

Optimal Power Management of an Electric Bicycle based on Terrain Preview and Considering Human Fatigue Dynamics

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Abstract—This paper proposes an optimal control approach to power management and pacing in an electric bicycle ride with the objective of minimizing travel time. We assume prior knowledge of upcoming terrain. Furthermore human pedaling force constraints are estimated by using a phenomenological fatigue dynamics model. Using upcoming terrain information, estimated rider’s state of fatigue (*SOF*), measured velocity, cadence, and state of charge (*SOC*) of the battery, the optimal solution to the problem is obtained via a three-state dynamic programming (*DP*) approach. The proposed solution guides the rider by suggesting an optimal reference velocity and also optimally adjusts the electric-human power split. The optimal solution is compared to aggressive and conservative rule-based strategies that we have devised. Simulation results show that travel time can be significantly reduced with the optimal control approach.

I. INTRODUCTION

In an electric bicycle an electric motor (EM), coupled with the wheel, provides riding assistance to the rider. Motor torque augmentation can assist the rider while starting, accelerating, or climbing uphill. The electrical motor can also provide regenerative braking torque. The energy recovered during coasting, braking or riding downhill, can be returned to the energy storage device for future use.

In most electric bicycles the level of electrical assistance is either set proportionally to rider’s pedaling effort or is adjusted directly by the rider via a control unit mounted on the bike. More advanced automatic power split strategies have been proposed recently in [1], [2], and [3]. In [3] the electric motor assists when the human rider is estimated to have lower metabolic efficiency and energy is regenerated back to the battery when human metabolic efficiency is higher. While the approach proposed in [2] and [3] is interesting and useful, it does not consider the pedaling force constraints imposed by human muscle fatigue. Moreover, like other existing literature, [3] is focused more on short events such as power boost during acceleration and therefore only considers instantaneous power demand. During long distance rides, such as in a time-trial, pacing and power-split strategies play important roles and can benefit from road terrain preview. Today with modern portable GPS-enabled devices, one can access the terrain information of a cycling path in advance. Terrain preview can add an anticipatory element to pacing and power-split strategies, helping the cyclist to distribute her/his effort more predictively. For example, before a steep uphill climb, the battery can be charged in anticipation, by

having the cyclist pedal harder. The climb can then be more easily negotiated with electric power assistance.

In this paper we propose to employ optimal control tools to calculate the best pacing and power split strategies in long-distance cycling on an electric bicycle. The pedaling force constraints imposed by rider’s fatigue are considered and we assume that the elevation profile of the cycling path is known in advance. The optimal pacing strategy can be displayed to the rider in the form of a target power (or velocity) profile; the EM will assist or regenerate such that human-electric power matches power demand. It is crucial to have a fairly accurate estimate of state of rider’s fatigue; otherwise, the rider cannot sustain the suggested pace. Furthermore, an accurate model enables the EM to contribute effectively when the rider is fatigued and needs to recuperate. Nevertheless, the human fatigue attributes are difficult to formulate analytically and most previous human fatigue models either focus on human efficiency [3] or consider static loading and do not address dynamic loading effects [4], [5]. In cycling however, the available pedaling force by the rider varies significantly as the state of human fatigue changes. To the authors’ best knowledge, there is no published work that considers both human fatigue and terrain preview in power management of electric bicycles.

In our previous work [6], a lumped dynamic model for rider’s fatigue and recovery was presented based on the models proposed in recent literature [7]. This lumped model of human fatigue dynamics was utilized in calculating an optimal pacing strategy for a conventional bicycle in [6]. An electric bicycle, which is the subject of current paper, provides more versatility in managing road loads due to the extra buffer provided by the battery; at the same time it presents more complexity in finding the optimal riding strategy due to increased number of dynamic states.

In this paper, the lumped dynamic model for rider’s fatigue and recovery is described briefly in Section II-B. The parameters of the fatigue model were estimated in [6] during multiple on-road tests and are not described in this paper. The models of the bicycle and battery are also introduced in Sections II-A and II-C. In Section III, the energy management problem is formulated as an optimal control problem using the terrain information, state of fatigue (*SOF*), velocity, cadence, and state of charge (*SOC*). We employ a dynamic programming approach to solve the optimal control problem where the goal is to minimize total travel time. To meet the optimal power distribution, the optimal human power can be suggested to the cyclist as a target. Even if the cyclist does not perfectly follow the

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recommended power, the strategy tunes its optimal power distribution solution based on the situation. Two rule-based strategies are introduced in Section IV for comparison purposes. Simulating the rule-based strategies as well as the dynamic programming approach shows in Section V that a century (100 mile) time-trial ride is accomplished in shorter time if the cyclist follows the suggestions of the optimal solution. Conclusions are presented in Section VI.

II. THE HUMAN-ELECTRIC-BICYCLE MODEL

A. Bicycle Model

In this paper, a simple longitudinal dynamic model for a medium size electric bicycle is used, in which both the rider and the battery provide the power to the bicycle. Neglecting the inertial effect of rotating wheels, the longitudinal bicycle model can be written as:

$$m_t \frac{dv}{dt} = \frac{\eta_g l_c}{r_g r_w} F_{rider} + \frac{T_{EM}}{r_w} - F_{aero} - F_{road} - F_b \quad (1)$$

where F_{rider} is the rider pedaling force, T_{EM} is the hub motor torque, $F_{aero} = \frac{1}{2} C_d \rho A v^2$ is aerodynamic drag, $F_{road} = m_t g (\mu \cos(\theta) + \sin(\theta))$ is the road force and F_b is the braking force. Bicycle velocity is v , ρ is the density of air, and θ is the road slope. Other parameters are measured or obtained from references and are shown in Table I.

TABLE I
PARAMETERS OF THE TEST BICYCLE.

Parameter	Value	Unit	Source
Bicycle mass (m_b)	19	kg	measured
Rider's mass (m_r)	81.6	kg	measured
Rolling Resistance Coeff. (μ)	0.0032	-	Table 6.4 of [8]
Drag Coefficient C_d	0.9	-	Table 5.1 of [8]
Frontal Area (A)	0.4	m ²	Table 5.1 of [8]
Wheel radius (r_w)	0.35	m	measured
Crank arm length (l_c)	0.1725	m	measured
Gearbox efficiency (η_g)	0.95	-	Table 9.4 of [8]

B. Human Fatigue Model

Muscles require supply of Adenosine Triphosphate to contract and generate power [9], and each muscle cell has a limited capacity for producing force. A fatigued cell can not produce force while it has the ability to recover [7]. Muscle fatigue and recovery is a complex dynamic process with respect to the external loading history. Furthermore, human fatigue modeling during dynamic movements is complex and the existing models are often high-order and highly nonlinear.

In [6], we proposed a low-dimensional human fatigue model, which combines the fatigue and recovery in a single state equation. It first describes the dynamics of the maximal available isometric force $F_{max,iso}$ as a function of applied isometric (static) force at each time, $F_{iso}(t)$:

$$\frac{dF_{max,iso}(t)}{dt} = -kF_{max,iso}(t) \frac{F_{iso}(t)}{MVC} + R(MVC - F_{max,iso}(t)) \quad (2)$$

where MVC is Maximum Voluntary Contraction and represents the maximal static force a person can apply when

rested. Here k is the fatigue coefficient and R is the recovery coefficient. Equation (2) has an equilibrium when $F_{iso} = F_{max,iso}$, and the equilibrium force $F_{th,iso}$ is:

$$F_{th,iso} = MVC \frac{R}{2k} \left(-1 + \sqrt{1 + 4 \frac{k}{R}} \right) \quad (3)$$

This is the force at which fatigue and recovery happen at the same rate and therefore an individual can continue to generate this threshold force for a long time. However, in this case $F_{max,iso}$ has reached its lowest level which means the individual is maximally fatigued. Having defined $F_{max,iso}$ and $F_{th,iso}$, we proposed the notion of State of Fatigue (SOF) as:

$$SOF(t) = \frac{MVC - F_{max,iso}(t)}{MVC - F_{th,iso}} \quad (4)$$

which is an index normalized between 0 and 1 and is an indication of rider's fatigue level; with 1 denoting maximal fatigue. The isometric fatigue model has three unknown parameters: k , R and MVC . These parameters vary for different people. We have repeated a number of "Maximum-Effort" experiments, reported in [6], for a particular cyclist to obtain these parameters. The resulting values are $k = 0.0153s^{-1}$, $R = 0.0063s^{-1}$, and $MVC = 1000N$, which are also used in this paper.

During dynamic movement, available muscle force decreases as muscle contraction velocity increases [9], [10]. In [6] we explained this phenomena in more detail and proposed to scale down the maximal isometric force as follows to approximate the available maximal force F_{max} :

$$F_{max}(t) = F_{max,iso}(t) \left(1 - \frac{\omega(t)}{\omega_{max}} \right) \quad (5)$$

where ω denotes a rider's cadence with its maximum value denoted by ω_{max} . In this paper we assume $\omega_{max} = 20$ rad/sec. We will use $F_{max}(t)$ as an estimate for the maximal pedaling force the cyclist can apply at time t .

C. Battery Model

The charge stored in the battery is represented by its State of Charge (SOC) which is a normalized index between 0 and 1; with 1 denoting full charge. The relationship between battery's charging and discharging power, P_{batt} , and its state of charge is described by the following differential equation [11]:

$$\frac{dSOC(t)}{dt} = -\frac{V_{oc} - \sqrt{V_{oc}^2 - 4R_{batt}P_{batt}}}{2R_{batt}C_{batt}} \quad (6)$$

where V_{oc} , R_{batt} , and C_{batt} are the battery's open-circuit voltage, internal resistance, and capacity respectively. Battery power is positive when it is discharging during power assist and is negative during regeneration when the battery is charging.

III. OPTIMIZATION STRATEGY

In this paper we focus on the objective of minimizing travel time in a long distance ride such as in a time trial. Therefore the objective function to be minimized is:

$$J = \int_{t_0}^{t_f} dt = \int_{x_0}^{x_f} \frac{dx}{v(t)} \quad (7)$$

where x_0 and x_f are starting and ending positions respectively. The goal is to find the two control inputs, battery and human power, that minimize the above objective function. Gear ratio is not treated as a control input; it is estimated following the procedure described in our previous work based on the shifting strategy of a professional rider [6]. Minimization of the objective function is subject to the dynamics of bicycle velocity v , rider's state of fatigue SOF , and battery's state of charge SOC , as described in Section II. There are also several inequality constraints that must be enforced:

$$\begin{aligned} \text{power request constraint:} & \quad \left| \frac{P_{demand} - P_{rider} - P_{EM}}{P_{demand}} \right| < R_{err} \\ \text{constant rider power limit:} & \quad 0 \leq P_{rider}(t) \leq P_{rider_max} \\ \text{rider dynamic force limit:} & \quad 0 \leq F_{rider}(t) \leq F_{max}(t) \\ \text{EM power limit:} & \quad P_{gen_max} \leq P_{EM}(t) \leq P_{mot_max} \\ \text{SOF limit by definition:} & \quad 0 \leq SOF(t) \leq 1 \\ \text{SOC limit by definition:} & \quad 0 \leq SOC(t) \leq 1 \\ \text{reasonable velocity range:} & \quad 0 \leq v(t) \leq v_{max} \\ \text{only if braking applied:} & \quad 0 \leq F_b \end{aligned} \quad (8)$$

where F_{rider} and P_{rider} are the rider's pedaling force and power respectively and F_{max} is the dynamic maximum force that the rider can provide as shown by Equation (5). The output power of the EM is P_{EM} ; P_{gen_max} and P_{mot_max} are the EM's maximum output powers in generating and motoring modes of operation respectively. The rider and the motor combined output power should equal the total power P_{demand} ; the first constraint allows a small relative error R_{err} between them because of the limited resolution of a numerical solution. We also enforce a maximum allowed velocity v_{max} . Constant parameters of inequalities in (8) are given in Table II. It should be noted that the P_{rider_max} is chosen as 500 Watt based on an actual rider's experience. In fact, the rider could sustain this level of power for 2-minute intervals.

TABLE II
CONSTRAINT VALUES

Parameters	Value	Unit
P_{rider_max}	500	Watt
P_{mot_max}	800	Watt
P_{gen_max}	-800	Watt
v_{max}	14	m/s
R_{err}	3%	-

We solve the above optimization problem numerically via dynamic programming which obtains the optimal control and trajectories based on Bellman's principle of optimality. All three states, as well as position, are quantized. Therefore, the

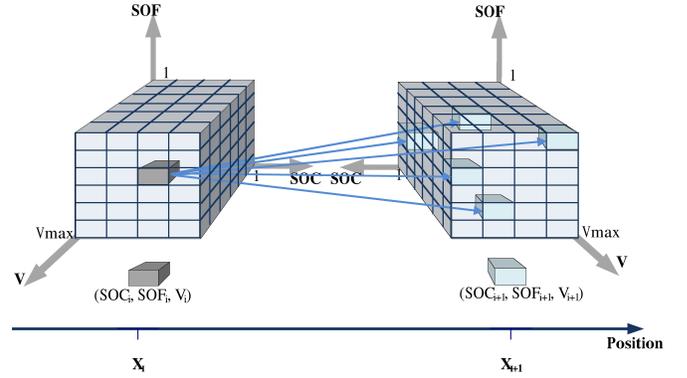


Fig. 1. Schematic of the DP grid and sample transitions

condition of the system at position x_i can be represented as (SOC_i, SOF_i, V_i) . Moving from one quantized position to the next, multiple transitions can be defined as (SOC_i, SOF_i, V_i) to $(SOC_{i+1}, SOF_{i+1}, V_{i+1})$. The number of these transitions depends on the quantization resolution; and an example that contains few transitions is shown in Figure 1. For each transition P_{demand} is calculated from Equation (1), P_{batt} is calculated from Equation (6) considering the efficiency of the electric motor, and P_{rider} is computable using the equations of Section II-B. Those transitions that do not meet the constraints given in (8) are disallowed. For admissible transitions the stage cost is calculated and stored and the process is carried out backward in position.

Due to the presence of three dynamic states, the computational requirement of DP is high and memory usage also grows rapidly with resolution. As a result, it is not feasible to compute the optimal solution in real time and on a handheld device. However, since the terrain information is fully obtained, the computation can be completed offline in advance on a computer cluster, given the parameters of the rider and the bicycle. Note that several other factors such as weather condition, wind speed, and rider's health conditions affect the cyclist's performance and are not considered when solving the optimal pacing problem; therefore the cyclist may deviate from the suggested optimal pace. Using the feedback nature of the DP pre-stored solutions, if the rider fails to follow the optimal suggestions, a new updated pacing strategy is available from the DP table based on the measured or estimated values of the three states.

The functional architecture of the proposed electric bicycle system is shown in Figure 2, where the optimal reference can be either velocity or power shown to the rider through a display. If the rider follows the reference, the optimal objective will be met.

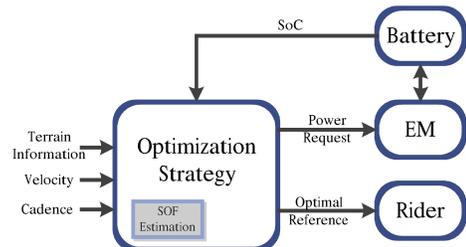


Fig. 2. Architecture of electric bicycle model

IV. RULE-BASED STRATEGIES

In order to evaluate the proposed optimal strategy, two types of rule-based strategies are introduced as described in the following subsections. The feasibility of operation of these strategies as well as their comparison to the optimal strategy are later demonstrated in Section V. Note that both strategies do not have terrain information a-priori, and it is assumed that their output can be suggested to the rider through a display.

A. Aggressive Strategy

The aggressive strategy suggests the rider to provide constant pedaling force to reach the maximum velocity unless the rider is very fatigued. Based on predefined rules, the EM is either used to augment the rider's pedal strokes or regenerate based on the velocity, SOF and SOC . The common sense rules used here for a typical rider are as follows:

For the rider:

- If the bicycle velocity is below a threshold (v_{min}), the rider will provide pedaling force.
- If SOF is above a threshold (SOF_{high}), which means that the rider is very fatigued, she will rest.
- While resting, if SOF is below a threshold (SOF_{low}), which means that the rider is refreshed, the rider stops resting and starts providing force.
- If the bicycle velocity is above a threshold (v_{max}), the rider will rest.

For the EM:

- If the bicycle velocity is below a threshold (v_{min}), the EM will provide power.
- If SOC is below a threshold (SOC_{low}), which means that the battery is almost depleted, and SOF is not higher than SOF_{med} , the EM will start recharging the battery.
- If SOF is above a threshold (SOF_{high}) and SOC is not lower than SOC_{med} , the EM will provide power to help the rider.
- If the bicycle velocity is above a threshold (v_{max}), the EM will be either idling or recharging the battery.

Table III shows the threshold values using in the rules.

TABLE III
THRESHOLD VALUES

Threshold	Value	Threshold	Value
SOF_{low}	0.2	SOC_{low}	0.25
SOF_{med}	0.45	SOC_{med}	0.4
SOF_{high}	0.85	SOC_{high}	0.95
v_{min}	5 m/s	v_{max}	14 m/s

The rules can be illustrated by state flow diagram as shown in Figure 3.

B. Conservative Strategy

Rather than providing a constant pedaling force, a rider following the conservative strategy provides pedaling force proportional to the level of fatigue. The strategy calculates

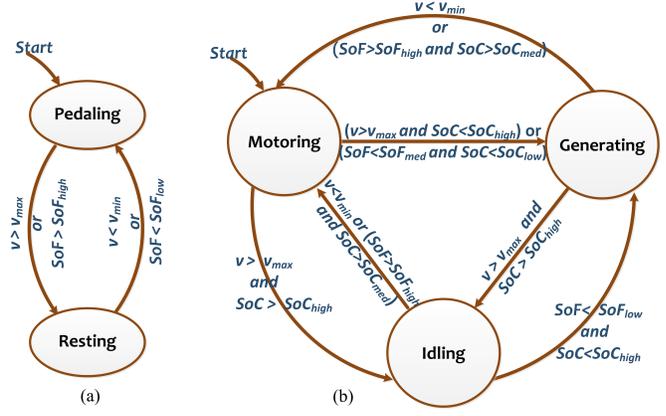


Fig. 3. State flow diagram

the needed power to maximize the velocity, and requests the splits of the power demand from the rider and battery.

A combined backward/forward approach as described in [12] was simplified for bicycle application and was implemented in MATLAB environment to simulate the bicycle with a conservative rider. The simplified steps to do the backward/forward simulation are as follows:

- The required tractive force (F_{demand}) to reach the maximum velocity (v_{max}) is calculated as follows:

$$F_{demand} = m_t \frac{dv}{dt} + F_{aero} + F_{road} \quad (9)$$

- The model of a conservative rider is set up in such a way that he/she provides only a fraction of the maximal force F_{max} based on SOF (see 10). The C_{SOF} scaling factor is determined as depicted in Figure 4(a). In fact, the level of the rider's aggressiveness can be selected by how C_{SOF} is defined versus SOF .

$$F_{rider} = C_{SOF} F_{max} \quad (10)$$

- The force that is requested from EM (F_{EM}) is calculated using two scaling factors as a function of SOC . Similar to the approach in [13], these scaling factors (C_{mot} and C_{gen}) determine the fraction of $P_{mot,max}$ and $P_{gen,max}$ that EM is permitted to provide for motoring or regenerating respectively; and F_{EM} is computed as:

If $F_{demand} \geq 0$ (motoring):

$$F_{EM} = \min \left\{ C_{mot} \frac{P_{mot,max}}{v}, F_{demand} - \frac{\eta_g l_c}{r_g r_w} F_{rider} \right\} \quad (11)$$

else (regenerating):

$$F_{EM} = \max \left\{ C_{gen} \frac{P_{gen,max}}{v}, F_{demand} \right\} \quad (12)$$

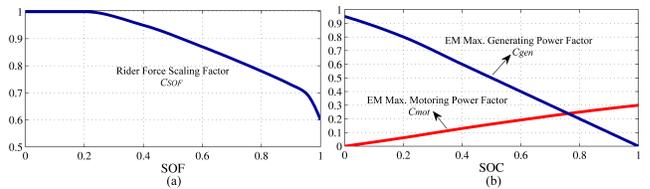


Fig. 4. Scaling factors of (a) the rider force (b) the EM Power Output

where C_{mot} and C_{gen} are selected intuitively for the application in this paper (see Figure 4(b)).

- The rider and EM will provide force based on (10), (11) and (12) to satisfy the demand (F_{demand}). If these two sources of energy cannot keep the bicycle moving then the rider is forced to rest for 2 minutes.

V. SIMULATIONS AND RESULTS

We simulate a 100 mile (century) ride to evaluate the performance of the proposed algorithms. The elevation profile is obtained from a real ride reported in our previous work [6]. The route begins in Waynesville, North Carolina, and includes approximately 10,000 feet of climbing. The specification of the simulated bicycle is given in Table I and the battery is a 26 Volt BionX Li-ion module.

Figure 5 shows the optimal solution calculated via DP and the corresponding constraints. Note that this is only a selected interval of the whole 100 mile ride with several remarkable peaks. It is assumed that the estimated *SoF* value is accurate and the rider is always following the optimal velocity. It can be seen that during each climb the state of fatigue increases and the state of charge of the battery decreases; during descents the battery is recharged and the rider also gets the opportunity to recuperate.

The results of the aggressive rule-based strategy are shown in Figure 6. It can be seen that in order to reach high speeds, the rider puts forward a lot of effort increasing her/his state of fatigue nearly to maximum and is therefore

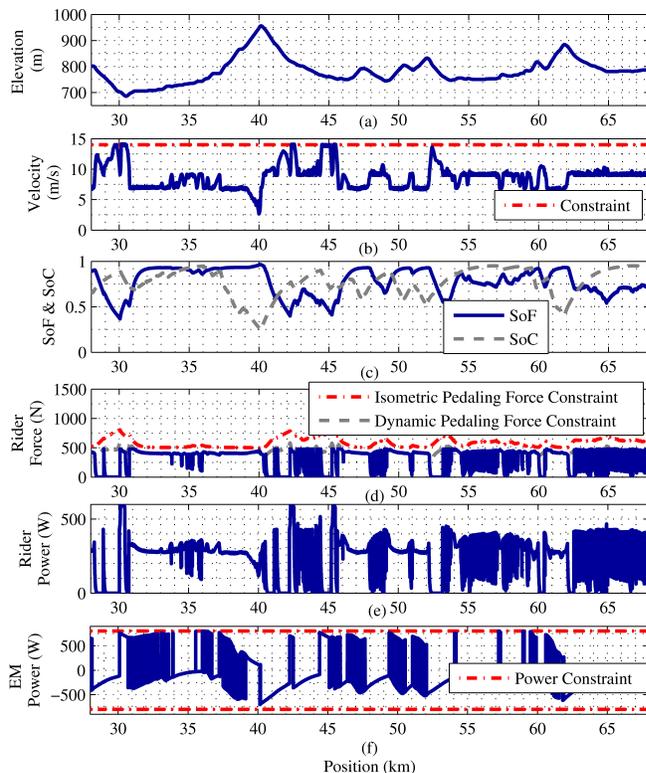


Fig. 6. Simulation results of aggressive rule-based strategy.

“overwhelmed” during “unanticipated” steep climbs. Neither of the two sources can provide enough force during climbs and the velocity drops to a very low value.

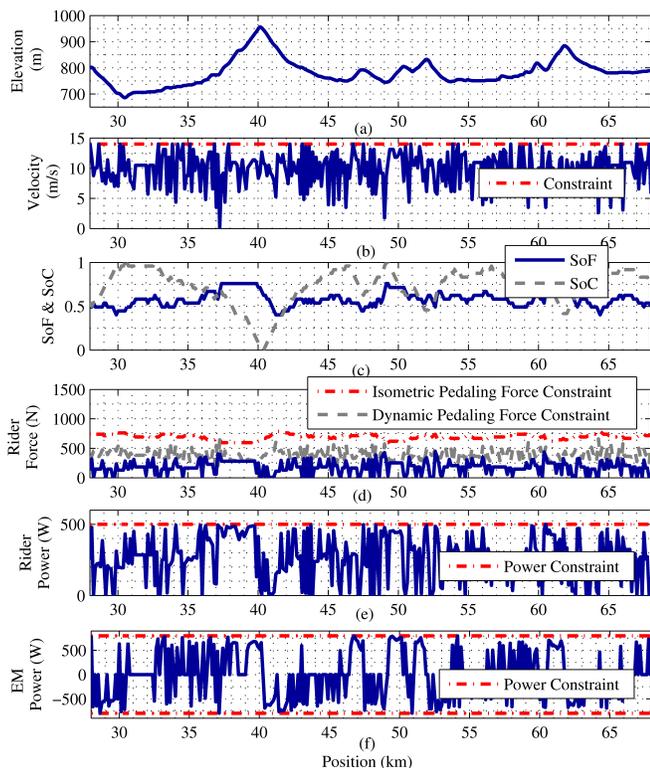


Fig. 5. Simulation results of optimal strategy.

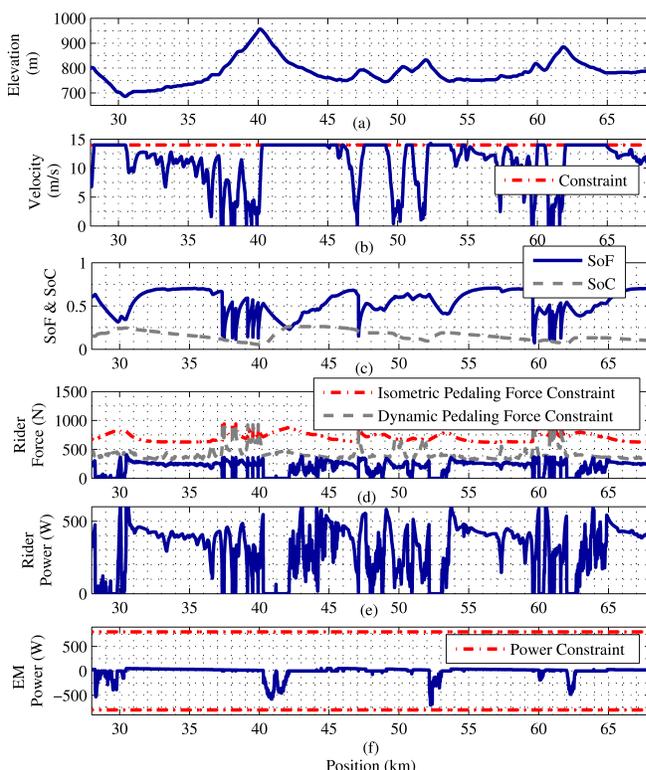


Fig. 7. Simulation results of conservative rule-based strategy.

The conservative rule-based strategy results are shown in Figure 7. According to the *SOF* profile, the rider is not as aggressive as the previous rule-based strategy. As a result, the battery is already almost depleted at 28 km and during the portion shown in Figure 7. The motor is unable to provide much assistance during the depicted portion of the ride; therefore multiple stops occur while climbing the highest hill. During each stop, the rider has to rest for two minutes, which wastes a large amount of time.

Different from rule-based strategies, the purposed optimal strategy always keeps the *SOF* in a reasonable range. In some situations, recharging the battery and recovering the rider's energy are more important than reaching the maximum velocity due to upcoming uphill (e.g. 40km to 45km section). In fact, being aware of the future elevation profile, the optimal solution not only manages the human energy consumption but also reduces the travel time significantly. This is shown by the results in Table IV comparing the ride time of the three strategies for the whole century path (100 miles).

TABLE IV
COMPARISON OF RIDE TIME ACROSS DIFFERENT STRATEGIES

Strategy	Travel Time (hour:minute)
Dynamic Programming	5:05
Aggressive Rule-Based	6:20
Conservative Rule-Based	6:50

The DP was implemented on Clemson's Palmetto computer cluster using MATLAB. In the DP implementation, the position and the three dynamic states are discretized with the following resolution: $dX = 125$ m; $dSOF = 0.042$; $dSOC = 0.024$; $dV = 0.44$ m/s. The DP code was not optimized for computation time; as a result, the simulation took approximately 50 hours on a 2.66GHz Intel Xeon 7542.

VI. CONCLUSIONS

In this paper, the pacing strategy for an electric bicycle is obtained based on an optimal control approach. This strategy accounts for the human fatigue dynamics and enables the cyclists to accomplish a time-trial in minimum possible time. In addition to the fatigue dynamics model, the strategy uses the terrain information of the cycling route to optimally distribute the power demand between two sources of energy (battery and human). It was assumed that the rider's share of power can be requested/suggested through a display or a mobile phone App.

The optimal control problem was solved using dynamic programming. The solution was evaluated on a century time-trial ride using simulations. In addition, two rule-based strategies were also presented with simulation results. The quantitative comparison between the travel times shows that the optimal solution results in the least total travel time. On the other hand, the qualitative comparison between the results of dynamic programming and rule-based strategies demonstrates the benefit of having prior knowledge of the upcoming terrain.

The dynamic programming solver was implemented on a computer cluster; the memory usage was too high for real time implementation. The simulation time was also long. However, the computation can be completed offline prior to the time-trial and stored as a look-up table. As a future work, a mobile phone App can be designed to guide the rider to pace optimally.

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