

# Simultaneous Mass and Time-Varying Grade Estimation for Heavy-Duty Vehicles

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## Abstract

In this paper two different approaches are proposed for simultaneous mass and grade estimation for Heavy-Duty Vehicles. In the first method an observer is used which can estimate mass and time-varying grade given their feasible range. The second approach is a recursive time-varying least square method with forgetting. Inclusion of multiple forgetting factors makes the algorithm suitable for simultaneous estimation of a constant and a fast time-varying parameter. Accurate mass estimation and good tracking of time-varying grade is demonstrated in simulations.

## 1 Introduction

Reliable on-line vehicle parameter estimation is important for reduced emissions, increased fuel efficiency and enhanced safety and driveability of Heavy-Duty Vehicles (HDV). Unlike passenger cars, an HDV's mass can vary significantly from trip to trip. Mild grades can be serious loadings for HDVs. An anti-lock brake controller relies on an estimate of mass and road grade for calculating vehicle's forward speed which is necessary for the calculation of wheel slip. In longitudinal control of platoons of mixed vehicles, knowledge of the participating vehicle mass and road grade is necessary for avoiding issuing infeasible acceleration and braking commands [1]. Moreover, mass estimation is essential to the engine control unit (ECU) for reduced emission, and to transmission control for reduced gear hunting. The closed loop experiments performed by Yanakiev et al. [2] indicate that the longitudinal controllers with fixed gains have limited capability in handling large parameter variations of an HDV. Therefore it is necessary to use an adaptive control approach with an implicit or explicit online estimation scheme for estimation of unknown vehicle parameters [1, 3, 4].

Examples of adaptive controllers for vehicle control applications can be found in the work by Liubakka et al. [5], Ioannou et al. [6], and Oda et al [7]. Yanakiev et al. [8, 9] developed an adaptive controller for longitudinal control of an HDV using direct adaptation of PIQ controller gains. Recently, Druzhinina et al. [10] have developed an adaptive control scheme for the longitudinal control of HDV's during braking. Within this scheme they provided simultaneous mass and road grade estimation. They demonstrated convergence in estimates for constant mass and piecewise constant road grade. This method is an indirect estimation method since estimation is achieved in closed-loop and as a by-product of a control scheme.

As HDV automation is increasing, there are more control functions that could benefit from on-line estimation of the vehicle mass and road grade. Moreover, many times estimates independent of a con-

troller are required. In other words a direct estimation scheme is more appealing. In an HDV's electronically controlled powertrain, direct estimation of mass can be accomplished using two primary methods: (a) processing acceleration data [1], (b) processing acceleration or deceleration data during a gear shift [11]. In both of these methods, changes in road grade can cause poor mass estimation by biasing the driveline torque that is available for acceleration or deceleration. Therefore, road grade variations combined with uncertainties in vehicle mass, present strong challenges for vehicle parameter estimation and control. One remedy to the problem is to use some kind of sensor to estimate grade and then use a parameter estimation technique to estimate mass. In [12] using an on-board accelerometer is proposed for grade estimation. The mass is then estimated based on this estimate of the grade. Bae et al. [13] use GPS readings to obtain road elevation and calculate the grade using the measured elevations. With the grade known, they estimate the mass with a simple least square method based on a simple longitudinal dynamics equation.

A model-based method, instead of a sensor-based scheme, can provide a cheaper alternative for simultaneous estimation of mass and grade or it can be used along with a sensor-based scheme to provide some redundancy. However, the fact that grade is time-varying demands an estimation method which is dynamic in nature and can estimate parameters as well as keep track of their variations.

In this paper we propose two independent methods for simultaneous estimation of mass and time-varying grade. Both methods are direct estimation methods which rely on a model of longitudinal dynamics of the vehicle, vehicle speed and engine torque measurements for estimation. Vehicle speed and engine torque are available in an HDV through the J1939 port. The first approach estimates mass and grade with a dynamic observer. The grade and mass are calculated such that they drive the observer state to zero. Good convergence is shown in simulations, when a reasonable range for mass and grade is known a-priori. In the second method we use a recursive least square method with forgetting. In this analysis we show that RLS with a single forgetting factor is not capable of keeping track of a constant mass and a time-varying grade. That motivates a discussion on the possibilities for including different forgetting factors for different parameters. It is shown in simulations that inclusion of multiple forgetting factors, overcomes the difficulties of the single-forgetting scheme. Good convergence in mass and grade estimates are shown in these simulations.

## 2 Vehicle Longitudinal Dynamics

The estimation algorithms introduced in this paper rely on a model of vehicle longitudinal dynamics. The dynamics of engine rotational speed,  $\omega$ , can be described based on the balance between the engine torque on the crankshaft,  $T_e$ , aerodynamic resistance torque,  $T_{aero}$ , road grade and rolling resistance torque,  $T_B$ , and torque due to appli-

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cation of friction brake,  $T_{fb}$ , i.e.,

$$J_t \dot{\omega} = T_e - T_{fb} + T_{\beta} - T_{aero}, \quad (1)$$

where  $J_t = Mr_g^2 + J_e$  is the effective vehicle inertia,  $J_e$  is the engine crankshaft inertia,  $M$  is the total mass of the vehicle, and  $r_g$  is the total gear ratio. The engine speed is proportional to the vehicle speed, i.e.  $v = \omega r_g$  as long as the wheels do not slip. The torque due to aerodynamic resistance is given by

$$T_{aero} = C_q v^2 r_g = C_q r_g^3 \omega^2,$$

in which  $C_q$  is the aerodynamic drag coefficient. The torque due to road grade ( $\beta$ ) and the coefficient of rolling resistance ( $\mu$ ) is given by

$$T_{\beta} = (-\mu g M \cos \beta + M g \sin \beta) r_g,$$

where  $g$  is the acceleration due to gravity. Here  $\beta = 0$  corresponds to no inclination,  $\beta > 0$  corresponds to a descending grade.

### 3 Method I: Estimation with an observer

As an alternative to parametrization-based approach, dynamic input observer for grade has been proposed in our previous work [14] based on the results in [15]. This observer alone, is not sufficient for our application because it requires the knowledge of the vehicle mass. In this section we propose an extension to the previous method. We augment an information set-based adaptation algorithm of the type studied in [16] which will enable simultaneous mass-grade estimation provided a-priori known lower and upper bounds on grade. These bounds can be obtained from the maximum and minimum allowable grades on a highway,  $\beta_{max}$  and  $\beta_{min}$ , based on construction guidelines [17].

We consider the torque due to unknown grade,  $T_{\beta}$ , as an unknown time-varying disturbance. Thus, the system (1) has the following form:

$$J_t \dot{\omega} = T_e - T_{fb} - C_q r_g^3 \omega^2 + W, \quad (2)$$

where  $W(t) = -r_g \mu g M \cos \beta(t) + r_g M g \sin \beta(t)$  is treated as an unknown function of time which is bounded, together with its time derivative, i.e.,  $|W(t)| \leq L$ ,  $|\dot{W}(t)| \leq \dot{L}$ , for some constant  $L > 0$ ,  $\dot{L} > 0$ .

The dynamic observer for  $W(t)$  can be defined in the following form [14]:

$$\dot{\hat{W}} = \alpha (r_g^2 M + J_e) \omega - \varepsilon, \quad (3)$$

where  $\alpha > 0$  is the observer gain and  $\varepsilon$  is the solution of the following differential equation:

$$\dot{\varepsilon} = -\alpha (-T_e + T_{fb} + C_q r_g^3 \omega^2 - \hat{W}). \quad (4)$$

Denoting the estimation error by  $e = \hat{W} - W$ , and considering the Lyapunov function  $V = \frac{1}{2} e^2$ , it can be shown that  $\dot{V} \leq -\alpha V + \frac{\dot{L}^2}{2\alpha}$ , which implies that the estimation error satisfies

$$|W(t) - \hat{W}(t)| \leq \sqrt{(W(0) - \hat{W}(0))^2 e^{-\alpha t} + \frac{\dot{L}^2}{\alpha^2}} \leq R_{\alpha}(t), \quad (5)$$

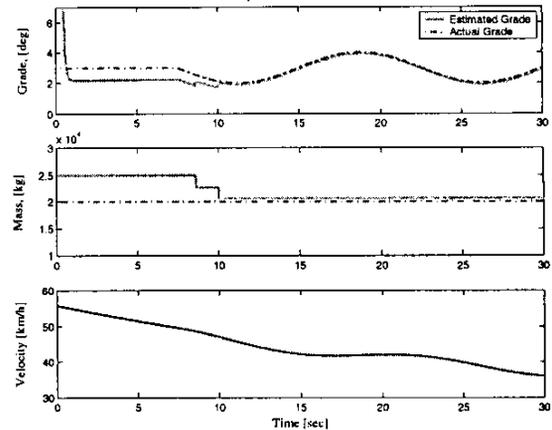
where

$$R_{\alpha}(t) = \max \left\{ \sqrt{(W_{min}(t) - \hat{W}(0))^2 e^{-\alpha t} + \frac{\dot{L}^2}{\alpha^2}}, \sqrt{(W_{max}(t) - \hat{W}(0))^2 e^{-\alpha t} + \frac{\dot{L}^2}{\alpha^2}} \right\},$$

and  $W_{min}$ ,  $W_{max}$  are a-priori known bounds on  $W(t)$  i.e.,

$$W_{min}(t) \leq W(t) \leq W_{max}(t).$$

Thus, if mass  $M$  were known, the estimation error (5) could be made arbitrarily small by amplifying the observer gain  $\alpha$ . The accurate estimate of the road grade  $\beta$  could be then backtracked from  $\hat{W}$ . Since



**Figure 1:** Grade, mass and vehicle speed. The observer-based method is used for estimation.

the mass  $M$  is unknown, we replace it by its estimate  $\hat{M}$  in (3) and generate the adaptation algorithm for  $\hat{M}$  using the approach of [16]. The mass estimation is based on a recursive procedure for generating a sequence of estimates  $\hat{M}(t_k)$  at discrete time instants  $t_k$ . The estimate for the mass  $M$  in the interval  $[t_{k-1}, t_k]$ ,  $\hat{M}(t_{k-1})$ , is adjusted only when the estimate  $W$  goes out of its known boundary (i.e.  $W_{min}$  and  $W_{max}$ ). That is, if the estimate  $\hat{W}(t_k)$  from (3) lies within allowable bounds for given  $\hat{M}(t_{k-1})$ , i.e. satisfies the following inequality

$$W_{min}(t_k) - R_{\alpha}(t_k) \leq \hat{W}(t_k) \leq W_{max}(t_k) + R_{\alpha}(t_k),$$

where

$$W_{min} = r_{min} \hat{M} r_g, \quad r_{min} = -\mu g \cos \beta_{min} + g \sin \beta_{min},$$

$$W_{max} = r_{max} \hat{M} r_g, \quad r_{max} = -\mu g \cos \beta_{max} + g \sin \beta_{max},$$

then  $\hat{M}(t_{k-1})$  is not updated, i.e.,  $\hat{M}(t_{k-1}) = \hat{M}(t_k)$ . Otherwise, if

$$\hat{W}(t_k) > W_{max}(t_k) + R_{\alpha}(t_k),$$

then the current estimate  $\hat{M}(t_{k-1})$  cannot be the true value of the mass  $M$  in the interval  $[t_{k-1}, t_k]$  and, therefore, it has to be updated to make  $\hat{W}(t_k)$  equal to  $W_{max}(t_k) + R_{\alpha}(t_k)$ :

$$\hat{M}(t_k) = \frac{\varepsilon(t_k) + R_{\alpha}(t_k) - \alpha J_e \omega(t_k)}{\omega(t_k) \alpha r_g^2 - r_{max} r_g}. \quad (6)$$

Similarly, if

$$\hat{W}(t_k) < W_{min}(t_k) - R_{\alpha}(t_k),$$

then the current estimate  $\hat{M}(t_{k-1})$  has to be updated as follows:

$$\hat{M}(t_k) = \frac{\varepsilon(t_k) - R_{\alpha}(t_k) - \alpha J_e \omega(t_k)}{\omega(t_k) \alpha r_g^2 - r_{min} r_g}. \quad (7)$$

where  $\varepsilon(t_k)$  is the solution of (4) at time  $t_k$ .

We tested through simulations the performance of the proposed estimation scheme. In particular, we consider the vehicle operation during a braking maneuver on a downhill grade. We assume that the

grade varies sinusoidally between 2 and 4 degrees. The real vehicle mass is 20,000 kg, while an initial guess for  $\hat{M}(t_0)$  is taken as 25,000 kg. The estimates of vehicle mass and road grade tend to vicinity of their true values in 7 seconds, as shown in Figure 1.

#### 4 Method II: Estimation with Recursive Least Square

It was shown in (1) that a linear relationship exists between engine torque, external loads on the vehicle and engine speed. The coefficients in this relationship are functions of mass and grade. Therefore in case of constant mass and grade, a simple recursive least square method can be used to estimate mass and grade online based on measured signals. However in a real scenario when the grade and possibly mass are changing during the course of a trip, the classical least-square method averages all the past information and calculates a constant estimate based on the averaged data which is not the right estimate. For estimation of time-varying parameters of a linear system, a modified version of least square method is generally used in which older information is gradually discarded by decreasing its weight in the estimation as time progresses [18, 19]. This method is called least square with exponential forgetting. In this section we first formulate the time-varying least square problem and then introduce a modification in parameter update scheme which will enable mass and grade estimation when they vary with different rates. Equation (1) can be reorganized to yield:

$$r_g^2 \dot{\omega} = (T_e - T_{fb} - T_{aero} - J_e \dot{\omega}) \frac{1}{M} + \frac{g r_g^2}{\cos \alpha} \sin(\beta - \beta_\mu) \quad (8)$$

where  $\tan \beta_\mu = \mu$ . We can rewrite the equation in the following linear form,

$$y = \phi^T \theta, \quad \phi = [\phi_1 \quad \phi_2]^T, \quad \theta = [\theta_1 \quad \theta_2]^T \quad (9)$$

Where

$$\theta = [\theta_1, \theta_2]^T = \left[ \frac{1}{M}, \sin(\beta - \beta_\mu) \right]^T$$

is the parameter of the model to be determined and

$$y = r_g^2 \dot{\omega}, \quad \phi_1 = T_e - T_{fb} - T_{aero} - J_e \dot{\omega}, \quad \phi_2 = \frac{g r_g^2}{\cos \alpha}$$

can be calculated based on measured signals and known variables.

$\hat{\theta}$  is selected such that it minimizes the least square loss function. When  $\theta$  is time-varying, the loss-function is defined as follows:

$$V(\hat{\theta}, k) = \frac{1}{2} \sum_{i=1}^k \lambda^{k-i} (y(i) - \phi^T(i) \hat{\theta}(k))^2 \quad (10)$$

where  $\lambda$  is a positive parameter smaller than 1 and is called the forgetting factor. It is introduced to discard older information in favor of newer information. This method is a pragmatic approach to capture the time-varying nature of the parameter,  $\theta$ . The problem is called least-square with exponential forgetting and  $\hat{\theta}$  can be calculated recursively as follows. More detailed derivation can be found in books on parameter estimation such as [18]:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L(k) (y(k) - \phi^T(k) \hat{\theta}(k-1)) \quad (11)$$

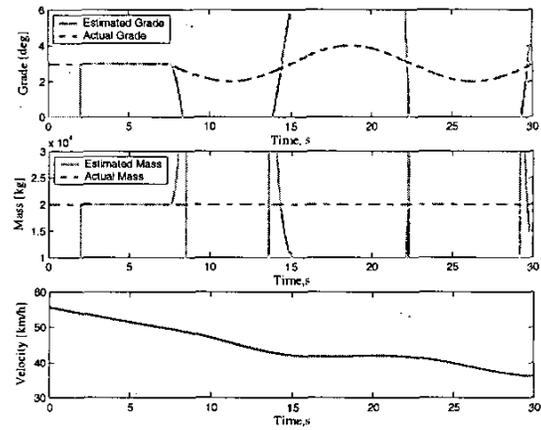
where

$$L(k) = P(k) \phi(k) = P(k-1) \phi(k) \left( \lambda + \phi^T(k) P(k-1) \phi(k) \right)^{-1}$$

and

$$P(k) = \left( I - L(k) \phi^T(k) \right) P(k-1) \frac{1}{\lambda}$$

The least-square method with exponential forgetting, as described above, is suitable for keeping track of the parameters when all vary with similar rates. In our case however, the first parameter,  $\theta_1$ , is only a function of mass and is constant. The second parameter,  $\theta_2$ , only depends on the grade and can vary relatively fast. In simulations, we observed that the described least square with forgetting does not converge when the grade is constantly changing. Figure 2 shows the performance of RLS with a single forgetting factor for sinusoidal variations in grade. The well-known phenomenon of estimator "blow-up" or "wind-up" can be seen during grade changes and errors in both mass and grade estimates become very large. The estimates converge back to the real values only when the grade becomes constant. Here a forgetting factor of 0.9 is chosen. We noticed that reducing the forgetting factor will only worsen the problem. When realistic measurement noise is introduced in the data, the performance is sacrificed even more. In [20, 21, 22] a vector-type forgetting scheme is



**Figure 2:** Poor estimation of mass and grade when RLS with a single forgetting is used. The forgetting factor is 0.99. Smaller forgetting factors worsens the performance.

suggested to account for different rate of changes of different parameters. Yoshitani and Hasegawa [23] have used this vector-type forgetting scheme for parameter estimation in control of strip temperature for the heating furnace. For a self-tuning cruise control Oda et al. [7] used this method for detecting step changes in the parameters of a transfer function. We employ a similar update scheme for the matrix,  $P(k)$ , which bypasses the limitation of a single forgetting factor. In this scheme the covariance matrix  $P$  is updated in the following way:

$$P(k) = \Lambda^{-1} \left( I - L(k) \phi^T(k) \right) P(k-1) \Lambda^{-1} \quad (12)$$

where  $\Lambda = \text{diag}(\lambda_1, \lambda_2)$  and  $\lambda_1$  and  $\lambda_2$  are the forgetting factors for the first and second parameters respectively. Choosing two values for  $\lambda_1$  and  $\lambda_2$  will allow more degrees of freedom in the update of the two entries of  $L(k) = [L_1(k), L_2(k)]$  and enhances the stability of the classical method quite noticeably. Before employing the vector-type forgetting, and to remedy the problems associated with different rates of variations, the authors had formulated a multiple forgetting method which has similarities to and differences from the above-mentioned scheme. It has shown very good convergence and tracking capabilities in simulation and experiments and the way it is formulated makes an intuitive sense. Since it provides some motivation on the concept

of multiple forgetting, we discuss the formulation and the structure of the problem in the next section.

#### 4.1 A Recursive Least Square Scheme with Multiple Forgetting

When working on the particular mass and grade estimation problem, the authors noticed that the difficulties in RLS with single forgetting stem from the following facts: 1. In the standard method it is assumed that the parameters vary with similar rates. 2. In the formulation of the loss-function defined in (10) and subsequently the resulting recursive scheme, the errors due to all parameters are lumped into a single scalar term. So the algorithm has no way to realize if the error is due to one or more parameters. As a result if there is drift in a single parameter, corrections of the same order will be applied to all parameters which results in over-shoot or undershoot in the estimates. If the drift continues for sometime it might cause poor overall performance of the estimator or even the so-called estimator "wind-up" or "blow-up". It is true that we are fundamentally restricted by the fact that the number of existing equations is less than number of parameters which we are estimating, but in a tracking problem we can use our past estimation results more wisely by introducing some kind of "decomposition" in the error due to different parameters. Therefore, our intention is to "separate" the error due to each parameter and subsequently apply a suitable forgetting factor for each. Without loss of generality and for more simple demonstration, we shall assume that there are only two parameters to estimate. We define:

$$V(\hat{\theta}_1(k), \hat{\theta}_2(k), k) = \frac{1}{2} \sum_{i=1}^k \lambda_1^{k-i} (y(i) - \phi_1(i)\hat{\theta}_1(k) - \phi_2(i)\theta_2(i))^2 + \frac{1}{2} \sum_{i=1}^k \lambda_2^{k-i} (y(i) - \phi_1(i)\theta_1(i) - \phi_2(i)\hat{\theta}_2(k))^2 \quad (13)$$

With this definition for the loss function the first term on the right hand side of (13) represents only the error of the step  $k$  due to first parameter estimate,  $\hat{\theta}_1(k)$  and the second term corresponds to the second parameter estimate,  $\hat{\theta}_2(k)$ . Now each of these errors can be discounted by an exclusive forgetting factor. Notice that  $\theta_1(k)$  and  $\theta_2(k)$  are unknown. We will later replace them with their estimates,  $\hat{\theta}_1(k)$  and  $\hat{\theta}_2(k)$ .

Here  $\lambda_1$  and  $\lambda_2$  are forgetting factors for first and second parameters respectively. Incorporating multiple forgetting factors provides more degrees of freedom for tuning the estimator, and as a result, parameters with different rates of variation could possibly be tracked more accurately. The optimal estimates are those that minimize the loss function and are obtained as follows:

$$\frac{\partial V}{\partial \hat{\theta}_1(k)} = 0 \Rightarrow \sum_{i=1}^k \lambda_1^{k-i} (-\phi_1(i)) (y(i) - \phi_1(i)\hat{\theta}_1(k) - \phi_2(i)\theta_2(i)) = 0 \quad (14)$$

Rearranging (14),  $\hat{\theta}_1(k)$  is found to be:

$$\hat{\theta}_1(k) = \left( \sum_{i=1}^k \lambda_1^{k-i} \phi_1(i)^2 \right)^{-1} \left( \sum_{i=1}^k \lambda_1^{k-i} (y(i) - \phi_2(i)\theta_2(i)) \right) \quad (15)$$

Similarly  $\hat{\theta}_2(k)$  will be:

$$\hat{\theta}_2(k) = \left( \sum_{i=1}^k \lambda_2^{k-i} \phi_2(i)^2 \right)^{-1} \left( \sum_{i=1}^k \lambda_2^{k-i} (y(i) - \phi_1(i)\theta_1(i)) \right) \quad (16)$$

For real-time estimation a recursive form is required. With some algebraic manipulations, and similar to derivation of regular RLS formulation [19], the recursive form can be written as follows:

$$\hat{\theta}_1(k) = \hat{\theta}_1(k-1) + L_1(k) (y(k) - \phi_1(k)\hat{\theta}_1(k-1) - \phi_2(k)\theta_2(k)) \quad (17)$$

where

$$L_1(k) = P_1(k-1)\phi_1(k) \left( \lambda_1 + \phi_1^T(k)P_1(k-1)\phi_1(k) \right)^{-1} \\ P_1(k) = \left( I - L_1(k)\phi_1^T(k) \right) P_1(k-1) \frac{1}{\lambda_1}$$

and similarly,

$$\hat{\theta}_2(k) = \hat{\theta}_2(k-1) + L_2(k) (y(k) - \phi_1(k)\theta_1(k) - \phi_2(k)\hat{\theta}_2(k-1)) \quad (18)$$

where

$$L_2(k) = P_2(k-1)\phi_2(k) \left( \lambda_2 + \phi_2^T(k)P_2(k-1)\phi_2(k) \right)^{-1} \\ P_2(k) = \left( I - L_2(k)\phi_2^T(k) \right) P_2(k-1) \frac{1}{\lambda_2}$$

In the two aforementioned equations  $\hat{\theta}_1(k)$ ,  $\hat{\theta}_2(k)$ ,  $\theta_1(k)$ , and  $\theta_2(k)$  are the unknowns. As is customary in similar problems,  $\theta_1(k)$ , and  $\theta_2(k)$  can be replaced with their estimates,  $\hat{\theta}_1(k)$  and  $\hat{\theta}_2(k)$ . Upon substitution for  $\theta_1(k)$  and  $\theta_2(k)$  and rearranging (17) and (18) we obtain:

$$\hat{\theta}_1(k) + L_1(k)\phi_2(k)\hat{\theta}_2(k) = \hat{\theta}_1(k-1) + L_1(k) (y(k) - \phi_1(k)\hat{\theta}_1(k-1)) \quad (19)$$

$$L_2(k)\phi_1(k)\hat{\theta}_1(k) + \hat{\theta}_2(k) = \hat{\theta}_2(k-1) + L_2(k) (y(k) - \phi_2(k)\hat{\theta}_2(k-1)) \quad (20)$$

For which the solution is,

$$\begin{bmatrix} \hat{\theta}_1(k) \\ \hat{\theta}_2(k) \end{bmatrix} = H^{-1} \begin{bmatrix} \hat{\theta}_1(k-1) + L_1(k) (y(k) - \phi_1(k)\hat{\theta}_1(k-1)) \\ \hat{\theta}_2(k-1) + L_2(k) (y(k) - \phi_2(k)\hat{\theta}_2(k-1)) \end{bmatrix} \quad (21)$$

Where  $H$  is

$$\begin{bmatrix} 1 & L_1(k)\phi_2(k) \\ L_2(k)\phi_1(k) & 1 \end{bmatrix}$$

Using the fact that  $P_1$  and  $P_2$  are always positive it can be proved that the determinant of  $H$  is always nonzero and therefore the inverse always exists. With some more mathematical manipulations, (21) can be written in the form of (11):

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L_{new}(k) (y(k) - \phi^T(k)\hat{\theta}(k-1)) \quad (22)$$

where  $L_{new}(k)$  is defined as follows:

$$L_{new}(k) = \frac{1}{1 + \frac{P_1(k-1)\phi_1(k)^2}{\lambda_1} + \frac{P_2(k-1)\phi_2(k)^2}{\lambda_2}} \begin{bmatrix} \frac{P_1(k-1)\phi_1(k)}{\lambda_1} \\ \frac{P_2(k-1)\phi_2(k)}{\lambda_2} \end{bmatrix} \quad (23)$$

The proposed method incorporates different forgetting factors for each parameter. To this end, it does what the vector-type forgetting method does. Eq. (22) is similar in form to the standard update of (11). However the gains of the standard and the proposed form are different. Specifically the former has a cross-term  $P_{12}(k-1)$ , while the latter does not. In other words the covariance matrix of the proposed method is diagonal. This will result in update of the two parameters proportional to  $P_1(k)$  and  $P_2(k)$ .

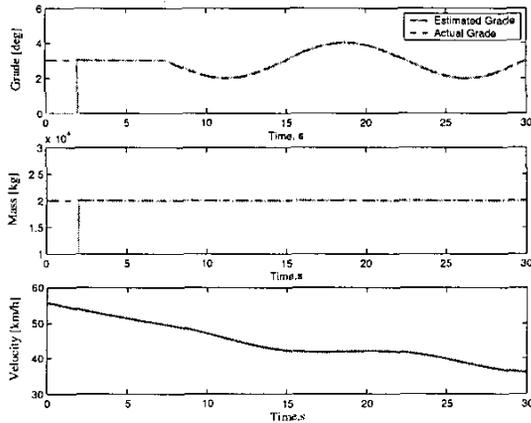
In short, introduction of the loss-function (13) with decomposed errors and different forgetting factors for each have two distinct implications:

- 1) Parameters are updated with different forgetting factors. That is done by scaling the covariances by different forgettings. This is more or less what is done in the RLS with vector-type forgetting as well. However our approach is based on minimization of a loss-function

while the vector-type approach introduces multiple forgetting factors in an ad-hoc fashion.

2) It decouples the updating step of the covariance of different parameters. This is different from standard RLS or RLS with vector-type forgetting. We believe that when the parameters are independent of each other this makes an intuitive sense.

Simulation analysis showed that the performance is similar to the RLS with vector forgetting when similar forgetting factors are used. We tested this algorithm in our application by simulations and with various grade change patterns. Every time initial estimates of mass and grade were calculated using the regular least-square method and based on the first few second batch of data. We assumed that the parameters were constant within this period. Once the initial estimates were obtained, the proposed recursive algorithm was employed for updating the estimates. Figure 3 shows the performance of the esti-



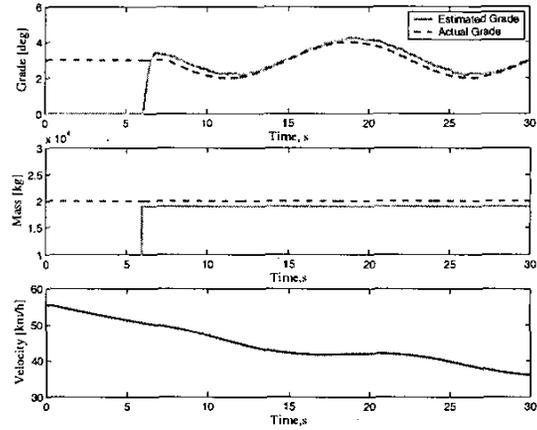
**Figure 3:** Grade and mass estimates versus the actual values. RLS with multiple forgetting factors is used for estimation.

mator when grade has sinusoidal variations. Mass is estimated within 0.04 percent and grade is estimated within 0.5 percent and with very small lag. Even with a much higher speed of variations, the estimator performs reasonably well. In simulation we observed that if the forgetting factors are chosen so that they roughly reflect relative rate of change of parameters, parameter changes are tracked well. In this example forgetting factors of 0.8 and 1.0 are chosen for grade and mass respectively. Unlike estimation with single forgetting, the estimation is very smooth and the estimates converge much faster. Because a forgetting factor of 1.0 is chosen for mass, the mass estimates are not as sensitive to changes in grade. A summary of some other scenarios is shown in Table 1. The results shown in this table are based on numerical data that is not noisy. Simulations with data contaminated by noise show that noise deteriorates the performance of the single forgetting estimation. The multiple forgetting scheme showed much better robustness in presence of noise.

To demonstrate the influence of noise, we added zero mean Gaussian white-noise, generated by a the Simulink Random number generator, to both torque and engine speed measurements. Noise powers of 10 and 0.01 were chosen for engine torque and engine speed signals respectively. The signals are sampled at 50 Hz. To avoid numerical problems in differentiating the noisy engine speed signal, we integrated both sides of (8) and applied the least square method to the resulting equation. Also a bigger batch span was used to get more

**Table 1:** Comparison of the performance of single and multiple forgetting RLS algorithms

Scenario	Single Forgetting	Multiple Forgetting
Constant grade Constant mass	good estimation	good estimation
Step changes in grade Constant mass	overshoots in estimates	good estimation
Linear change of grade Constant mass	bad estimation	good estimation
Sinusoidal change of grade constant mass	bad estimation	good estimation



**Figure 4:** Trajectories of grade and mass estimates versus the actual values when simulated noise is added to measurements.

accurate initial estimate. The results in Figure 4 show that mass and grade are estimated within five and fifteen percent respectively.

#### 4.2 Comparison with a Kalman Filter

When a model of a system with varying parameters is available, a Kalman filter approach can be used to obtain "optimal" estimates for the parameters relying on the model and measurements. In our case such a model does not exist. Mass is known to be constant but variations of grade can not be modelled. An ad hoc estimation approach could be using a Kalman filter with the assumption that parameter variation is described by a random walk process:

$$\theta(k+1) = \theta(k) + v_1(k) \quad E[v_1(k)v_1^T(k)] = R_1(k) \quad (24)$$

The measurement noise is assumed to be Gaussian white:

$$y = \phi^T(k)\theta(k) + v_2(k) \quad E[v_2(k)v_2^T(k)] = R_2(k) \quad (25)$$

The Kalman estimator will resemble equation 11 but with the gains updated as follows [24]:

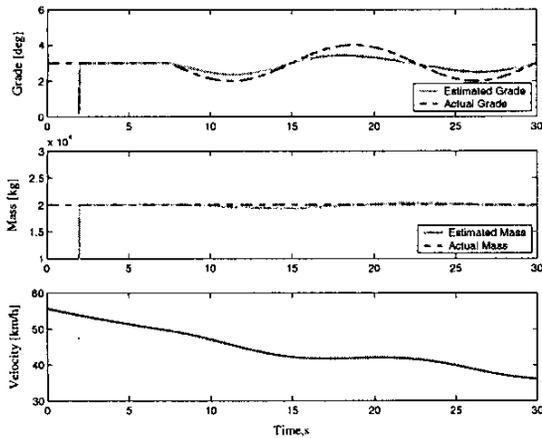
$$L(k) = P(k-1)\phi(k) \left( R_2(k) + \phi^T(k)P(k-1)\phi(k) \right)^{-1}$$

and

$$P(k) = (I - L(k)\phi^T(k))P(k-1) + R_1(k).$$

However due to the difference between the assumed model and the actual variation in parameters, the estimates are usually poor. It was observed in simulations that depending on the scenario, this assumption could cause large deviations from actual parameter values or slow down the convergence. Figure 5 shows estimation results when a Kalman filter was used. The covariance matrix  $R_1$  is chosen to be

diag(0.000001,0.01) as a tuning parameter and  $R_2$  is chosen to be zero assuming no measurement noise. It can be seen that the mass and grade are not estimated as well as previous schemes.



**Figure 5:** Kalman filter estimation of grade and mass versus the actual values.

### 5 Concluding Remarks

Two algorithms for the problem of simultaneous mass-grade estimation for heavy-duty vehicles are proposed. Both methods use engine speed and engine torque readings to estimate mass and time-varying grade. In the first method an observer is designed for estimation of mass and grade. It is shown through a Lyapunov function and also in simulations that the estimates converge to their actual values, given a-priori knowledge of their feasible range. In the second method use of recursive least-square with vector-type forgetting is proposed. We show in simulations that a single forgetting factor cannot estimate parameters with different rates of variation. Ways to incorporate more than one forgetting factor for estimation of multiple parameters with different rates of variation are discussed and the effectiveness of the algorithm with multiple forgetting in estimating a constant mass and time-varying grade is shown with simulations. It is shown that if the chosen forgetting factors reflect relative rate of variation of the parameters, both parameters can be estimated with good accuracy. In the second method no bound for parameter values was assumed while the first method finds the parameters in a pre-specified range. The first method ensures both estimates remain within their feasible range under all driving conditions and even when inputs are not persistently exciting. A robust solution in the second method can be achieved with persistent excitations. To avoid poor results during periods of low excitations, bounds on estimates can be enforced, similar to the first method, to ensure that the estimates remain in their feasible range. While with persistent excitations, the least square approach guarantees convergence of both mass and grade estimates, the observer based approach may allow deviations from true values as long as the estimates remain in their feasible range. Also measurement noise might trigger unwanted updates in estimates. A combination of the two methods can provide the desired redundancy for robust estimation.

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