PREDICTIVE CRUISE CONTROL WITH PROBABILISTIC CONSTRAINTS FOR ECO DRIVING

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ABSTRACT

In predictive adaptive cruise control systems, a major challenge is estimating the future driving pattern of the lead car. This paper proposes an adaptive cruise control system that acts more smoothly and fuel efficiently by utilizing probabilistic information of velocity transition of the front car. The car following problem is formulated in a chance constrained model predictive control framework in which the inter-vehicle gap constraints are enforced probabilistically. The probability distribution of the position of the front car is estimated through a Markov Chain Monte Carlo (MCMC) simulation. The position probability distribution is then utilized to convert the chance constrained MPC problem to a deterministic linear MPC problem. Two case studies with two real driving cycle profiles are presented to show the potential improvement in fuel economy.

1 Introduction

Today's advances in telematics and traffic information technology can enable safer, smarter, and greener driving patterns [1, 2]. Vehicles that rely on real-time traffic information systems and communicate wirelessly to the infrastructure and neighboring vehicles will be able to manoeuver more predictively, enhancing their safety and fuel efficiency. Examples are use of cooperative vehicle wireless communication for lane change, platooning, and obstacle avoidance [3, 4]. In [5, 6] it is shown how advanced telematics can be an enabler for better energy management of hybrid and plug-in hybrid vehicles.

Moreover information from individual vehicle received wirelessly can help estimate the state of macroscopic traffic flow [7] and perhaps even enable higher resolution microscopic prediction of motion of neighboring vehicles. Such information can in turn be used by individual vehicles, running in an adaptive cruise control mode, to reduce their velocity transient and stops and goes [8, 9] leading to improved fuel economy, emissions, and ride comfort.

The adaptive cruise control (ACC) systems in production cars today are intended to reduce driver's workload and improve safety. Different from a conventional cruise control system, an ACC system uses a radar to measure the distance to the front vehicle and can adjust the velocity to maintain a safe distance. While most ACC designs are based on instantaneous measurement of inter-vehicular gap, it is shown that predictive control strategies can result in smoother car following [10–12]. In such predictive adaptive cruise control systems, a major challenge is estimating the future driving pattern of the lead car. Some recent work have proposed using information about upcoming terrain [13, 14] and information about the state of traffic signals [15, 16] to enhance the fuel economy in the adaptive cruise control mode.

To deal with the uncertainty of motion of the lead car, in this paper we employ a chance constrained model predictive control framework for the ACC in which the inter-vehicle distance constraints are imposed probabilistically. The velocity of the lead car is predicted stochastically using a Markov chain assumption; i.e. it is assumed that the velocity at the next sampling time only depends on the current step velocity and is independent of the past. The probability distribution of the velocity of the lead car over the prediction horizon is then found via Monte Carlo simulations. The lead vehicle is assumed to share this information with neighboring vehicles via wireless communication. Knowledge of the velocity distribution of the lead car enables convert-

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ing the chance constrained MPC problem to a deterministic linear MPC problem for which efficient real-time solution methods exist.

The rest part of this paper is organized as follows: Section 2 introduces the car following model and vehicle longitudal dynamics. In Section 3, the model predictive cruise control problem with stochastic constraints is presented. Simulation case studies are given in Section 4 yielding the summary in Section 5.

2 The Plant Model

2.1 Car Following Model

Different methods in the literature model a car following its leading car, e.g. safe-distance models, stimulus-response models, and psycho-spacing models [17]. In this work we assume a safe-distance following model in which the follower maintains a velocity dependent gap with the lead car:

$$D(t) = r(t) - x(t) \geq L_{min} + Tv, \tag{1}$$

where $D(t)$ is the following distance; $r(t)$ and $x(t)$ are the position of the front car and following car respectively; $v = \dot{x}$; L_{min} is the minimum following distance at $v = 0$ and usually is estimated as $L_{min} = \frac{1000}{\rho_{jam}}$, where ρ_{jam} (*vehicles/km*) is the density of traffic jam; and T is the reaction time constant.

2.2 Vehicle Kinematics

The vehicle longitudinal motion is modeled based on simple kinematic relationships and a first order lag between the acceleration command and the actual acceleration:

$$\begin{cases} \dot{x} = v \\ \dot{v} = a \\ \dot{a} = -\frac{1}{\tau}a + \frac{1}{\tau}u(t) \end{cases}, \tag{2}$$

where $u(t)$ is the control input and could be understood as the steady-state acceleration command and τ is a time constant. The state vector is denoted by $Z = [x \quad v \quad a]^T$. The discretized state-space model can be written as :

$$Z(k+1) = A_d Z(k) + B_d u(k), \tag{3}$$

where A_d and B_d are the discretized system matrices.

3 Control Problem Formulation

We use a model predictive approach for the cruise control of the follower. To clarify the notations between the real states and predictive states, we use $Z(i+k|k)$ denoting the i step prediction of Z from time step k. The state dynamics of the following car are predicted by Eq.(3) as a function of control input u as:

$$Z(i+k|k) = \bar{A}(i)Z(k) + \bar{B}(i)U, \tag{4}$$

where $U = [u(0+k|k) \quad \ldots \quad u(i+k|k) \quad \ldots \quad u(N_c - 1 + k|k)]^T$ and N_c is the steps of prediction horizon; $\bar{A}(i) \in R^{3\times 3}$, $\bar{B}(i) \in R^{3\times N_c}$, and they are functions of A_d and B_d.

In model predictive control, the optimization is carried out in a manner of moving horizon. At each time step k, a cost J as a function of predicted states, control inputs, and the reference input is optimized. Meanwhile, the solution needs to satisfy the constraints imposed from systems design requirements. The optimization yields a series of optimal control inputs $u^*(i+k|k), i = 0, 1, \quad \ldots \quad N_c - 1$, but only the first step control input $u^*(0+k|k)$ is applied. Then the same optimization process is repeated with instantaneously measured initial conditions. Note that in our problem, the cost function and the constraints may be the function of the preceding car position $r(t)$, which is uncertain. With assumption of exact knowledge of $r(t)$, we can solve it as a standard MPC problem. In practice, however, $r(t)$ is not exactly known in advance and needs to be predicted. This could be done either in a deterministic way or stochastic way. In this study, we propose to predict $r(t)$ in a stochastic way yielding stochastic MPC. In order to evaluate the proposed methodology, a passive car following model with a feedback control law is also developed as a baseline to compare.

3.1 Cost Function

The goal is to reduce fuel consumption and keep the car-following performance as well. Instead of minimizing the fuel consumption directly, which is nonlinear, we will minimize the acceleration over the prediction horizon. The effectiveness of this cost function to reduce fuel consumption is tested separately by using a commercialized vehicle powertrain simulation software-Powertrain Simulation and Analysis Toolkits(PSAT) developed by Argonne National Laboratory [18]. Moreover, we penalize the car following error at the end of each prediction horizon N_p. This can be achieved by the following cost function:

$$\begin{aligned} J = &\sum_{i=0}^{N_c-1} a^2(i+k|k) \\ &+ q\{r(N_p+k|k) - x(N_p+k|k) - T\dot{x}(N_p+k|k) - L_{\min}\}, \end{aligned} \tag{5}$$

where $\ldots(\ldots + k|k)$ denotes the prediction of a variable at instant k over the horizon; q is the penalty coefficient for car following error. Other form cost function can be found through [10, 11]. Note that we have a quadratic cost for acceleration but only a linear cost for tracking error. The gap constraints introduced later will ensure the linear term remains positive. By defining $P = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ and $R = [1 \quad T \quad 0]$, we have:

$$J = \sum_{i=0}^{N_c-1} (PZ(i+k|k))^T (PZ(i+k|k)) + q\{r(N_p+k|k) - RZ(N_p+k|k) - L_{\min}\} \tag{6}$$

By substituting the Eq.(4) into Eq.(5), the cost function is reformed as:

$$J = U^T \Lambda U + (\Gamma - qR\bar{B}(N_p))U + C_1, \tag{7}$$

where $\Lambda = \sum_{i=0}^{N_c-1} (P\bar{B}(i))^T P\bar{B}(i)$,

$\Gamma = \sum_{i=0}^{N_c-1} 2(P\bar{B}(i))^T P\bar{A}(i)Z(k)$,

$C_1 = \sum_{i=0}^{N_c-1} (P\bar{A}(i)Z(k))^T (P\bar{A}(i)Z(k)) + q(r(N_p+k|k) - RA(N_p)Z(k) - L_{\min})$.

As we can see, Λ is a constant; Γ changes as the real-time measurement $Z(k)$ varies, but it is a constat at each step in the prediction horizon. C_1 is independent of the control input u and as a result, this term can be dropped from the cost function. Note that the coefficient matrices Λ and Γ are independent of the position of the preceding car because a linear cost for car follow error is used in Eq.(5). If the car following error cost has quadratic form such as [19], the cost function will be coupled with the position of the preceding car.

3.2 Constraints

With the cost function in a quadratic form, we now need to consider the constraints imposed from vehicle system limitations and safety requirements such as the maximum and minimum acceleration constraints, non-negative velocity, maximum velocity constraints, and safety car following distance. Specifically, the constraints for the acceleration are formulated as:

$$a_{\min} \leq a(i+k|k) \leq a_{\max}, \tag{8}$$

for $i = 0,1,2...N_p - 1$, where a_{min} and a_{max} are the minimum and maximum allowable acceleration respectively. The control input constraints are set the same as the acceleration constraints i.e. $a_{\min} \leq u(i+k|k) \leq a_{\max}$ for $u(i+k|k)$, $i = 0,1,2...N_c - 1$ considering the control inputs are the steady-state accelerations.

The constraints for the velocity are expressed as:

$$v_{min} \leq v(i+k|k) \leq v_{max}, \tag{9}$$

for $i = 1,2,...N_p$, where v_{min} and v_{max} are minimum and maximum velocity, and v_{min} is set as 0 in our study.

At the same time, the car following distance $D(t)$ in Eq.(1) should be kept in a reasonable range for which the constraints are given by:

$$L_{min} \leq r(i+k|k) - x(i+k|k) - T\dot{x}(i+k|k) \leq L_{\max}, \tag{10}$$

where L_{max} is the maximum offset of the following distance. All these constraints should be satisfied at each sampling time resulting in total constraint number as $2N_c + 6N_p$.

An assumption for the representative of constrains as in Eq.(10) is that $r(i+k|k)$ is known a-priori or can be deterministically predicted. However, in practice, the future position $r(i+k|k)$ is not exactly known. By assuming $r(i+k|k)$ is a random variable, in an alternative way, we can consider the constraints are satisfied at a given probability. For example, the minimum safety distance constraint in Eq.(10) is rewritten as:

$$P\{r(i+k|k) - x(i+k|k) - T\dot{x}(i+k|k) \leq L_{min}\} \leq 1 - \alpha(\cdot), \tag{11}$$

where $\alpha(\cdot)$ is a non-constant value as a function of parameters of prediction step i and the acceleration of the preceding vehicle $a_r(k)$ discussed in Section 4. The solution for Eq.(11) is

$$L_{min} + x(i+k|k) + T\dot{x}(i+k|k) \leq r_k^{1-\alpha}(i), \tag{12}$$

where $r_k^{1-\alpha}(i)$ is a value at which the cumulative distribution function of the position is equal to $1-\alpha$, namely:

$$P\{r(i+k|k) \leq r_k^{1-\alpha}(i)\} = 1 - \alpha$$

Similarly, we can convert the maximum following distance constraints in Eq.(10) as probability constraints. Consequently, the constraints in Eq.(10) are converted as:

$$r_k^{\beta}(i) - L_{\max} \leq x(i+k|k) + T\dot{x}(i+k|k) \leq r_k^{1-\alpha}(i) - L_{min}, \tag{13}$$

where β is a given satisfying probability value for the maximum following offset.

3.3 Stochastic Prediction of the Position $r(i+k|k)$

For the design of model predictive cruise control, one of the challenges is to predict the uncertain future input. For immediate prediction($\approx 0-5s$), this may be done by Kalman filter and autoregressive moving average algorithms (ARMA). For short term prediction($\approx 0-300s$), prediction with event occurrences e.g. break light propagation, vehicle cut-in, local traffic jam, sudden change of speed limits, or even road elevation change, may increase the prediction accuracy. In our discussion, a Markov chain model is developed to predict the uncertainty of the preceding vehicle driving condition in a short horizon and the distribution of the position $r(i+k|k)$ is found through Monte Carlo simulation.

Markov chain Monte Carlo (MCMC) is a powerful means for generating random samples that can be used in computing statistical estimates[20]. [21–23] introduced the Markov chain construction methods for velocity, acceleration, and driver's power demand transition in the application of energy management of hybrid electric vehicles. In our discussion, we only consider the velocity transition excluding the acceleration as a stochastic process with property of Markov chain assuming the state value of next step only depends on its previous step. Another option is to combine the velocity and acceleration as the transition state and construct the Markov chain. However, this would require large training data and is not suitable for a preliminary study.

The transition probability matrix is trained from history driving data as a square matrix $P_{N_v \times N_v}$, where N_v is the number of the discretized point for the velocity. The one step transition probability from any state indexed as m to another state indexed as n is $P_{m,n}$. For multiple e.g. i step transition, the transition matrix is calculated as $P_i = P^i$.

With known state transition probability and current state value, the distribution of the future position in predication horizon is also known, which is obtained from Monte Carlo simulation. Monte Carlo methods are a class of computational algorithms that rely on simulated random sampling to compute their results. Specially, in our problem, different possible realizations, usually referred as scenario tree [24], of the velocity $v_r(i+k|k)$ of preceding car are simulated according to Markov chain transition. The vehicle position is the integral of the velocity and it has the following form in disretized space:

$$r(i+k|k) = r(k) + \frac{1}{2}\sum_{i=0}^{n-1}(v_r(i+k|k) + v_r(i+1+k|k))\Delta t(i),$$

where $\Delta t(i)$ is the sampling time at the ith step prediction.

The distribution of $r(i+k|k)$ is approximated from the simulation of large size of pseudo-random number by Metropolis-Hastings algorithm [20]. Monte carlo simulation takes computation time but this process could be done off line.

3.4 Predictive control with prescient information and passive control with state feedback

3.4.1 Predictive control with precise prescient information With precise prescient position of the car ahead at step k in the prediction horizon, $r(i+k|k)$ in Eq.(10) is assumed to be exactly known. This is non-causal, but the performance with this level preview is expected as the benchmark-the best one that the controller can achieve. The simulation results later coincide this expectation.

3.4.2 Passive following model Without any knowledge of future driving condition, the car behind follows the preceding in a passive way and the safety distance is kept. This is done by a feedback control law $u(t) = k_1\Delta v + k_2\Delta s - k_3\frac{\Delta v^2}{2\Delta s}(\Delta v < 0) + k_4$, where $\Delta v = v_r(t) - v(t)$, $\Delta s = r(t) - x(t) - T\dot{x}(t) - L_{min}$, and k_1, k_2, k_3, k_4 are coefficients as functions of Δv. More details about the design of a feedback law in adaptive cruise control could be found through [25]. We refer this model as a passive following model.

4 Case Study and Result Analysis

4.1 Pre-setting of simulation data

The proposed strategies are evaluated through two driving cycles shown in Fig.1, which were obtained by driving vehicle from Clemson, SC to Highland NC with the same driver. The data was retrieved from Garmin GPS 20x receiver with a sampling time of 1 second. In stochastic MPC, the probability matrix P of the state transition is trained from the first driving cycle and the MPC with the same P is applied to the two cycles. For the final evaluation of different control strategies, instead of the cost function J, fuel economy-miles per gallon (MPG), emission(CO_2), and tracking distance $D(t)$ are used. The value of fuel economy and emission for different control strategies are obtained from simulations in PSAT, which uses complex models to simulate the dynamics of the vehicle powertrain yielding accurate simulation results. The tracking distance indicating the tracking ability is evaluated by the average following distance as well as maximum and minimum distance.

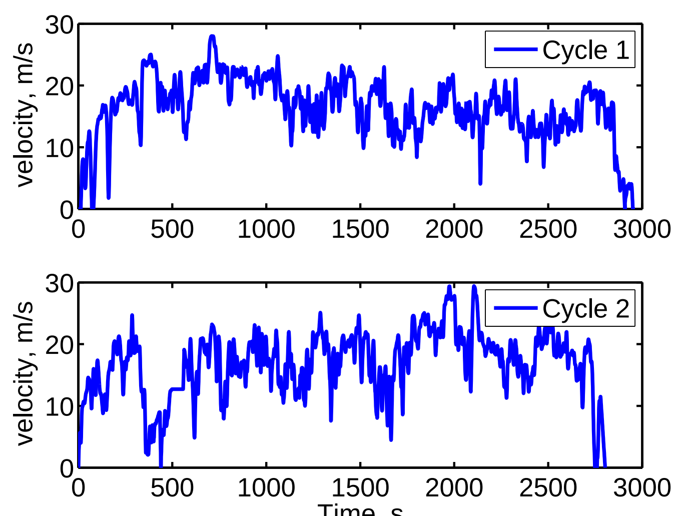

Figure 1. Two cycle profiles from the real driving of the same driver

The probability α of the constraint satisfaction has the form as $\alpha(\cdot) = \lambda_0\lambda_1^{i-1}$, where i is the prediction step and λ_0 as well as λ_1 is set as $\lambda_0 = 0.96$ and $\lambda_1 = 0.96$. As we see later, if we change λ_0 according to the current acceleration, better results will be presented. The setting of β is the same as α, but it is not adjusted by acceleration of the preceding car. All other main parameters used in the simulation are listed as follows:

Table 1. Simulation Parameters.

L_{min}(m)	L_{max}(m)	T(s)	p	N_c	N_p	$\Delta t(i)$(s)	τ(s) [11]
6	200	1.5	0.2	10	10	1	0.45

4.2 Simulation and Analysis

The simulation results of cycle 1 are shown and discussed first followed by that of cycle 2. Fig. 2 showed the optimal vehicle velocity from strategies of passive control (P-control), MPC with stochastic input (MPC-STO), and MPC with prescient knowledge (MPC-PRE) and Table 2 summarized the performance including the fuel economy, emission, and tracking ability. The zoomed figures show that all strategies, like a filter, smooth the velocity profile of the following car, thus the fuel economy of these strategies are better than the preceding car. Among the three, the velocity profile from prescient MPC is the smoothest one yielding a fuel economy improvement of 32% compared with the preceding car. Also, the average tracking distance (34m) is less than the stochastic MPC (48m) and passive following (39m). The excellent performance of MPC with prescient information indicates the improvement margin for other control methods. Even though the fuel economy of stochastic MPC(12.1%) is better than that of passive following (11.4%), the larger tracking distance (48m) degrades its advantages.

By analyzing the design of stochastic MPC, we found the factors leading to large following distance are the ignorance of acceleration in the probability transition matrix P and strict setting for α. One option is to add acceleration as additional state in transition matrix, however, this will yield very large size of P and need more training data. As another alternative method, we can set the value of α by the current acceleration. Specifically, the current acceleration of the reference vehicle $a_r(k)$ is classified as hard deceleration, normal deceleration, normal acceleration, hard acceleration. λ_0 in α is relaxed as $0.96, 0.7, 0.4, 0.2$ depending of the level of acceleration. By doing so, the fuel economy of adjusted MPC-STO is improved up to 15.5%, the average following distance is reduced, and so does the maximum following distance.

In the above simulations for MPC-STO, the velocity transition matrix P is trained from cycle 1 itself. In a further validation of the proposed MPC-STO, we run all the simulations above again in cycle 2 with the transition matrix P from cycle 1 and the results are summarized in Table 3. To fairly compare the performance of MPC-STO and passive following, the parameters in passive following is tuned such that the tracking distance is similar with MPC-STO. By checking the new simulation data, it indicates that all the findings from cycle 1 hold for cycle 2.

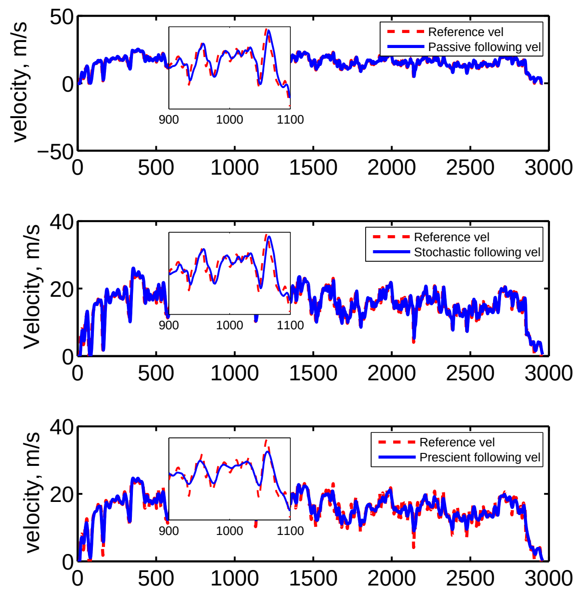

Figure 2. Velocity profiles from different control strategies: the first one is from passive control, the second one from MPC with stochastic prediction, and the third one from MPC with prescient driving condition.

Table 2. Performance comparison of different control methods. 1) P-control: passive control, 2) MPC-PRE: MPC with prescient knowledge of future uncertainty, 3) MPC-STO: MPC with prediction of the velocity as a Markov chain, 4) MPC-STO(adj): MPC-STO adjusted by the acceleration of the lead vehicle.

Control method	MPG (normalized)	CO2 (normalized)	E(D(t)), min(D(t)), max(D(t))(m)
Ref. Veh.	1	1	-
P-Control 1)	1.114	0.89	39.34, 6.00, 71.41
MPC-PRE 2)	1.320	0.76	34.03, 6.00, 55.52
MPC-STO 3)	1.121	0.88	48.56, 6.00, 93.32
MPC-STO(adj) 4)	1.155	0.86	38.30 6.00, 74.34

Table 3. Performance comparison for different control methods for cycle 2.

Control method	MPG (normalized)	CO2 (normalized)	E(D(t)), min(D(t)), max(D(t))(m)
Reference Veh.	1	1	-
P-Control	1.075	0.93	39.42, 6.00, 73.54
MPC-PRE	1.317	0.76	35.40, 6.00, 57.59
MPC-STO(adj)	1.145	0.87	39.40, 6.00, 84.54

Table 4 also summarizes all the information required for different control strategies. Passive control needs the minimum information-current velocity and position of the reference vehicle. MPC-PRE has the most restrictive requirement-the prescient driving condition for prediction horizon at the beginning of each step. MPC-STO requires the history driving information from the reference vehicle.

Table 4. Comparison of different control methods for the requirement of driving information. $r(t)$: reference vehicle position at current time step t, $v_r(t)$: current reference vehicle velocity, $a_r(t)$: current reference vehicle acceleration, $a_r(\tau), \tau < t$: historical time series acceleration, $a_r(\tau), \tau > t$: (prediction of) future acceleration, P: velocity transition matrix for reference vehicle.

Method	Information Requirement					
	$r(t)$	$v_r(t)$	$a_r(t)$	$a_r(\tau)$ $\tau < t$	$v_r(\tau)$ $\tau > t$	P
P-Control	$\surd$	$\surd$	$\times$	$\times$	$\times$	$\times$
MPC-PRE	$\surd$	$\surd$	$\times$	$\times$	$\surd$	$\times$
MPC-STO	$\surd$	$\surd$	$\times$	$\times$	$\times$	$\surd$
MPC-STO(adj)	$\surd$	$\surd$	$\surd$	$\times$	$\times$	$\surd$

We should note that the improvement result strongly depends on the driving cycles and the results may be different for different cycles. Testing the proposed algorithms with the data from the same driver in general needs further investigation. Also, even though we consider the safety constraints in Eq.(10), additional safety constraints could be added if necessary.

5 Conclusion

The adaptive cruise control system of a car was formulated in a model predictive control framework with probabilistic gap constraints. Solution of this problem required prediction of motion of the front car; this was done based on a Markov chain assumption for the front car velocity transition. The probability distribution of the velocity over the prediction horizon was calculated using Monte Carlo simulations and allowed converting the chance constrained MPC problem to a deterministic one. The resulting controller was tested in two case studies in which an improvement in fuel economy as compared to a passive car following model was shown. Future work can consider the velocity and acceleration as the transition states and test with more real driving data.

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