12th IFAC Symposium on Transportation Systems
Redondo Beach, CA, USA, September 2-4, 2009

Predictive Use of Traffic Signal State for Fuel Saving

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Abstract: This paper proposes preview of upcoming traffic light information to minimize idle at the lights and reduce fuel use. An optimal control algorithm is formulated for each equipped vehicle that uses short range radar and traffic signal information predictively to schedule an optimum velocity trajectory for the vehicle. The objectives are timely arrival at green light with minimal use of braking, maintaining safe distance between vehicles, and cruising at or near set speed. Preliminary simulation results show considerable reduction of fuel use.

Keywords: traffic light timing, model predictive control, adaptive cruise control

1. INTRODUCTION

Poor traffic signal timing is believed to account for an estimated 10 percent of all traffic delay (about 300 million vehicle-hours) on major roadways alone USDT (2007). Advanced traffic signal control methods such as traffic-actuated signals and signal synchronization are very costly to implement and maintain; just the annual cost of signal timing updates is estimated at 217 million dollars a year according to NTSRC (2008). Even with these measures in place, we often cruise at full speed toward a green and have to come to a sudden halt whenever the light turns red. This lack of information about "future" state of the traffic signal increases fuel use, engine and brake wear, and sometimes trip time. In an ideal situation if the future state of a light's timing and phasing is known, the speed could be adjusted for a timely arrival at green.

While maybe unrealistic a few years ago, communicating traffic signal state to the vehicles in advance is not far-fetched today. In fact researchers are now experimenting with broadcasting red light warnings to vehicles to improve intersection safety. The required navigation and information broadcast technology is available today and is expected to be more widely deployed in near future.

This paper focuses on employing upcoming light time and phase information within the vehicle's adaptive cruise control system to minimize wait time at stop lights and fuel use. To achieve this goal an optimal control algorithm will be formulated for each equipped vehicle that uses short range radar and traffic signal timing information to schedule an optimum velocity trajectory for the vehicle. The objectives are timely arrival at green light with minimal use of braking, maintaining safe distance between vehicles, and cruising at or near set speed. Figure 1 shows a schematic of this proposed concept.

Adaptive cruise control is now in production and a well-matured technology. Many ideas on intelligent transportation system (ITS) have been explored extensively during the 1990s within intelligent highway initiatives in the US, Japan, and Europe [Vahidi and Eskandarian (2003)]. Voluntary use of future signal and traffic information has only recently attracted attention under CICAS (Cooperative Intersection Col-

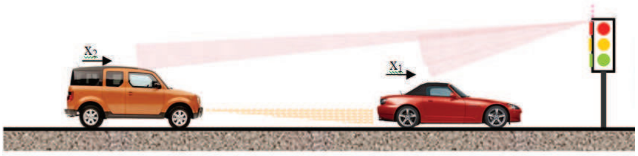

Fig. 1. Schematic of telematics-based predictive cruise control.

lision Avoidance Systems) initiative mainly for improving intersection safety [Sengupta et al. (2007); Chan and Bougler (2005)]. Optimal traffic management at intersections has been mainly studied from a signal-timing optimization perspective e.g. signal synchronization [Brockfeld et al. (2001); Huang and Huang (2003); Gershenson (2005)]. More recently and for futuristic autonomous vehicles, Dresner et al. [Dresner and Stone (2008); VanMiddlesworth et al. (2008)] have proposed replacing traffic lights and stop signs by intelligent lights: Via a two way communication protocol, the autonomous vehicles call the intersection ahead to reserve a time-space slot to pass. It is suggested in [Dresner and Stone (2008)] that this setup has the potential to improve.

To the best knowledge of the authors, the proposed Predictive Cruise Control (PCC) concept is first in its kind that utilizes the adaptive cruise control function in a predictive manner to simultaneously improve fuel economy and reduce signal wait time. The proposed predictive speed control mode differs from current adaptive cruise control systems in that i) besides maintaining a safe gap between vehicles, it minimizes use of brakes, thus reducing brake wear and kinetic energy loss, ii) can work in stop and go traffic, and more importantly iii) receives a timing signal from an upcoming traffic light in advance to safely and smoothly speed up or down to a timely arrival at green light whenever possible, therefore reducing idling at red.

These sometimes conflicting objectives are unified under a model predictive control (MPC) framework. The proposed MPC formulation allows tracking a target speed, calculated based on traffic signal information, with minimum brake use. At the same time it enforces several physical constraints including a safe distance to the front vehicle. Simulation of complex stop and go situations is facilitated relying on MPC as the "driving brain" of each vehicle. The result is a potentially powerful simulation tool that can be extended to multi-vehicle simulations with each vehicle modeled as an intelligent agent. This may be another contribution of this work over the existing microscopic and macroscopic models developed (mainly by physicists and computer scientists) over the past few decades for traffic simulation [Nagel et al. (2003); Nagel (1996); Hoogendoom and Bovy (2001)]. Many underlying functions or rules required to determine procession of vehicles in these models limit embedding systematic optimization routines in them.

Section II describes the MPC design followed by the approach taken to accurately estimate the fuel economy and CO_2 emissions of the vehicle. Simulation results for a single-vehicle scenario, driven with and without PCC, are presented in section III followed by Conclusion in section IV.

2. METHODOLOGY

One of the analytical challenges unique to this optimal control problem is dynamic switching of lights to red and green. This type of motion constraints renders the feasible solution space non-convex. Solution of a non-convex optimization problem is computationally intensive and may not converge to the global optimum. We resolve this issue by handling the problem at two levels: i) a set of logical rules that calculates a reference velocity for timely arrival at green lights combined with ii) a model predictive controller that tracks this target velocity. The resulting solution may be sub-optimal but is real-time implementable. A simple model of the vehicle will be used at the supervisory level for velocity planning; but the fuel economy and drivability will be later evaluated using a detailed model of the powertrain.

2.1 Reference Velocity Planning

A reference velocity v_{target} is determined based on driver's set cruise speed, and also the signal received from the upcoming traffic light. The basic idea is to safely i) increase v_{target} , up to a maximum allowable, when there is enough green time to pass, or otherwise ii) decrease v_{target}, down to a minimum allowable, to arrive at the next green. All will be done considering driver's set cruise control. The objective is to minimize stop time at red.

It is assumed that the approximate distance to the next traffic light(s) is known at each time and shown by d_i where the subscript i denotes the light number in a sequence of traffic lights, i.e. d_1 is the approximate distance to the first upcoming light and d_2 to the second light at each time. The light(s) update and broadcast an expected sequence of their green and red times regularly. Suppose g_{ij} is start of the j^{th} green of the i^{th} traffic light and r_{ij} is start of the j^{th} red of the i^{th} light. For example light number 1 broadcasts, at regular intervals, a sequence

$$[g_{11},\ r_{11},\ g_{12},\ r_{12},\ g_{13},\ \cdots] = [40,\ 100,\ 150,\ 200,\ 240,\ \cdots]$$

which implies the first traffic light is currently red, it will turn green in 40 seconds, red in 100 seconds, green again in 150 seconds, and so forth. Figure 2 shows a schematic of the map formed at each time step based on the information received from the lights. Equipped vehicles can use the remaining distance to the next light(s) and the green and red sequence to set their target speed. This target speed (slope of each path) cannot

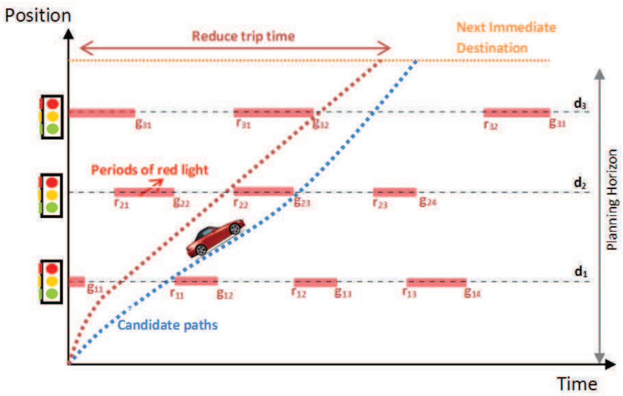

Fig. 2. Schematics map of red lights distributed over space-time. The graphics shows how a PCC car passes two consecutive intersection without having to stop at a red.

exceed the speed limits or the speed set by the driver. Other constraints such as acceleration constraints, maintaining safe distance to the front vehicle, and minimizing use of brakes are handled separately by a dynamic optimization scheme (details in section II-B).

The following steps are followed to determine the target speed at each step k:

(1) For a vehicle to pass during the first green of the first light, its velocity should be in the interval $[\frac{d_1}{r_{11}}, \frac{d_1}{g_{11}}]$. This is only feasible if this interval has an intersection with the feasible speed interval of $[v_{min}, v_{max}]$. If this intersection is empty, passing through the first green without stopping at red is deemed infeasible. In that event, feasibility of passing during the next green interval is checked and the process is repeated until for some i^{th} interval $[\frac{d_1}{r_{1i}}, \frac{d_1}{g_{1i}}]$ has an intersection with $[v_{min}, v_{max}]$. This intersection is mathematically characterized by:

$$[\frac{d_1}{r_{1i}}, \frac{d_1}{g_{1i}}] \cap [v_{\min}, v_{\max}] \qquad (1)$$

and determines the range of speed that ensures passing the first light without having to stop at a red.

For example assume the speed limits are $[v_{min}, v_{max}] = [5, 20]m/s$ and the distance to the first traffic light is $1000m$. The first light broadcasts,

$$g_{11} = 5s, \quad r_{11} = 25s, \quad g_{11} = 40s, \quad r_{11} = 100s$$

then

$$[\frac{d_1}{r_{11}}, \frac{d_1}{g_{11}}] = [40, 200]\ m/s$$

does not meet the speed limit. The second interval

$$[\frac{d_1}{r_{12}}, \frac{d_1}{g_{12}}] = [10, 25]\ m/s$$

intersects with the feasible speed at $[10, 25]\ m/s$ Therefore, if the velocity of the vehicle is chosen between $10\ m/s$ and $20\ m/s$, the vehicle passes the first light without having to stop.

(2) If passing without stop at the first light is determined to be feasible, the process in step 1 is repeated for the second traffic light by checking the intersections

$$[\frac{d_2}{r_{2i}}, \frac{d_2}{g_{2i}}] \cap [v_{\min}, v_{\max}]$$

and picking the first non-empty one.

(3) Next, the intersection of the feasible range of speeds determined in step 1 and that of step 2 is calculated. A non-empty solution $[v_{low}, v_{high}]$ indicates feasibility of passing the two lights without having to stop at a red.

(4) The process is continued by checking the next lights until a stop at red becomes unavoidable. The last feasible range $[v_{low}, v_{high}]$ is an appropriate target velocity. To ensure shortest trip time we set $v_{target} = v_{high}$.

Note that the target velocity is updated at each sampling time and therefore may change at each instant based on vehicle's position and the most recent information from the lights. This set of rules is not necessarily "optimal", but helps break down a fundamentally non-convex optimization problem to a simpler real-time implementable one. Tracking this target velocity, maintaining a safe distance to the front vehicle, and minimizing use of brakes are handled by this optimization scheme described next.

2.2 Optimal Tracking of the Reference Velocity

A simple model of the vehicle is used at the supervisory level for calculating the vehicle acceleration based on effective traction force of the engine f_{engine} or braking force f_{brake} and the road forces f_d. For the i^{th} vehicle with mass m_i , the longitudinal dynamics is:

$$m_i \frac{d^2 x_i}{dt^2} = f^i_{engine} - f^i_{brake} + f^i_d \tag{2}$$

where f^i_d lumps the road forces including aerodynamic drag, rolling resistance, and road grade forces:

$$f^i_d = -c_D v_i^2 - m_i g(\sin(\theta) + \mu\cos(\theta)) \tag{3}$$

where c_D is a "lumped" drag coefficient, μ is the coefficient of rolling resistance, and g is gravitational acceleration. The f^i_d term is treated as a measured disturbance and updated at each sample time. Equation (2) can be written in the following state-space discretized form:

$$\begin{aligned} z_i(k+1) &= Az_i(k) + B_u u_i(k) + B_w w_i(k) \\ y_i(k) &= Cz_i(k) \end{aligned} \tag{4}$$

where $z_i = [x_i \quad v_i]^T$ is the state vector, $u_i = [f^i_{engine} \quad f^i_{brake}]^T$ is the control input, and $w_i = [f^i_d]$ is the measured disturbance. The main output of interest are $y_i = [x_i \quad v_i]^T$; however other outputs need to be introduced to handle the gap inequality constraint described later. The matrices $A \in \mathbb{R}^{2\times 2}$, $B_u \in \mathbb{R}^{2\times 2}$, $B_w \in \mathbb{R}^{2\times 1}$, and $C \in \mathbb{R}^{2\times 2}$ are the discretized system matrices. The engine and brake forces are manipulated for tracking the target speed as closely as possible while maintaining a safe distance to the front vehicle. These objectives along with the desire to minimize use of service brakes can be unified in a Model Predictive Control (MPC) framework. The control performance index at each step k for the i^{th} vehicle is defined as:

$$J_i(k) = \sum_{j=k}^{k+P-1} [w_1(v_i(j) - v_{target}(j))^2 + w_2(f^i_{brake}(j))^2] \tag{5}$$

Here w_1 and w_2 are simply penalty weights for each term. The above index penalizes deviations of vehicle speed v_i from the target speed v_{target} and also reduces use of brake force over a future prediction window of P steps. Reduced use of service brakes in the cost function indirectly contributes to fuel savings. Fuel use is not explicitly penalized; this allows use of the simpler first-order vehicle model for control design. Fuel savings will be later evaluated using a detailed model of the vehicle's powertrain. The trip time is minimized by setting v_{target} equal to maximum feasible speed as explained in the previous section.

The speed limit, engine and brake force limits, and the minimum safe following distance are imposed as pointwise-in-time inequality constraints. The constraints should be satisfied over the future prediction horizon $\forall j \in \{k, k+1, \cdots, k+P-1\}$. The speed limit constraint is,

$$v_{min} \leqslant v_i(j) \leqslant v_{max} \tag{6}$$

where v_{min} and v_{max} are speed limits and should also be smaller than the driver set speed. Bounds on the traction force are represented by,

$$\begin{aligned} 0 \leqslant f^i_{engine}(j) \leqslant f^{max}_{acceleration} \\ 0 \leqslant f^i_{brake}(j) \leqslant f^{max}_{deceleration} \end{aligned} \tag{7}$$

where $f^{max}_{acceleration}$ and $f^{max}_{deceleration}$ depend on tire and road condition and also maximum engine and braking torque capability. The minimum safe following distance between follower vehicle i and lead vehicle $i+1$ depends on the follower vehicle speed and can be written as:

$$\alpha v_i(j) + \beta \leqslant x_{i+1}(j) - x_i(j) \tag{8}$$

where β is a "static gap" parameter and determines the minimum gap needed when the vehicles are stopped and α is a "dynamic gap" parameter providing extra gap with increased speed. The need to stop at a red light is also imposed by the constraint in (8) where x_{i+1} is set to the position of the traffic light or that of a front vehicle stopped at that light.

The cost function (5) subject to the model equation (4) and inequality constraints (6), (7), and (8) is minimized at each sample time to determine the sequence of next $N \leqslant P$ control inputs $U_i(k) = [u_i(k) \quad u_i(k+1) \quad \cdots \quad u_i(k+N-1)]$ over the future horizon P. When $N < P$ the remaining control moves $[u_i(k+N) \quad u_i(k+N+1) \quad \cdots \quad u_i(k+P-1)]$ are assumed to be zero. According to the standard MPC design , only the first entry of the control sequence $U_i(k)$, is applied to the vehicle, the optimization horizon is moved one step forward, the model and constraints are updated if necessary, and the optimization process is repeated to obtain the next optimal control sequence $U_i(k+1)$ [Maciejowski (2002); G. C. Goodwin and Dona (2005); Bemporad (2006)].

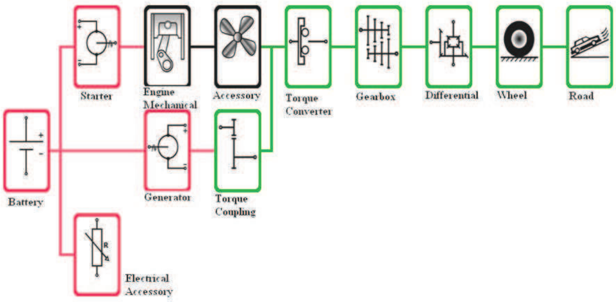

Fig. 3. Schematic of a PSAT powetrain model

2.3 Evaluation of Fuel Savings Potential with a Detailed Powertrain Model

The MPC solution generates a constraint-admissible velocity profile that follows the set target speed as closely as possible. In order to estimate the fuel economy of the vehicle when following this optimal velocity trajectory, a production vehicle is selected and its powertrain model is assembled from the extensive database of Powertrain System Analysis Toolkit (PSAT). PSAT developed by Argonne National Laboratory [PSAT (2002)] is a powerful simulation tool for evaluating the fuel economy of conventional and hybrid vehicles when following a prescribed velocity cycle. Its physics-based component models combined with empirical maps obtained from production vehicles allow high-fidelity evaluation of fuel economy. Figure 3 shows schematics of a PSAT powertrain. This is a conventional (non-hybrid) powertrain with an automatic transmission. The models for torque converter, transmission, and vehicle dynamics are all very detailed and include several dynamic states and switching modes. Details such as electrical accessory loads, the starter, generator, etc. are not overlooked and modeled for simulation accuracy.

PSAT is a “forward-looking” causal simulation tool in which the vehicle speed is determined by the combined influence of road loads and engine (or brake) torque at the wheels. The resulting velocity is compared to the prescribed desired velocity; the difference is fed to a driver model which in turn determines a torque demand. The torque demand is met by the engine (or brake) torques and the above simulation loop is repeated. The engine fuel rate is determined using an empirical engine map and as a function of engine speed and engine torque. The fuel rate is integrated over the whole cycle time to determine the amount of fuel used.

3. SIMULATION RESULTS

Simulations are performed to determine the potential impact on fuel economy and trip time of a single vehicle when future traffic signal information is predictively used within the adaptive cruise control system of the vehicle. Future research will extend the simulations to the case of multiple equipped vehicles and their impact on the flow and fuel economy of mixed traffic.

The simulations are ran first with the Predictive Cruise Control (PCC) off which serves as a baseline for comparison and then with PCC on during which advanced information of the lights phasing and timing is available. The comparison baseline is a vehicle without advanced access to signal phasing and timing information. For a fair comparison, the baseline vehicle is assumed to operate in adaptive cruise control mode as well [1] . The baseline vehicle tracks a target velocity using the MPC strategy explained in section 2.2. Its controller minimizes (5) subject to the same model equation (4) and inequality constraints (6), (7), and (8). However the target velocity v_{target} is always equal to the driver set speed for the base-line vehicle. The need for a timely stop at red light is enforced through the constraint (8) and by fixing x_{i+1} to the position of the light as soon as the light turns yellow or if an upcoming light is found to be red (thus no advanced phase and time information).

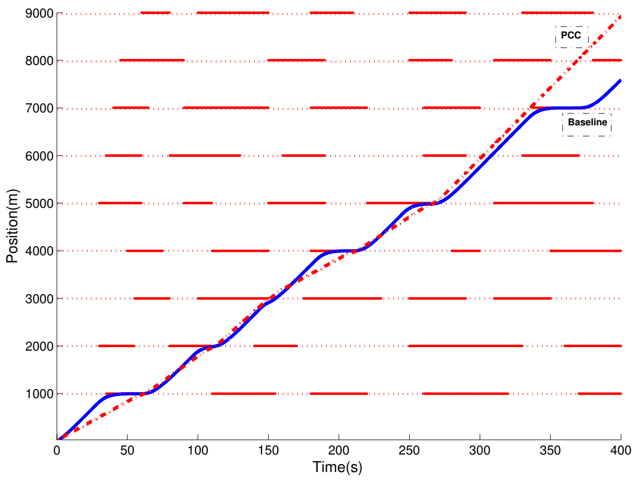

Fig. 4. Trajectory of PCC and baseline vehicles with respect to the red-light map.

A sequence of 8 traffic lights spaced at 1 km intervals is assumed for this simulation study. Because real data for the traffic lights' timing is not available at the time of this simulation, the light timing and phasing is chosen arbitrarily but in a reasonable range. The light timing and phasing is assumed to be fixed and independent of the incoming traffic. Future work can consider situations of synchronized or traffic-actuated lights. Figure 4 summarizes the light timing information. Also on this graph we show the trajectory of PCC and baseline vehicles.

In all simulations the driver set speed is 30 m/s, the maximum speed is $v_{max} = 30$ m/s, and the minimum speed v_{min} is zero. The vehicle mass is assumed to be 1000 kg. The parameters of the supervisory level controller are summarized in Table 1.

Table 1. MPC Parameters

parameter	description	value	units (SI)
T_s	sample time	0.2	s
P	prediction horizon	8	s
N	control horizon	2	s
W_1	penalty weight 1	3000	$(m/s)^{-2}$
W_2	penalty weight 2	150	N^{-2}
α	dynamic gap parameter	0.2	s
β	static gap parameter	1	m
$f^{max}_{acceleration}$	max positive traction	3000	N
$f^{max}_{deceleration}$	max negative traction	6800	N

Figure 5 shows the velocity profile, control inputs, and the distance traveled by the baseline vehicle. Zero portions of the

[1] Adaptive cruise control assumption can be thought of as a systematic mean to model a driver behavior in flowing traffic. In other words the comparison is not limited only to ACC equipped vehicles.

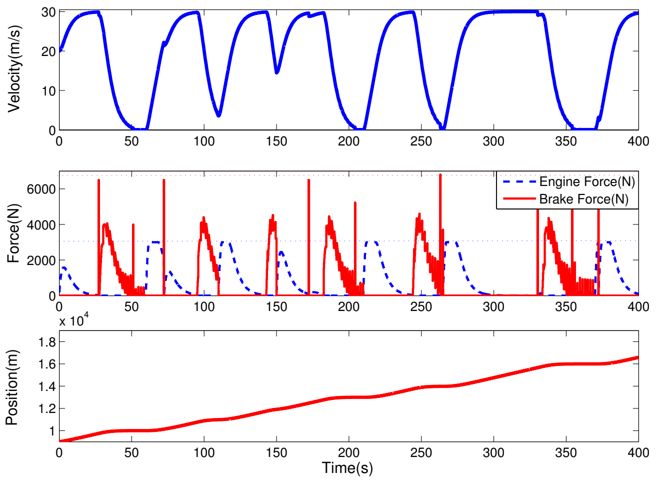

Fig. 5. Velocity, Control Inputs and the Position for a vehicle without advanced signal information.

velocity profile show that the vehicle stops at multiple red signals. In a period of 400 seconds, the vehicle travels the distance of 7.59 km and passes 8 lights. The average velocity is therefore 18.97 m/s. During the same time and with the same initial conditions the PCC-equipped vehicle can travel a much longer distance of 8.92 km as shown in Figure 6. By predictive use of signal information, the PCC vehicle schedules its velocity to a timely arrival at a green light whenever possible. As a result the average velocity is 22.30 m/s which is a 17.5 percent improvement over the baseline vehicle. During the simulation the minimum and maximum speed constraints as well as all other constraints are met.

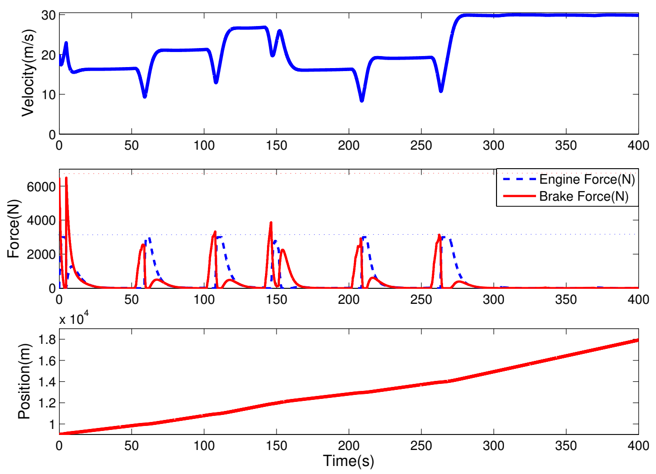

Fig. 6. Velocity, Control Inputs and the Position for a vehicle with advanced signal information.

Next an economy-sized passenger vehicle with the mass of 1000 kg and 5-speed automatic transmission was selected for evaluating the fuel economy and CO_2 emissions in PSAT. The vehicle has a 1.7 L 4-cylinder gasoline engine with the maximum power of 115 hp. The detailed vehicle modeled is assembled in PSAT v6.2. The velocity profiles shown in the first subplot of Figures 5 and 6 are fed as inputs to the PSAT simulation environment. A driver-model follows this input velocities very closely. Table 2 summarizes the statistics of the resulting velocity and acceleration. The calculated fuel economy and CO_2 emissions are shown in Table 3. The PCC-equipped vehicle uses 59 percent less fuel with 39 percent less CO_2 emissions than the vehicle with the conventional ACC for the same travel time. This is while the PCC vehicle travels a much longer distance.

Table 2. Drive-cycle statistics for PCC and baseline vehicles.

PCC vehicle	Maximum	Average	Standard Deviation	Unit
Speed	29.97	22.30	6.07	m/s
Acceleration	2	0.28	0.59	m/s^2
Deceleration	-3.23	-0.35	0.65	m/s^2
Baseline vehicle c	Maximum	Average	Standard Deviation	Unit
Speed	30	18.97	11.20	m/s
Acceleration	2.00	0.64	0.72	m/s^2
Deceleration	-2.79	-1.23	0.90	m/s^2

Table 3. PSAT simulation results for an economy-size vehicle.

Value	PCC	Baseline
Fuel Economy (miles/gallon)	30.00	18.77
CO_2 Emissions(g/mile)	292	480

These preliminary results are encouraging. The scenario of many consecutive traffic lights might be uncharacteristic of rural routes, but quite common in city and suburban driving. Of course the gain from PCC depends on timing and phasing of traffic lights and the distance between them which will be explored in more depth in the next step of this research. Via multi-vehicle simulations, our future research will also investigate how PCC-equipped vehicles impact the fuel economy, emissions, and flow of mixed traffic.

4. CONCLUSION

The Predictive Cruise Control (PCC) concept proposed in this paper shows the potential to reduce fuel use and trip time of future vehicles by utilizing preview information of traffic signal timing and phase. More specifically in one simulated single-vehicle scenario the trip time was reduced by 17.5 percent with 59 percent less fuel when the signal information was predictively utilized. Communicating the signal state to vehicles has been recently proposed for improving intersection safety. The positive results in this paper demonstrate that signal-to-vehicle communication technology can also help relieve traffic congestion and reduce fuel consumption and greenhouse gas emissions of future vehicles. By this, we hope to encourages further research and innovation towards more intelligent intersection control systems.

From an analytical perspective, formulation of the trip optimization in a model predictive control framework is novel and lends itself well to many traffic-imposed hard constraints. The MPC formulation allows systematic extension of this work to the case with multiple vehicles in mixed traffic which is our plan for future work.

REFERENCES

Bemporad, A. (2006). Model predictive control design: New trends and tools. In *Proceedings of the IEEE Conference on Decision and Control*, 6678–6683.

Brockfeld, E., Barlovic, R., Schadschneider, A., and Schreckenberg, M. (2001). Optimizing traffic lights in a cellular automaton model for city traffic. *Physical Review E.*, 64(5), 056132. doi:10.1103/PhysRevE.64.056132.

Chan, C.Y. and Bougler, B. (2005). Evaluation of cooperative roadside and vehicle-based data collection for assessing intersection conflicts. In *Proceedins of IEEE Intelligent Vehicles Symposium*, 165–170.

Dresner, K. and Stone, P. (2008). A multiagent approach to autonomous intersection management. *Journal of Artificial Intelligence Research*, 31, 591–656.

G. C. Goodwin, M.M.S. and Dona, J.A. (2005). *Constrained Control and Estimation*. Springer.

Gershenson, C. (2005). Self-organizing traffic lights. *Complex Systems*, 16(1), 29–53.

Hoogendoom, S. and Bovy, P. (2001). State-of-the-art of vehicular traffic flow modelling. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 215(4), 283–303.

Huang, D.W. and Huang, W.N. (2003). Traffic signal synchronization. *Physical Review E.*, 67(5), 056124.

Maciejowski, J.M. (2002). *Predictive Control with Constraints*. Prentice Hall.

Nagel, K. (1996). Particle hopping models and traffic flow theory. *Physical Review E.*, 53(5), 4655.

Nagel, K., Wagner, P., and Woesler, R. (2003). Still flowing: Approaches to traffic flow and traffic jam modeling. *Operations Research*, 51(5), 681–710.

NTSRC (2008). National Traffic Signal Report Card. `http://www.ite.org/REPORTCARD/`.

PSAT (2002). Powertrain System Analysis Toolkit. `http://www.transportation.anl.gov/software/PSAT/index.html`.

Sengupta, R., Rezaei, S., Shladover, S., Cody, D., Dickey, S., and Krishnan, H. (2007). Cooperative collision warning systems: Concept definition and experimental implementation. *Journal of Intelligent Transportation Systems: Technology, Planning, and Operations*, 11(3), 143–155.

USDT (2007). The Intelligent Transportation Systems for Traffic Signal Control Deployment Benefits and Lessons Learned. `http://www.its.dot.gov/jpodocs/repts_te/14321_files/a1019-tsc_digital_n3.pdf`.

Vahidi, A. and Eskandarian, A. (2003). Research advances in intelligent collision avoidance and adaptive cruise control. *IEEE Transactions on Intelligent Transportation Systems*, 4(3), 143–153.

VanMiddlesworth, M., Dresner, K., and Stone, P. (2008). Replacing the stop sign: unmanaged intersection control for autonomous vehicles. In *The Fifth Workshop on Agents in Traffic and Transportation Multiagent Systems*, 94–101. Estroil, Portugal.