FOR PLANNING FUEL OPTIMAL VEHICLE VELOCITY

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ABSTRACT

A vehicle's untimely arrival at a local traffic wave with lots of stops and goes increases its fuel use. This paper proposes predictive planning of the vehicle velocity for reducing the velocity transients in upcoming traffic waves. Macroscopic evolution of traffic pattern along the vehicle route is first estimated by combining a traffic flow model. The fuel optimal velocity trajectory is calculated by solving an optimal control problem with the spatiotemporally varying constraint imposed by the traffic. Preliminary simulation results indicate the potential for improvement in fuel economy with a little compromise on travel time.

1 Introduction

Frequent stops and goes in traffic waves increase the fuel usage and emissions of passenger and commercial vehicles. Many of such stop and go conditions occur due to *lack of information* about the upcoming traffic pattern down the road. Many drivers choose to aggressively speed up to near the speed limit, only to be forced to abruptly decelerate their vehicles when faced with the slower traffic ahead of them and then perhaps idle or crawl in slow-moving traffic. If the upcoming traffic pattern is somehow "revealed" to the drivers in advance, the opportunity exists to adjust the velocity more predictively to reduce harsh deceleration and idling or crawling intervals. Such planning of velocity could lower fuel use and emissions, improve the ride, and reduce brake and engine wear.

In the connected vehicles of today, the vehicle navigation system or handheld wireless devices put real-time traffic information at our fingertips. Via these devices it is possible to now retrieve *coarse* traffic flow information en route to our destination via several online traffic information services. For example Google Maps provides real-time traffic information around 30 major U.S. cities now; the traffic layer of Google Earth also shows the average velocity of vehicles at numerous nodes on a road. In the near future it may be technically plausible to get *higher resolution* information on the immediate state of traffic i.e. the velocity of individual nearby vehicles. In the Mobile Millennium project [1], a collaboration between Nokia, UC Berkeley's California Center for Innovative Transportation (CCIT), California Department of Transportation (CALTRANS) and NAVTEQ in-vehicle cellular phones are used as traffic sensors and form traffic velocity fields that can then be transmitted back to participating vehicles. One can envision access to a complete map of surrounding traffic, once such technologies are more widely deployed.

This level of information can potentially transform the way we drive our vehicles. In fact many navigation systems today suggest alternate shortest-time routes based on traffic conditions. Several research papers have addressed the routing problem based on traffic information [2, 3]. The authors believe that there is more that can be done: By calculating a speed profile that reduces time spent behind a local traffic jam, fuel can be saved with a little compromise on trip time. To the best knowledge of the authors such an investigation is missing in the literature; its results if positive can cut the cost of running fleets of commercial heavy trucks and provide an eco-friendly option for passenger cars. The proposed approach relies mostly on software and information and needs minimal hardware investments. This impacts not only the high-tech vehicle of the future but the current fleet when equipped with add-on accessories such as a smart

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phone or intelligent speed assistant systems[4].

This paper formulates such predictive planning of velocity and demonstrates its impact on fuel economy and emissions of passenger and commercial vehicles via several simulation case studies. We cast the problem as an optimal control problem with the goal of reducing velocity transients while also penalizing trip time. This optimal control problem will be solved numerically by a two dimensional dynamic program. A key to this work is realizing the traffic-imposed constraints on the velocity which requires spatiotemporal estimation of traffic velocity. In this paper we focus on highway trips where a longer traffic preview horizon along with coarser traffic information can be effective. For this type of planning knowledge of current state of traffic is not sufficient and a predictive traffic model may be used to estimate the evolving pattern of traffic down the road.

Because a feedfoward traffic estimator that predicts evolution of traffic along the vehicle route is a key part of this work, it is discussed in detail next in Section 2.1. This is followed by formulation of the optimal velocity planning problem in Section 2.2. The numerical dynamic programming solution process is described in Section 2.3. Section 2.4 describes the model used for evaluation of fuel economy. Several simulation case studies are presented in Section 3 followed by Conclusions.

2 Methodology

2.1 Spatiotemporal Prediction of Traffic

There is a vast body of work by traffic engineers, physicists, and computer scientists on traffic modeling. Excellent and thorough review of such models can be found in [5–7]. Microscopic traffic models use simple car-following rules to model procession and interaction of individual vehicles. These models are described by a system of ordinary differential equations or in the cellular automata approach by rules for advancing individual vehicle in a fine grid in discrete time steps. The drawback of microscopic models is the high computational load as the number of vehicles increases. Macroscopic models use the analogy of traffic flow to fluid flow and formulate spatiotemporal evolution of speed and traffic density using coupled partial differential equations (PDEs). Aggregating a large number of vehicles into a continuum macroscopic model has the advantage of being much faster computationally. At the same time via macroscopic models it is possible to capture complex traffic phenomena such as a congestion wave, instabilities, and phase transitions of traffic flow [8]. Macroscopic models have also been used for calculating average travel times, fuel consumption and emission levels [9], for short-term forecasts of traffic flow for rerouting [10, 11], and for design of traffic flow control systems [12, 13]. More recently in [14], a gas-kinetic traffic model is used to estimate the future velocity of a plug-in hybrid vehicle. Missing from the literature are methods that help plan the velocity of an individual vehicle to reduce the possibility of its untimely arrival at a local traffic wave.

One way of forming a spatiotemporal traffic map is through these existing gas-kinetic PDE models of traffic to predict its evolution. Specifically in inter-city highway driving use of a predictive model is important due to the long planning horizon. The evolution of average traffic velocity at each point and time $v_t(x,t)$, and average traffic density $\rho_t(x,t)$, are predicted by the following coupled PDEs [5]:

$$\frac{\partial \rho_t}{\partial t} + \frac{\partial(\rho_t v_t)}{\partial x} = q(x,t) \tag{1}$$

$$\frac{dv_t}{dt} = \frac{\partial v_t}{\partial t} + v_t \frac{\partial(v_t)}{\partial x} = \frac{v_e(\rho_t) - v_t}{\tau} - \frac{1}{\rho_t}\frac{\partial P}{\partial x} + \frac{\eta}{\rho_t}\frac{\partial^2 v_t}{\partial x^2} \tag{2}$$

The first equation describes balance of vehicles; there $q(x,t)$ models the incoming or outgoing traffic at discrete road junctions. The second equation models dynamics of the velocity, characterized by a traffic pressure P expressed as $P = c_0^2 \rho_t$ and traffic viscosity η in which both η and c_0 are constants. The function $v_e(\rho_t)$ captures the speed-density relationship at steady-state. Details on the choice of parameters P and η and the function $v_e(\rho)$ can be found in [8, 15, 16]. Approximate boundary and initial conditions and the ramp inputs $q(x,t)$ can be retrieved from real-time traffic information systems as well as time of the day, season, holidays, current and forecast weather, accidents, or events such as school schedules, sports games and concerts, and even uniquely local variables [17]. The set of coupled PDEs will be solved using a finite-difference approach in real-time to determine traffic-imposed constraints in the future path of a vehicle. For inner-city driving, the *immediate* traffic-imposed bounds on speed can be obtained via infrastructure-to-vehicle communication [18] or via ad-hoc [19,20] vehicle-to-vehicle communication networks.

2.2 Optimal Control Problem Setup

The average traffic velocity $v_t(x,t)$ estimated above will be an upper limit to the velocity each vehicle can assume at position x at time t. The goal is to find a velocity profile that i) meets this traffic-imposed speed limit (and the speed limits of the road) and ii) lowers fuel use without too much compromise on trip time. In other words, the slope of each feasible path is upper-bounded by the spatiotemporally varying limit $v_t(x,t)$ imposed by traffic. The problem of finding the optimal speed trajectory $v(x,t)$ can be formalized as an optimal control problem which will be solved numerically. The cost function is:

$$\min_{v(x,t)} J = \int_{x_i}^{x_f} \|L(v(x,t))\|_Q^2 \frac{dx}{v(x,t)} \tag{3}$$

subject to road speed limits $[v_{min}, v_{max}]$, traffic-imposed bound on speed $v_t(x,t)$, and driver set speed v_{set}:

$$v_{min} \leqslant v(x,t) \leqslant min(v_{max}, v_t(x,t), v_{set}) \tag{4}$$

and with acceleration and deceleration constraints imposed on $\dot{v}(x,t)$. In (3), x_i and x_f are the origin and destination and $\| \bullet \|_Q$ denotes the weighted 2-norm with the diagonal penalty weighting matrix Q. Appropriate choice of the the integrand $L(v(x,t))$ is an open problem. For example the choice $L(v) = \left[\dot{m}_f \; NO_x \; (v(x,t) - v_{set}) \right]^T$ penalizes the fuel rate $\dot{m}_f$ and NO_x emissions, while also penalizing deviations from the driver set speed. The latter ensures travel time is not compromised. Another choice is to explicitly penalize trip time by selecting $L(v) = \left[\dot{m}_f \; NO_x \; 1 \right]^T$ which will result in trip time $t_f - t_i$, appearing in the cost function. However inclusion of fuel rate and emissions in the cost function add to the complexity of this optimal control problem, because it requires inclusion of a detailed model of the vehicle powertrain. Therefore in this first investigation we use a simpler form $L(v) = \left[\dot{v}^2 \; 1 \right]^T$ to penalize trip time and velocity transients $\dot{v}$ (accelerations and decelerations) which indirectly contribute to increase in fuel use. This correlation of the cost function with the fuel economy is tested in the simulations with PSAT-a commercialized vehicle powertrain simulation software developed by Argonne National Laboratory and explained in detail in part 2.4. Another benefit of using this cost function is that it is independent of a specific powertrain and could be directly transferred among different vehicles. The other factor increasing the fuel use is idling at zero speed; penalizing the total trip time should cut unnecessary idling. The solution $v(x,t)$ can then be issued as a reference to the low-level vehicle controller. Alternatively the velocity $v(x,t)$ can be suggested to the driver as the eco-friendly speed by a mobile phone or intelligent speed assistant system which has been experimented in Australia and Europe [21]. In part 2.4, the PSAT software including a driver model and independent of the optimal control process was used to test the effectiveness of the suggested velocity profile.

2.3 Numerical Solution Via Dynamic Programming

The optimal control problem posed above cannot be solved analytically due to the spatiotemporally varying constraints imposed on its optimization variables along with several other pointwise-in-time constraints. In this work we solve this problem numerically using a dynamic program.

The vehicle kinematics is represented by the following two-state dynamic equations:

$$\begin{cases} \dot{x} = v \\ \dot{v} = u \end{cases} \tag{5}$$

where x and v are position and velocity of the vehicle respectively and u is its acceleration which is selected as a control input. Since the final optimal velocity trajectory is evaluated in powertrain simulation software PSAT, we ignore the the dynamics of the powertrain and driver's reaction time at the upper level control. Therefore $L = \left[u^2 \; 1 \right]^T$ is set in the cost function (3). In addition to the velocity constraint (4), we impose the acceleration constraint on the input u:

$$a_{\min} \leq u(x,t) \leq a_{\max} \tag{6}$$

where $a_{\max}$ is the positive maximum allowable acceleration and $a_{\min}$ is the negative maximum allowable deceleration.
The cost function can be written as follows:

$$J = \int_{x_i}^{x_f} u^2 \frac{dx}{v(x,t)} + \phi(t_f, t_i) \tag{7}$$

where $\phi(t_f, t_i)$ is a terminal cost on trip time and proportional to $t_f - t_i$ by a penalty weight.

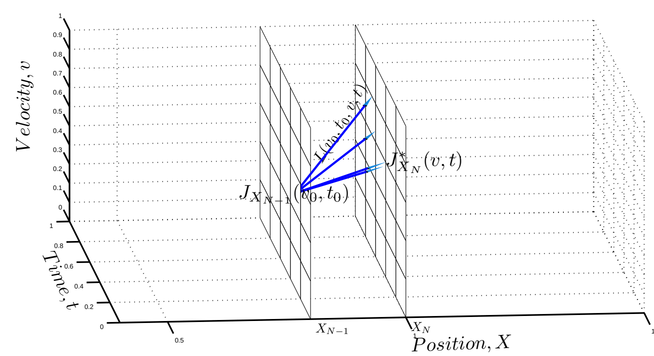

Figure 1. Schematic of the DP grid and value function iteration.

The cost function in (7) is rewritten in discretized space calculated backward:

$$J = \sum_{n=0}^{N_{\max}} \frac{u^2(x_n, t_{x_n})}{v(x_n, t_{x_n})} \Delta x + \phi(t_f, t_i) \tag{8}$$

We also define the the cost function $J_{X_N}(v,t)$ as the cost-to-go from position x_N to the final position which is a function of variables v and t:

$$J_{X_N}(v,t) = \sum_{n=N}^{N_{\max}} \frac{u^2(x_n, t_{x_n})}{v(x_n, t_{x_n})} \Delta x + \phi(t_f, t_i) \tag{9}$$

The optimal cost-to-go from position x_N to the final position will then be:

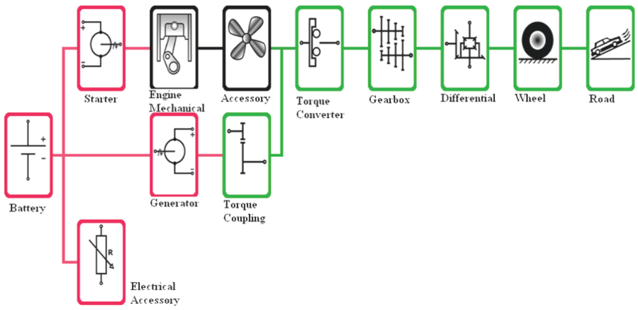

Figure 2. Schematic of a PSAT powetrain model

$$J^*_{X_N}(v,t) = \min_u \sum_{n=N}^{N_{\max}} \frac{u^2(x_n,t_{x_n})}{v(x_n,t_{x_n})}\Delta x + \phi(t_f,t_i) \qquad (10)$$

The optimal acceleration u^* can be found relying on Bellman's optimality principle and by value function iterations backward-in-position as shown in Figure 1. Given the optimal cost-to-go $J^*_{X_N}$ iterations over each node on the planar grid at x_{N-1} will yield the optimal cost-to-go $J^*_{X_{N-1}}$:

$$J^*_{X_{N-1}}(v,t) = \min_{u(x_{N-1})} (J^*_{X_N}(v,t) + \frac{u^2(x_{N-1})}{v(x_{N-1})}) \qquad (11)$$

and also determines the optimal control $u^*(x_{N-1})$. The process is continued backward-in-position until the sequence of optimal control inputs over the entire trip is determined.

2.4 Evaluation of Fuel Savings Potential with a Detailed Powertrain Model

The above solution is a constraint-admissible velocity profile that follows the set target speed as closely as possible. In order to estimate the fuel economy of the vehicle when following this optimal velocity trajectory, a production vehicle is selected and its powertrain model is assembled from the extensive database of Powertrain System Analysis Toolkit (PSAT). PSAT developed by Argonne National Laboratory [22] is a powerful simulation tool for evaluating the fuel economy of conventional and hybrid vehicles when following a prescribed velocity cycle. Its physics-based component models combined with empirical maps obtained from production vehicles allow high-fidelity evaluation of fuel economy. Figure 2 shows schematics of a PSAT powertrain. This is a conventional (non-hybrid) powertrain with an automatic transmission. The models for torque converter, transmission, and vehicle dynamics are all very detailed and include several dynamic states and switching modes. Details such as electrical accessory loads, the starter, generator, etc. are not overlooked and modeled for simulation accuracy.

PSAT is a "forward-looking" causal simulation tool in which the vehicle speed is determined by the combined influence of road loads and engine (or brake) torque at the wheels. The resulting velocity is compared to the prescribed desired velocity; the difference is fed to a driver model which in turn determines a torque demand. The torque demand is met by the engine (or brake) torques and the above simulation loop is repeated. The engine fuel rate is determined using an empirical engine map and as a function of engine speed and engine torque. The fuel rate is integrated over the whole cycle time to determine the amount of fuel used.

3 Simulation Results

Four simulation case studies were conducted to determine the potential impact on fuel economy and trip time of a vehicle when future state of traffic is available either via predictive databases or through the model formulated in Section 2.1. In the first two cases the average traffic speed along the route is assumed to be known in advance. In the next two cases, the speed of traffic is estimated using a PDE model of traffic with assumed boundary conditions and inputs.

For the fuel economy evaluation, two different vehicles have been considered: a passenger vehicle and a mid-size truck. The passenger vehicle is an economy-sized car with 5-speed automatic transmission, 1000 kg mass and 115 hp maximum power. The midsize truck has 6-speed automatic transmission, 8500 kg mass, and 500 hp maximum power. The fuel economy evaluation process is done in PSAT v6.2. In all simulations the maximum acceleration is assumed to be 2 m/s^2 which is a conservative estimate of maximum acceleration capability of a midsize vehicle. Assuming braking on dry asphalt, the friction coefficient of $\mu_b = 0.69$ yields the maximum possible deceleration of 6.7 m/s^2. However, to exclude aggressive driving, maximum braking deceleration of 3 m/s^2 is assumed.

3.1 Velocity Planning Based on Given Traffic Velocity Distribution

Two simple simulation cases were generated first, to better illustrate the idea proposed in this paper. For these cases, the traffic velocity distribution was assumed to be given for a 9 km road and for a time span of 1000 seconds. Figures 3 and 4 show in numbers the traffic velocities assumed for each time-space segment. Figure 3 represents a traffic wave that moves forward in space and Figure 4 represents a backward traffic wave. Both situations can happen in the real world. Such traffic velocity maps may be formed using historical traffic data or relying on real-time traffic monitoring (and prediction) services.

Superimposed on these velocity maps are trajectory of i) a conventional vehicle that does not have access to traffic preview and therefore travels at the speed limit unless faced with traffic ii) a vehicle that predictively plans its speed using traffic preview with low weight (0.5) on trip time iii) a vehicle that predictively plans its speed using traffic preview with high weight (50) on trip time. All simulations start with the initial velocity of 25 km/h. The road speed limit is $v_{max} = 60$ km/h and the minimum allowable velocity in all of elements is $v_{min} = 0$. Fuel economy of each case is evaluated in PSAT for the passenger vehicle described in the previous section.

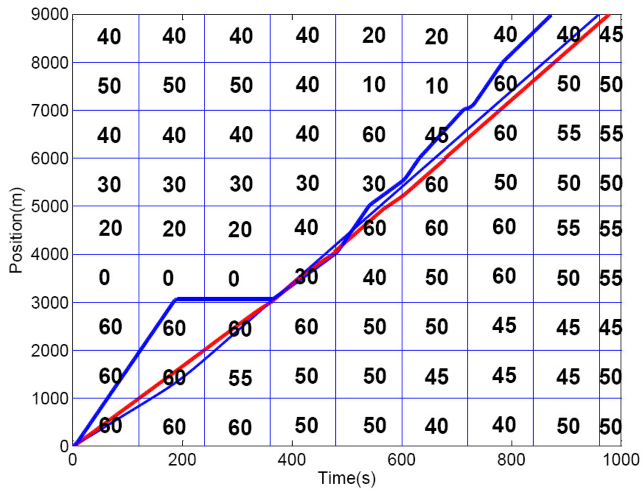

Figure 3. Trajectory of conventional vehicle (solid blue) versus predictive vehicles for *forward congestion* wave. Solid red and dashed blue line corresponds to the normalized time penalty of 0.5 and 50 respectively. Numbers inside each block show the traffic velocity assumed for each point in space-time.

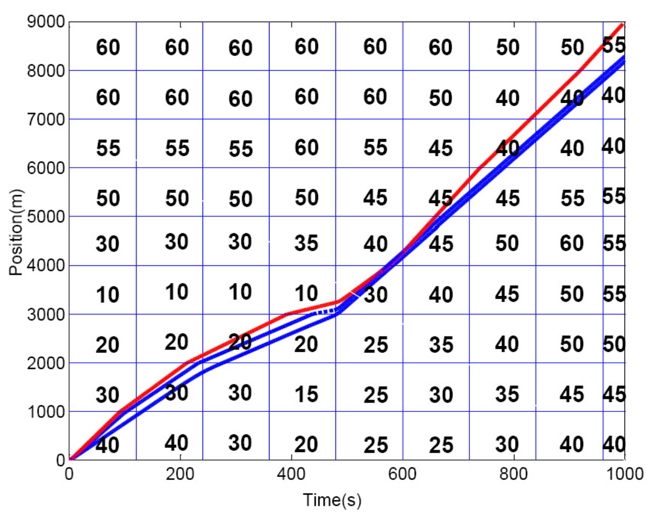

Figure 4. Trajectory of conventional vehicle (solid blue) versus predictive vehicles for *backward congestion* wave. Solid red and dashed blue line corresponds to the normalized time penalty of 0.5 and 50 respectively. Numbers inside each block show the traffic velocity assumed for each point in space-time.

Table 1 summarizes the trip time and fuel economy results for case I with forward traffic wave. As can be seen from the numbers, the PCC-equipped vehicle can save up to 21 percent fuel over the conventional vehicle (56 mpg versus 46 mpg). This is with an 8 percent increase in trip time (855 seconds versus 795 seconds).

Table 1. Case I results.

	Conventional	Predictive ($W_t = 0.5$)	Predictive ($W_t = 50$)
Fuel Econ. (mpg)	46	56	54
Trip Time (sec)	795	855	845

Results for the backward traffic wave of case II are summarized in Table 2. In this case, the vehicle with preview can save up to 8 percent fuel as compared to the conventional vehicle (54 mpg versus 50 mpg). This is at the cost of 6.5 percent increase in trip time. We also observe less fuel savings in case II with a backward wave as compared to case I with a forward wave. This could have been expected: When the traffic wave moves forward and away from us, we have the opportunity to avoid it by slowing down, while the reverse is not necessarily true.

Table 2. Case II results.

	Conventional	Predictive ($W_t = 0.5$)	Predictive ($W_t = 50$)
Fuel Econ. (mpg)	50	54	52
Trip Time (sec)	910	970	966

3.2 Velocity Planning Based on Macroscopic Traffic Flow Model

In the next two simulation cases the traffic speed profile is generated using the PDE traffic model presented in Section 2.1. The parameters of the system of partial differential equations 1 and 2 are summarized in Table 3. The resolution of the velocity distribution surface is 900 by 450 which corresponds to 20 meters and 1 second along position and time vectors.

Table 3. Macroscopic traffic model parameters

Parameter	Value	Unit
τ	0.01	s
v_f	20	m/s
ρ_{jam}	0.2	$vehicle/m$
α	6000	$1/m$

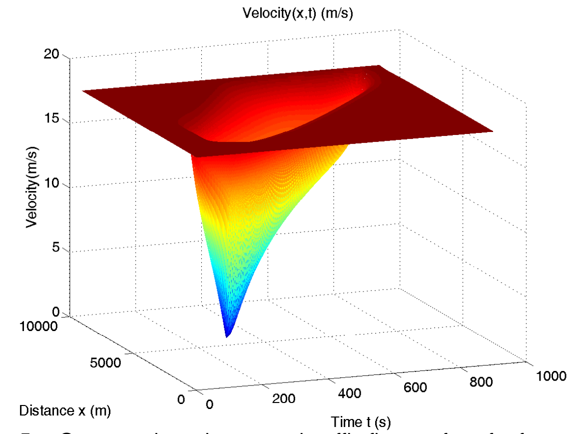

Figure 5. Generated spatiotemporal traffic flow surface for forward congestion wave.

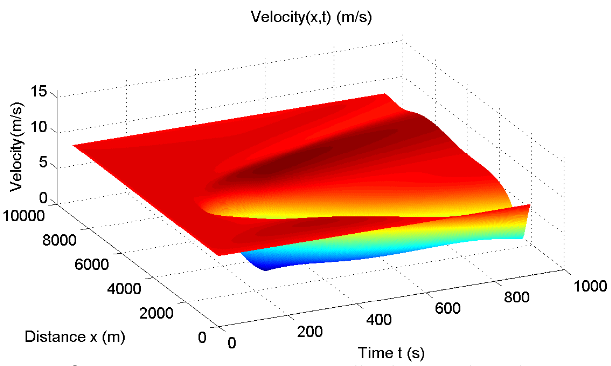

Figure 6. Generated spatiotemporal traffic flow surface for backward congestion wave.

In order to generate three dimensional spatiotemporal traffic flow surfaces for both backward and forward wave cases, the macroscopic traffic flow governing equations (system of PDEs in equations 1 and 2) need to be provided with a set of boundary and initial conditions for each case. Two sets of boundary and initial conditions have been assumed such that upstream, downstream and initial steady speed of traffic flow are 18 m/s and 10 m/s for forward and backward waves respectively. To create a transient congestion wave, a ramp input flow $q(x,t)$ is injected into equation (1) and represents a flow of cars entering the road from a side ramp. Here we assume the ramp is at position 3200 m and the ramp input is only nonzero during the time interval $[100, 200]$. In this time interval we assume two constant flow rates of 0.022 and 0.024 vehicle/m/s which result in forward and backward congestion waves respectively.

Given the traffic flow information calculated above, the vehicle velocity is calculated by solving numerically the optimal control problem posed in Section 2.2 . The results are compared to those of the conventional vehicle which is expected to move with the traffic stream. In other words, a conventional vehicle trajectory moves on the solution surface of Figure 5 or 6.

Tables 4 and 5 summarize the statistics of the resulting velocity profiles of conventional and preview vehicles for forward and backward congestion wave cases respectively. Also shown

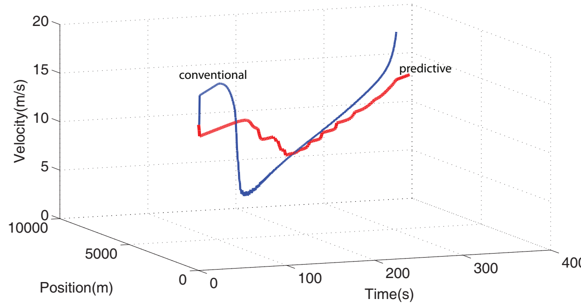

Figure 7. Trajectory of the vehicle with and without preview when faced with the forward traffic wave.

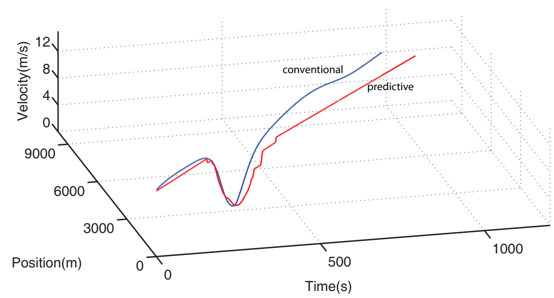

Figure 8. Trajectory of the vehicle with and without preview when faced with the forward traffic wave.

is the fuel economy when following these trajectories, calculated for a passenger and a midsize heavy vehicle. The results show that in forward congestion waves, up to 12 percent fuel saving for a passenger vehicle and 8 percent fuel saving for the heavy vehicle is obtained when the traffic information is predictively utilized. The savings are less in backward congestion wave. In all cases there is a compromise on trip time.

Table 4. Conventional and preview vehicles in forward congestion wave.

	Conventional	Preview	Unit
Max. Velocity	18.0	15.0	m/s
Min. Velocity	8.4	5.1	m/s
Trip Time	735	765	s
Fuel Economy (Passenger Vehicle)	49.26	55.00	mpg
Fuel Economy (Heavy Vehicle)	7.04	7.93	mpg

Table 5. Conventional and preview vehicles in backward congestion wave.

	Conventional	Preview	Unit
Max. Velocity	11.7	10.0	m/s
Min. Velocity	1.8	1.8	m/s
Trip Time	956	1058	s
Fuel Economy (Passenger Vehicle)	54.20	56.10	mpg
Fuel Economy (Heavy Vehicle)	7.97	8.07	mpg

4 Conclusion

In this paper we proposed the idea of predictively planning a vehicle's speed for reducing the velocity transients in upcoming traffic waves in order to reduce its fuel consumption. It was assumed that future state of traffic in space and time can be estimated and used as an spatiotemporal upper bound on how fast a vehicle can travel. One possible method for estimation of velocity that was proposed in this paper is using real-time traffic information as initial conditions to a macroscopic traffic model represented by a set of coupled nonlinear partial differential equations.

An optimal control problem was cast with the estimated traffic flow surface as the upper bound on the velocity and with the target of improving fuel economy. The validity of the approach was investigated in four different simulation case studies; in two cases spatiotemporal distribution of traffic speed was assumed and in the remaining cases the PDE traffic model was solved to generate the traffic surface. The fuel evaluation simulations showed up to 21 percent improvement of fuel economy was possible in some scenarios when the future state of traffic was known. This improvement was achieved at the cost of 6 to 8 percent increase in trip time. The simulations also showed that traffic preview may be more beneficial to fuel economy when the traffic congestion moves "forward" in space.

The methodology and simulation result reported here are for the purpose of quantifying the potential of traffic flow information for fuel saving of a vehicle. The realization of the potential fuel economy improvement still depends on the accuracy of the traffic predictions. More work is needed to forward the proposed method as a real-time oriented approach.

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