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Optimal Pacing in a Cycling Time-Trial Considering Cyclist's Fatigue Dynamics

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Abstract— **Optimal pacing of one's effort during a cycling time-trial or even during leisurely long bicycle rides can be a challenge not only for a novice rider but also for the experienced. The rider's level of fatigue, upcoming elevation changes, and varying wind speed all contribute to the problem complexity. This paper formulates the pacing strategy for a bicycle time-trial as an optimal control problem with the goal of finishing in minimum time while considering pedaling force constraints imposed by velocity and rider's fatigue. A phenomenological dynamic model for a rider's fatigue is constructed and the model parameters are estimated using experimental data from road tests. Assuming prior knowledge of the route elevation profile, the optimal control problem is solved using dynamic programming which generates a feedback strategy: Given measured bicycle velocity and the estimated rider's state of fatigue, the solution suggests a pacing strategy that if followed can reduce total travel time. Preliminary simulation results based on experimental data from a century (100 mile) ride show the potentials of the proposed approach.**

I. INTRODUCTION

Proper pacing is a key consideration for professional riders in cycling time-trials and contributes considerably to the end result [1]. Even a casual cyclist may be willing to know how to ride a path to reach a pre-determined destination more efficiently by managing his/her effort level. An even pace (constant power) may be the best strategy for cycling on flat roads with no wind [2]; but elevation and wind almost always vary in a road time-trial. The optimal strategy is then very much a function of future elevation profile, wind speed, and the cyclist's fatigue level [1]. In the past effectiveness of varying rider's power in parallel to changes in road loads has been shown in simulations using heuristic pacing strategies [1], [3].

Pacing in a cycling time-trial can be formulated more systematically, as an optimal control problem and may be solved in real-time given a reliable model of rider's fatigue, cadence-maximal force relationship, route information, and sufficient computing power. With recent advances in mobile device and backend computing, much easier online access to GPS road data, and availability of inexpensive bicycle probe sensors integrated with power meters, it may be possible to compute a near-optimal riding strategy for a rider in real-time. The suggested pace (power or velocity recommendation) can be displayed to the rider on the screen of the mobile device which can be easily mounted on the bicycle.

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An optimal control approach to pacing was recently presented in [4] but only accounted for limited *energy* and *power* resources of a cyclist via a simplified physiological model. We believe that to calculate optimal pacing, one needs to also consider the time-varying upper bound on the rider's pedaling *force* which is a function of the rider's state of muscle fatigue. In fact in many instances, a cyclist exhaustion is due to muscle fatigue as opposed to lack of reserve energy or power. This makes the problem quite challenging since human muscle fatigue process is complicated to accurately model or to quantitatively measure.

Most of the existing work on describing muscle fatigue only consider static loading and do not address dynamic loading effects such as those experienced during a bicycle ride. Some papers such as [5] model the concentration of species and their dynamics during muscle activation to predict the generated force of an individual muscle but require precise measurements for calibration of their many parameters. Others have proposed models capturing only the behavior of muscles in bulk. Such a model in [6] can correctly describe the fatigue activation and recovery process for hand muscles. It remains an open problem, if such lumped models of human fatigue dynamics are effective in calculating an optimal pacing (or exercise) strategy for an individual.

In this paper we first construct a lumped dynamic model for rider's fatigue and recovery based on models proposed in recent literature (Section II). The parameters of the model for an individual rider are estimated using test data from a "maximal-effort" experiment on a steep hill in Section III. The assumption is that this model can estimate the maximum force the rider can produce under similar conditions. Various other factors including: weather, diet, sleep and fatigue from other activities affect the cyclists riding capabilities and are not considered in this paper. In Section IV a minimum time optimal control problem is formulated with the goal of finding the optimal power output (or velocity) of the cyclist, given his fatigue dynamics model and with knowledge of upcoming terrain. Wind influence is not considered in this work nor the opportunity to optimally select gears. Gear switching strategy is captured using a mathematical fit to the rider's data and replicated in simulations. The optimal control problem is solved and simulated for a century (100 mile) ride, results are presented in Section V and analyzed against data from a casual ride on the same route. Section VI concludes with directions for future research that improve applicability of results to real-world cycling.

II. Construction of a Fatigue Dynamics Model

A. Background

As muscles contract and expand, they require a steady supply of Adenosine Triphosphate (ATP), which is commonly referred to as the energy "currency" of cells [7]. The rate of ATP production in a cell is a function of its oxygen supply. When cardiovascular and respiratory systems are unable to maintain supply of oxygen during intervals of increased exercise intensity, muscles cannot produce force solely by aerobic methods. Fortunately, cells have the capacity to produce force and energy anaerobically in absence of oxygen. During anaerobic ATP production, lactic acid accumulates in the muscles, causing the feeling of muscle fatigue and reduces muscle force generation capacity as also shown by Nobel laureate Archibald Hill in his seminal work [8]. While build-up of fatigue is clearly a dynamic process and a function of muscle load history, very few studies present a dynamic model for it. Among those that do, some focus on dynamics of species in a single muscle over a short interval and include a large number of parameters that are difficult to calibrate [5], [9]. Other proposed dynamic models have at least three dynamic states [6], [10], [11] for each muscle group and therefore too complex for the optimal control problems that we propose. The models presented in [12], [13] are single-state; however they consider muscle fatigue and recovery separately and model isometric (static) contractions only.

In this paper we construct a low-dimensional fatigue and recovery model that can capture a person's state of muscle fatigue during dynamic movements. We note that perception of muscle fatigue is also influenced by our motor cortex as recently emphasized in a series of articles by Noakes et al. [14], [15] and also in [16]. This paper focuses mainly on *peripheral fatigue* that is rooted in muscle physiology as opposed to *central fatigue* which is signaled by the motor cortex. Only when subjects exert their maximal force, they are mostly prone to the influence of central fatigue. Many of our test data are at sub-maximal force levels during which, it is known, peripheral muscle fatigue plays the major role [17], [18].

B. A Fatigue Dynamics Model for Isometric Contractions

The maximal force that one can apply, to pedal a bicycle for instance, is a decreasing function of both muscle contraction velocity and the fatigue level. We propose to separate these two effects, by first constructing a model of the maximal force during isometric (zero-velocity) contraction as a function of the fatigue only. This fatigue-based maximal force will be then scaled as a function of contraction velocity, as explained in Section II-C.

In this paper we employ a single-state dynamic model to capture variation of maximum available isometric force, as a function of applied force history for an individual. Lumping together the influence of main physiological factors that contribute to fatigue is justified by findings of [15]. Furthermore, only a rough estimate of a cyclist's "state of fatigue" is sufficient, given the nature of the problem addressed in this paper. Various other factors including weather condition, cyclist's diet, sleep, and fatigue from other activities will also affect riding capabilities and are not considered at this time.

Maximum Voluntary Contraction (MVC) is the maximum isometric force an individual person can generate, when rested. During continuous exertion, maximum produced isometric force $F_{max,iso}$ drops below the MVC level. The decay of $F_{max,iso}$ is an almost-exponential function of time. And its time constant is proportional to the ratio between MVC and a constant applied force F_{iso}, as shown by an interesting survey of several experimental results in [19]. More generally when the applied force F_{iso} varies with time, the experimental survey in [19] supports the following model [12]:

$$\frac{dF_{max,iso}(t)}{dt}|_{fatigue} = -kF_{max,iso}(t)\frac{F_{iso}(t)}{MVC} \quad (1)$$

where k is a constant but different for each person. Not reflected in this model is the process of recovery during rest when $F_{iso} = 0$. In [13] it is shown that recovery of muscle groups is also an almost-exponential increase towards MVC and can be modeled by:

$$\frac{dF_{max,iso}(t)}{dt}|_{recovery} = R(MVC - F_{max,iso}(t)) \quad (2)$$

where R is a constant recovery coefficient. But fatigue and recovery cannot be really separated temporally. As explained in [6], at each time a group of muscle fibers are in activation mode, some are fatigued, and some recovering. In other words, fatigue and recovery take place simultaneously. To reflect this simultaneous occurrence, we propose to linearly combine Equations (1) and (2) and to capture fatigue and recovery in a single-state equation:

$$\frac{dF_{max,iso}(t)}{dt} = -kF_{max,iso}(t)\frac{F_{iso}(t)}{MVC} + R(MVC - F_{max,iso}(t)) \quad (3)$$

According to this model, if one always exerts the maximum force, i.e. when $F_{iso} = F_{max,iso}$, the equation has an equilibrium where the derivative of $F_{max,iso}$ vanishes. This is supported by our preliminary experimental observations. We refer to equilibrium force as the threshold force $F_{th,iso}$:

$$F_{th,iso} = MVC \cdot \frac{R}{2k}(-1 + \sqrt{1 + 4\frac{k}{R}}) \quad (4)$$

This is the force at which fatigue and recovery happen at the same rate and therefore an individual can continue to generate this threshold force for a long time. However, in this case $F_{max,iso}$ has reached its lowest level which means the individual is maximally fatigued. We propose the notion of State of Fatigue SoF:

$$SoF(t) = \frac{MVC - F_{max,iso}(t)}{MVC - F_{th,iso}} \quad (5)$$

which is a normalized index between 0 and 1, observing that $F_{th,iso} \le F_{max,iso} \le MVC$. An SoF of 1 indicates that the subject is maximally fatigued and can only provide the threshold force while Maximum Voluntary Contraction

(MVC) is possible at $SoF = 0$. Note that so far, we assumed zero-velocity muscle contraction. This assumption is not valid during dynamic exercise and adjustments are necessary as described next.

C. Maximal Available Force During Pedaling

It is well-known that the steady-state force that a muscle can produce is a decreasing function of muscle contraction velocity [7]. Back in 1938, Hill showed through experiments on an isolated muscle [20] that this relationship is hyperbolic, that is:

$$(F_{max} + a)(v + b) = (F_{max,iso} + a)b$$

where v is the contraction velocity, F_{max} is maximal isotonic (dynamic) force versus $F_{max,iso}$ which is the isometric maximal force, and a and b are positive constants. While this is now a well-accepted relationship, it only describes the force-velocity relationship for an isolated muscle. For a group of skeletal muscles working together, as for instance in a leg press task or in pedaling, a linear rather than hyperbolic relationship between force and velocity has been observed [21], [22]. Equivalently it has been shown that the maximum power output is a parabolic function of cadence during a cycling exercise [23], which is consistent with a linear force-velocity relationship. One explanation for this observed linear relationship (as opposed to hyperbolic) can be found in [21].

We adopt this linear relationship between maximal isotonic force and velocity (cadence) and we propose to scale down the maximal isometric force $F_{max,iso}$, which is imposed by fatigue, to approximate the available maximal dynamic force F_{max} as follows:

$$F_{max}(t) = F_{max,iso}(t)\left(1 - \frac{\omega(t)}{\omega_{max}}\right) \tag{6}$$

where ω denotes a rider's cadence with its maximum value denoted by ω_{max}. In this paper we assume $\omega_{max} = 20$ rad/sec. There are two underlying assumptions in Eq. (6): i) muscle fatigue does not influence the linearity of force-velocity relationship and ii) contraction velocity does not directly influence the fatigue dynamics as shown in Section II-B. Future experimental work is needed to determine how strong these assumptions are. Most current papers study fatigue under isometric conditions only and therefore do not really address the interactions during dynamic exercise. An interesting study [24] suggests muscle fatigue induced by dynamic exercise will have a larger influence on muscle power output at higher than at lower muscle contraction velocities, which can be a good starting reference for future work in understanding direct interactions between fatigue and contraction velocities.

III. Experiments for Model Calibration

A. Test Setup and the Bicycle Model

In order to develop a model for a cyclists state of fatigue, a Specialized Tarmac Expert bicycle was outfitted with the following equipment: a CycleOps PowerTap power meter, a Garmin Edge 500 cycling computer and a dual velocity and cadence sensor manufactured by Wahoo Fitness. All these devices communicate wirelessly using the ANT+ protocol, which has been widely adopted by the fitness industry. A power meter is a device that calculates the power using strain gauges mounted inside the hub of the bicycles rear wheel, and is used by many cyclists to gauge workout intensity. Another useful metric for cyclists is cadence, which is the number of revolutions per minute of the bicycle's crank arm. The Garmin cycling computer records velocity, power, cadence, GPS location and altitude data at 1 Hz. The data is stored in the cycling computer's memory and can be uploaded to a computer offline. The parameters of the bicycle are listed in Table I and its gear ratios in Table II.

TABLE I

PARAMETERS OF THE TEST BICYCLE.

Parameter	Value	Unit	Source
Bicycle mass (m_b)	9.1	kg	measured
Rider's mass (m_r)	81.6	kg	measured
Rolling Resistance Coeff. (μ)	0.0036	-	Table 6.4 of [25]
Drag Coefficient C_d	0.9	-	Table 5.1 of [25]
Frontal Area (A)	0.4	m^2	Table 5.1 of [25]
Wheel radius (r_w)	0.35	m	measured
Crank arm length (l_c)	0.1725	m	measured
Gearbox efficiency (η_g)	0.95	-	Table 9.4 of [25]

TABLE II

GEAR RATIOS AND TEETH NUMBERS FOR THE TEST BICYCLE.

chain ring	cogs 11	12	13	14	15	17	19	21	23	25
53	4.82	4.42	4.08	3.79	3.53	3.12	2.79	2.52	2.30	2.12
39	3.55	3.25	3.00	2.79	2.60	2.29	2.05	1.86	1.70	1.56

The following longitudinal model is assumed for the bicycle based on Newton's second law, and neglecting the inertial effect of rotating wheels[1]:

$$m_t \dot{v} = \frac{\eta_g}{r_g} \frac{l_c}{r_w} F_{rider} - \frac{1}{2} C_d \rho A v^2 - m_t g \left(\mu \cos(\theta) + \sin(\theta)\right) - F_b, \tag{7}$$

where variable names are those defined in Table I and r_g is the selection of the gear ratio from values in Table II. Additionally $m_t = m_b + m_r$ is the total mass, g=9.81 m/s^2 is gravitational acceleration, ρ is density of air and assumed to be invariant at all elevations. The road slope is θ with positive/negative sign denoting uphill/downhill slopes. The rider force exerted on the the pedal is F_{rider} and total friction braking force on wheels is F_b. In all our experiments, the cyclist rode with shoes clipped in, which to some extent justifies our assumption in Eq. (7) that flow of power to the wheel is continuous (as opposed to intermittent).

The power meter estimates the rider power P_{wheel} at the wheel which allows us to calculate the pedaling force estimate, $\hat{F}_{rider}$, as follows:

$$\hat{F}_{rider} = \frac{\hat{r}_g}{\eta_g} \frac{r_w}{l_c} \frac{P_{wheel}}{v} \tag{8}$$

[1]The effective inertia is $m_e = m_t + 2\frac{I_w}{r_w}$ where I_w is the inertia of each wheel. Here we have assumed that wheel inertial effect is negligible.

where $\hat{r}_g$ is the estimated gear ratio and is obtained by division of wheel rotational speed $\omega_{wheel} = \frac{v}{r_w}$ by cadence ω_{pedal}. The estimated gear ratio often had several erroneous spikes, in particular during coasting which were filtered by a comparison to values in Table II.

B. Parameter Estimation

The fatigue model constructed in Section II has three unknown parameters k, R, and MVC. To obtain the three parameters, we arranged a "Maximal-Effort" experiment in which a rider climbs up a steep hill 5 times. The hill was chosen such that the rider had to ride at nearly his maximal force during each climb. A series of five exercise trials was conducted on a hill with a moderate gradient (3-6%). The base and the summit of the climb were marked and the cyclist was instructed to begin each test with a rolling start of approximately ten miles per hour (4.5 m/s). Once each trial began, the cyclists goal was to reach the summit as quickly as possible. Between trials, the cyclist was allowed 90 seconds to recover and coast down the hill to the start. The goal was to examine how the cyclist's power and force output decayed during each successive test. The experiment was not controlled for other factors influencing fatigue including: weather, diet, sleep and fatigue from other activities.

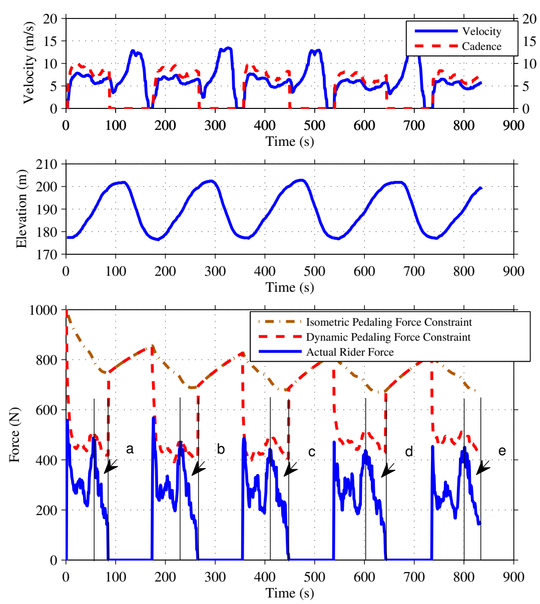

Fig. 1. Rider force from test data and estimated maximum force.

Figure 1 shows the velocity, cadence, elevation, actual rider force, and the estimated isometric and dynamic maximal forces. At the time of this preliminary test, there was no direct way of directly measuring rider's maximal force capacity. But when the rider force was rapidly decaying at time intervals (a)-(e), we hypothesized that he was deeply engaged in generating his maximal force. We formulated an optimization to minimize the difference between the actual and maximal force F_{max}:

$$\min_{k,R,MVC} \sum_{S\in\{a,b,c,d,e\}} \int_S \|F_{max}(t) - F_{rider}(t)\|^2 dt \qquad (9)$$

subject to equality constraints imposed by Eq. (3) and (6). We also imposed 700N$\leqslant MVC \leqslant$1000N as a reasonable range for the rider's maximum voluntary contraction. Additionally constraints are imposed on values of k and R to limit them between 10 times and 1/10 of values reported in [13] to discard unreasonable solutions. This optimization problem was solved using the *fmincon* nonlinear optimization function in MATLAB with the resulting values of $MVC = 1000$N, $R = 0.0063\text{s}^{-1}$, and $k = 0.0153\text{s}^{-1}$. The estimated maximum force can be seen in Figure 1 as the dashed curve.

IV. Optimal Pacing Problem Formulation

A cyclist has two degrees of freedom in managing road loads: i) choice of his or her effort determines the bicycle speed and ii) gear selection allows managing the balance between one's pedaling force and cadence. The focus of this paper is only on choice of optimal pace (velocity or power); choice of gear is "learned" from our test cyclist data and used in this paper's simulations. Because the gear selection is very much a function of bike's speed and road grade we propose the following linear approximation:

$$r_g(t) \approx a_1 v(t) + a_2\theta(t) + a_3 \qquad (10)$$

and parameters a_i are obtained from a least square fit to the rider's actual data. The estimated gear ratio can be then quantized to the closest values in Table II. We observed that this approximate model mimics, reasonably well, the same rider's gear choices on different days and routes.

The optimal control problem is to systematically calculate a cyclist's pace as a function of time such that some performance objective is optimized. One can think of many different objective functions depending on the exercise goal: maximizing calorie burn, minimizing time, etc. In this paper we focus on a minimum time optimal control problem (time-trial), therefore:

$$\min_{u(t)}\{J = \int_{t_0}^{t_f} dt = \int_{x_0}^{x_f} \frac{dx}{v(t)}\} \qquad (11)$$

where x_0 and x_f denote start and end positions respectively. The decision variable is $u(t)$, here chosen to be $F_{rider}(t)$ which influences the velocity through Eq. (7)[2]. Several pointwise-in-time constraints are also present and enforced:

$$\begin{array}{ll} \text{constant power limit:} & 0 \leqslant P_{rider}(t) \leqslant P_{max} \\ \text{force limit from Eq.(3),(6):} & 0 \leqslant F_{rider}(t) \leqslant F_{max}(t) \\ \text{only if braking applied:} & 0 \leq F_b \\ SoF \text{ limit by definition:} & 0 \leqslant SoF(t) \leqslant 1 \\ \text{reasonable velocity range:} & 0 \leqslant v(t) \leqslant v_{max} \end{array} \qquad (12)$$

Next, after showing test data from a century ride, we evaluate a simulated rider's performance when optimally paced at the same century ride.

[2]One can also use rider's power instead of force as the decision variable; the two are not independent given that gear selection is imposed.

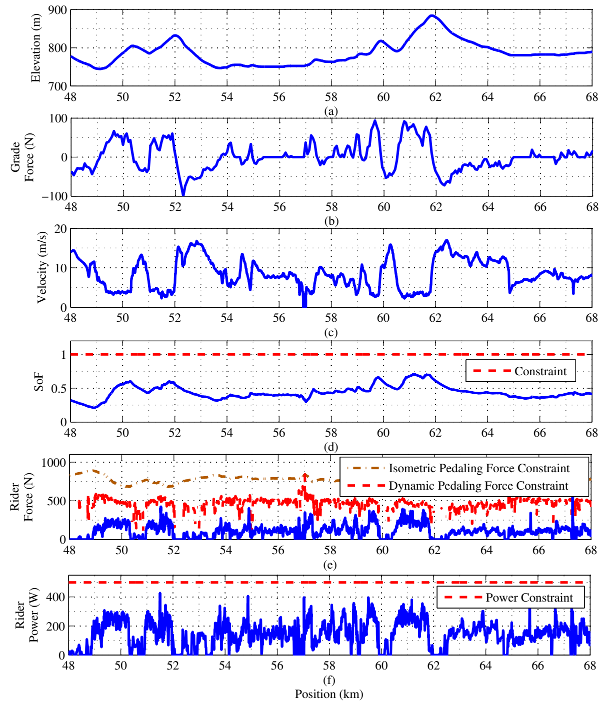

Fig. 2. Actual cyclist data from part of a century ride. The rider had been instructed to ride conservatively and not at full capacity in order to finish.

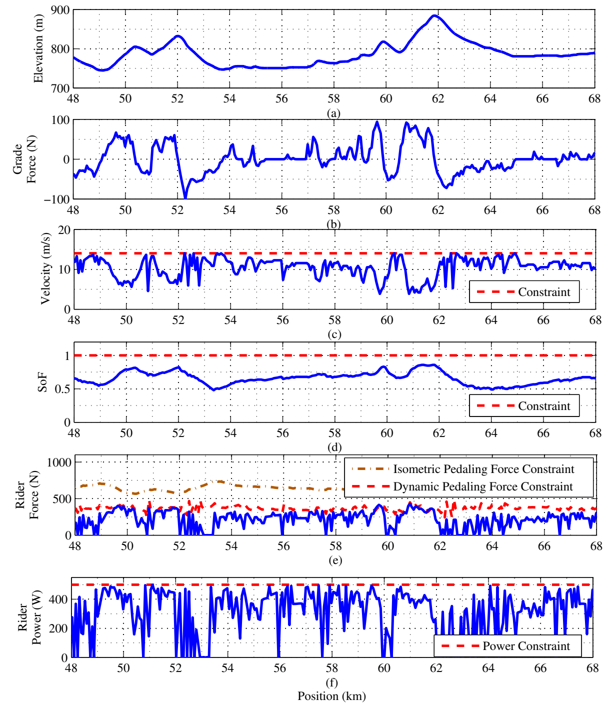

Fig. 3. Simulated cyclist results along the same path as in Figure 2. The simulated cyclist was “optimally paced”.

V. RESULTS

A. Actual Rider Data

Century data was collected at the third annual Blue Ridge Breakaway on August 18, 2012. This was by the same rider for whom the fatigue model had been calibrated and with the bike and equipment described in Section III. The 100-mile (≈168 km) ride began in Waynesville, North Carolina, and included approximately 10,000 feet of climbing. The elevation on the parkway reached 6000 feet above sea level, while the elevation of Waynesville, NC and Clemson, SC are 2,700 feet and 750 feet above sea level, respectively. Since this was an endurance event, the rider was instructed to ride very conservatively in order to finish.

Figure 2 shows a part of the data gathered during this century ride. Also shown in this Figure are maximum force and state of fatigue calculated based on actual pedaling effort and using equations (3)-(6). It can be observed that the actual rider's effort never reaches the maximum force and fatigue limits. This is in part because the rider had had limited knowledge of the route in the form of a small cue card provided at the start. As a result, the rider rode conservatively as supported by the power curve in subplot (f). Additionally we note that the current fatigue model may be a bit optimistic in predicting pedaling capacity.

B. Simulated Rider Results

The optimal control problem of Section IV was solved numerically, using the method of Dynamic Programming (DP), and for the entire century route. The fatigue model and bicycle parameters remain the same as before. The maximum rider power P_{max} was chosen at 500 Watts. This choice was based on actual rider's experience that he could maintain this level of power for periods of two minutes or less. The velocity is upper bounded to $15m/s$. A $(n_v = 40) \times (n_{SoF} = 210)$ grid was generated for the two states of velocity and fatigue at every 70m position interval along the 168000m route ($n_x = 2400$). This fine grid size for a long ride required 34GB of random-access memory (RAM) which forced us to run the DP computations on Clemson's Palmetto computer cluster with nodes with sufficient memory. The simulation was run on a 2.66GHz Intel Xeon 7542 node which had access to 512GB of RAM. The DP simulation time for the entire century was approximately 11.75 hours. This was a MATLAB implementation and not optimized for computation time. Sparser grids were also tested and generated reasonable results with much lower memory use and shorter computation times.

Figure 3 shows the results for the simulated rider for the same portion of the course as in Figure 2. This is only a short section of the 168 km century in order to illustrate the results more clearly. Because the actual rider was riding a century for the first time and very conservatively, the actual and simulated rider's performance cannot be compared quantitatively. Moreover, the simulated cyclist had complete knowledge of the course and was able to act predictively. As shown in subplot (d) of Fig. 3, the simulated cyclist got closer to the state of fatigue limit at the end of the each climb, whereas the actual cyclist did not push toward this limit partly because he did not know, in advance, the length of the uphill ride. Note also that the simulated cyclist's SoF increases slightly during and after the descent at 63 km while it had remained steady at 0.5 for the actual cyclist. Velocity during most climbs and descents is higher than in Fig. 2. In particular, note that speed is increased during descents.

This could be easily achieved through a more aerodynamic position and pedaling a little harder. Many cyclists often do not push to their limit during downhill rides preferring to coast rather than pedal. However, the rider's power output, while always within a reachable range, may be unrealistically high on average, based on previous riding experiences of the test subject. It is our goal to characterize the rider's power limits more accurately in the near future.

VI. Conclusions

This paper proposed the idea of utilizing an optimal control approach for optimally pacing a cyclist on a time-trial while accounting for body fatigue dynamics. A fatigue dynamics model was constructed and was calibrated using data from a test subject riding an instrumented bicycle. This model determined the time-varying upper bound to a simulated rider's pedaling force in a 168 km (century) ride. With the assumption of prior knowledge of the upcoming terrain, dynamic programming was employed to solve a minimum-time optimal control problem for the entire century and determined the optimal pace. Qualitative comparison of results to data from an actual rider show similar trends, while also provides insight on how performance of the actual rider could be improved.

The results are promising, but they may not still show the full potential of the fatigue model. Sometimes power seems to be a major bottleneck and our near term goal is to construct and evaluate a better model for dynamics of power constraints. The century ride took place on a highway where the road slopes were at most 3-6%. In country roads, commonly traversed by cyclists, the slopes may be a lot more demanding and muscle fatigue may be more of a bottleneck. In such cases, the predictive nature of our fatigue model may provide further predictive insights for pacing.

Currently the computational and memory demand of the dynamic programming implementation is high. However significant reduction in computational load is expected with coarser grid size and perhaps with implementation in the C programming language. Besides, we note that the DP solution can be generated offline on a computing cluster, given information about the rider and the path. The rider can then be coached based on DP optimal results, prior to a race. Alternatively, a look-up table of the pre-calculated DP solution can be employed to provide real-time feedback to the rider based on GPS position, measured velocity, and estimated fatigue level. We plan to develop a mobile phone App to demonstrate feasibility of this approach.

VII. Acknowledgement

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