A Two-Stage Lyapunov-Based Estimator for Estimation of Vehicle Mass and Road Grade

Michael L. McIntyre, *Member, IEEE*, Tejas J. Ghotikar, Ardalan Vahidi, Xubin Song, *Member, IEEE*, and Darren M. Dawson, *Senior Member, IEEE*

***Abstract*—This work proposes a two-stage estimation strategy to determine a heavy-duty vehicle's mass and road grade. The estimation strategy uses standard signals available through the vehicle control area network. The first stage of this approach utilizes an adaptive least-squares estimation strategy to determine the vehicle's mass and an estimate for a constant road grade. Due to the time-varying nature of the road grade, a nonlinear estimator that provides a more-accurate estimate of the road grade is then developed. Simulation and experimental results show, under a set of qualifying conditions, that both mass and road grade can be estimated with good accuracy.**

***Index Terms*—Least squares methods, mass estimation, nonlinear estimation, road grade estimation.**

I. Introduction

IN MODERN vehicle control systems, model-based parameter estimation, which uses standard signals available through the vehicle control area network (CAN), is a cheaper alternative to sensor-based estimation. In addition, a model-based parameter estimation approach, along with a sensor-based method, can be used to provide the needed system redundancy. In particular, there is increasing interest in the automotive industry for model-based estimation of vehicle mass and road grade, which can be used in transmission shift scheduling and vehicle longitudinal control, cruise control, hill holding, and traction control. The engine control unit can also utilize an accurate estimate of the road grade for estimating the engine torque, which may reduce the need for inline torque meters [21]. In controlling heavy-duty vehicles (HDVs), an accurate estimate of the mass is even more important due to the trip-to-trip load variation, which is often up to a 500% change from loaded to unloaded [3], [4]. A small change in road grade is serious loading for an HDV and affects its torque response [2]. The limited capabilities of fixed-gain controllers in handling the large parameter variations of HDVs have been shown in the past [25]. The application of adaptive controllers that compensate for vehicle parameter variations has been presented in a number of references, including [6], [8], [10], and [18].

The importance of vehicle mass and road grade estimation has led to considerable research in this area in the past few years. Different sensor- and model-based approaches have been proposed: In sensor-based methods, the time-varying grade is typically estimated using sensors such as an accelerometer [11] and a Global Positioning System receiver [2]; then, a conventional parameter estimation algorithm is utilized for the estimation of mass [22]. In model-based methods, the longitudinal dynamics model of the vehicle, along with data from the vehicle CAN (signals such as engine torque, vehicle speed, engine speed, and gear ratio), is utilized to estimate unknown system parameters. In particular, model-based methods for the simultaneous estimation of vehicle mass and road grade have been proposed in [7], [17], and [19]. Simultaneously estimating the vehicle mass and road grade is a challenging task mainly due to the time-varying nature of the road grade, which complicates the estimation process. To address this problem, Vahidi *et al.* [17], [19] proposed and experimentally tested a recursive least-square mass and road grade estimation strategy that utilizes multiple forgetting factors to reflect a constant mass and time-varying road grade. In a different approach presented in [16], a Lyapunov-based input observer is used to generate an estimate of the road grade, given an initial estimate of the vehicle mass. The mass estimate is adjusted accordingly to ensure that the grade estimate remains between *a priori* known lower and upper bounds. All these estimation techniques require persistent excitation, which may be lacking in typical driving scenarios. To actively generate persistent excitation needed for successful estimation, Winstead and Kolmanovsky [20] and [21] developed a speed cruise control where the engine torque is actively adjusted to improve the persistent excitation condition and the estimation of vehicle mass and road grade.

In this paper, we propose a new two-stage approach for the estimation of vehicle mass and time-varying road grade. In the first stage, a least-squares estimator based on the vehicle longitudinal dynamics model is developed, which determines an estimate of the vehicle mass and a constant estimate of the road grade. Due to the time-varying nature of the road grade (which complicates the estimation of mass), a nonlinear estimator (see [9] and [23]) is then developed to provide a more-accurate estimate of the road grade. Specifically, the

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M. L. McIntyre is with the Department of Engineering, Western Kentucky University, Bowling Green, KY 42101-1082 USA (e-mail: michael.mcintyre1@wku.edu).

T. J. Ghotikar and A. Vahidi are with the Department of Mechanical Engineering, Clemson University, Clemson, SC 29634-0912 USA (e-mail: tghotik@clemson.edu; avahidi@clemson.edu).

X. Song is with the Eaton Corporation, Southfield, MI 48076 USA (e-mail: xubinsong@eaton.com).

D. M. Dawson is with the Department of Electrical and Computer Engineering, Clemson University, Clemson, SC 29634-0915 USA (e-mail: darren.dawson@ces.clemson.edu).

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HDV's mass is first determined by the adaptive least-squares estimator; then, the mass estimate is utilized by the nonlinear estimator to provide an estimate of the time-varying road grade. Experimental results are presented, illustrating the validity of the estimation strategy when the persistence of excitation is guaranteed. A better filtering technique augmented with the least-square estimator for the mass reduces the numerical difficulties that arise due to the noisy acceleration signal and is an improvement over the integration technique proposed in [17] and [19]. In addition, the second-stage nonlinear estimator provides more accurate tracking of the time-varying road grade.

This paper is organized as follows: In Section II, the dynamic system model for a vehicle, along with the required assumptions for this analysis, is defined. In Section III-A, the least-squares estimator is designed, and an analysis is presented, verifying that, under a set of sufficient conditions, the mass and road grade are accurately estimated. In Section III-B, the nonlinear estimator is developed, along with an analysis that verifies that, under a set of sufficient conditions, the road grade is accurately estimated. Simulation and experimental results are presented in Sections IV and V, respectively. Concluding remarks are provided in Section VI.

II. Vehicle System Model

A. Longitudinal Dynamics

The vehicle's longitudinal dynamics model presented in [19] is used as follows:

$$\dot{v} = \frac{(T_e - J_e\dot{\omega}_e - k_{\text{aero}}v^2 r_g)}{r_g}\frac{1}{M} - \frac{g}{\cos(\beta_\mu)}\sin(\beta+\beta_\mu). \tag{1}$$

In (1), $v(t)$ and $\dot{v}(t)$ are the vehicle's longitudinal velocity and acceleration, respectively; $T_e(t)$ is the engine torque; J_e is the engine crankshaft inertia; $\dot{\omega}_e(t)$ is the engine's rotational acceleration; k_{aero} is the aerodynamic drag coefficient; $r_g(t)$ is the wheel radius divided by the total gear ratio; M is the total mass of the vehicle; g is the acceleration due to gravity; $\beta(t)$ is the road grade; and β_μ is defined by $\tan(\beta_\mu) = \mu$, where μ is the coefficient of the rolling resistance. The following assumptions frame the subsequent estimator development and analysis.

Assumption 1: For a realistic road profile, road grade $\beta(t)$ is continuous with respect to the longitudinal position; given that the vehicle's longitudinal velocity $v(t)$ is also a continuous function of time, we infer $\dot{\beta}(t) \in \mathcal{L}_\infty$.

Assumption 2: Signals $v(t)$, $\dot{\omega}_e(t)$, $T_e(t)$, and $r_g(t)$ are assumed to be measurable.

Assumption 3: The values of drag coefficient k_{areo}, engine crankshaft inertia J_e, and the coefficient of rolling resistance μ are known *a priori* and are assumed to be constant with respect to time.

Assumption 4: Road grade $\beta(t)$ is assumed to be a slowly varying function of time, i.e., $\dot{\beta}(t) \approx 0$), whereas the vehicle mass is assumed to be an unknown constant.

Assumption 5: It is assumed that the clutch is always fully engaged and that the friction brakes are never applied. These assumptions are not valid during the gearshift and braking periods. Unfortunately, in a standard setting, the service brake pressure and, subsequently, the brake torques cannot accurately be determined. Developing an accurate model of the transmission and gearshift process is an option but overcomplicates the model used for estimation. To address these issues, pre- and postconditioning of the signals and the estimates are proposed to handle periods of gearshift and braking.

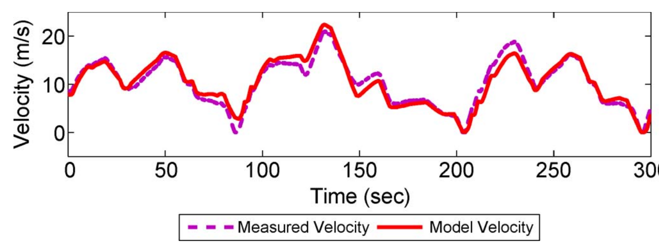

Fig. 1. Comparison of the model velocity with the velocity from J1939.

In an effort to further develop an estimation strategy, the dynamic model from (1) can be written as follows:

$$a \triangleq W\theta \tag{2}$$

where $a(t) \triangleq \dot{v}(t)$, and $W \triangleq [W_1 \quad W_2] \in \mathbb{R}^{1\times 2}$ is the known regression vector, with $W_1(t)$ and $W_2 \in \mathbb{R}$ given by

$$W_1 \triangleq \frac{(T_e - J_e\dot{\omega}_e - k_{\text{aero}}v^2 r_g)}{r_g} \tag{3}$$

$$W_2 \triangleq \frac{g}{\cos(\beta_\mu)}. \tag{4}$$

In (2), $\theta \triangleq [\theta_1 \quad \theta_2]^T \in \mathbb{R}^2$ is the unknown parameter vector, with the elementsdefined as follows:

$$\theta_1 \triangleq \frac{1}{M} \tag{5}$$

$$\theta_2 \triangleq \sin(\beta+\beta_\mu). \tag{6}$$

B. Model Validation

The validity of the model and its parameters was determined using several sets of experimental data obtained at Eaton Corporation. The net engine torque was made available from J1939, and the vehicle velocity and engine speed were obtained from the J1939 port. The clutch and transmission statuses were available through the clutch control and transmission control units, respectively. The values for the gear ratios, wheel radius, coefficient of rolling resistance, drag coefficient, and engine inertia, and the actual mass and road grade profiles were also provided by Eaton.

The right-hand side of (1) with the known parameters and given signals was integrated over time to find the vehicle velocity, and this velocity was compared with the velocity from J1939. Due to the uncertainty in braking torques, only portions with no braking were used. Fig. 1 compares the modeled velocity to the actual J1939 velocity for one of the data sets provided by Eaton. In general, the modeled and actual velocity are in good agreement. The differences are most probably due to simplifying assumptions during the gearshift period.

III. Estimator Design

A. Adaptive Least-Squares Estimator

To facilitate the estimator design, a *prediction error* $\varepsilon(t)$ is defined as follows:

$$\varepsilon \triangleq a_f - \hat{a}_f \tag{7}$$

where $a_f(t)$ is the filtered longitudinal acceleration of the vehicle and can be written as follows:

$$\dot{a}_f = -\beta_o a_f + \beta_o a \tag{8}$$

where $a_f(t_0) = 0$, $\beta_o > 0$ is constant, and actual acceleration $a(t)$ is calculated by backward differentiation of the velocity signal. In (7), $\hat{a}_f(t)$ is the estimate of the filtered longitudinal acceleration defined as follows:

$$\hat{a}_f \triangleq W_f \hat{\theta} \tag{9}$$

where $W_f(t) \in \mathbb{R}^{1\times 2}$ is the filtered regression vector given by

$$\dot{W}_f = -\beta_o W_f + \beta_o W \tag{10}$$

where $W_f(t_0) = [0 \quad 0]$, and β_o was introduced in (8). In (9), $\hat{\theta}(t) = [\hat{\theta}_1 \quad \hat{\theta}_2]^T \in \mathbb{R}^2$ is the estimate vector of the unknown parameters. From (2), (8), and (10), it is possible to write the following:

$$\dot{a}_f + \beta_o a_f = \dot{W}_f \theta + \beta_o W_f \theta. \tag{11}$$

Taking the time derivative of (9) and utilizing (10), we get

$$\dot{\hat{a}}_f + \beta_o \hat{a}_f = \frac{d}{dt}(W_f \hat{\theta}) + \beta_o W_f \hat{\theta}. \tag{12}$$

By subtracting (12) from (11) and utilizing (7) and (10), the resulting expression can be written as follows:

$$\dot{\varepsilon} + \beta_o \varepsilon = \frac{d}{dt}(W_f \tilde{\theta}) + \beta_o W_f \tilde{\theta} \tag{13}$$

where $\tilde{\theta}(t) \in \mathbb{R}^2$ is the estimated parameter error signal defined as

$$\tilde{\theta} \triangleq \theta - \hat{\theta}. \tag{14}$$

From (13), it is clear that an unmeasurable form of the prediction error $\varepsilon(t)$ can be written as

$$\varepsilon = W_f \tilde{\theta}. \tag{15}$$

A continuous-time least-squares update law is employed to estimate the unknown parameters described by [12]

$$\dot{\hat{\theta}} \triangleq -K_{\text{ls}} \frac{P_{\text{ls}} W_f^T \varepsilon}{1 + \gamma W_f P_{\text{ls}} W_f^T} \tag{16}$$

where K_{ls} is a constant diagonal gain matrix, γ is a positive constant gain, $\varepsilon(t)$ was defined in (7), W_f was defined in (10), and $P_{\text{ls}}(t)$ is the covariance matrix. Matrix $P_{\text{ls}}(t)$ is generated by the covariance propagation equation, which is described as follows [12]:

$$\dot{P}_{\text{ls}} \triangleq -K_{\text{ls}} \frac{P_{\text{ls}} W_f^T W_f P_{\text{ls}}}{1 + \gamma W_f W_f^T} \tag{17}$$

where $P_{\text{ls}}(t_0) = k_0 I_2$, $k_0 \in \mathbb{R}^+$ is a constant gain, and $I_2 \in \mathbb{R}^{2\times 2}$ is the standard 2×2 identity matrix. The vehicle's estimated mass $\hat{M}(t) \in \mathbb{R}$ is given by

$$\hat{M} \triangleq \frac{1}{\hat{\theta}_1} \tag{18}$$

and the estimated road grade $\hat{\beta}(t) \in \mathbb{R}$ is obtained by

$$\hat{\beta} \triangleq \sin^{-1}(\hat{\theta}_2) - \beta_\mu. \tag{19}$$

Remark: From (18), special care needs to be taken to avoid $\hat{\theta}_1(t) = 0$. To achieve this condition, the projection algorithm, as described in [12, Sec. 2.3.1], must be utilized. This algorithm takes $\hat{\theta}_1(t)$ and $\dot{\hat{\theta}}_1(t)$ into account to keep $\hat{\theta}_1(t) > 0$ while maintaining stability and convergence of the least-squares estimation strategy.

Theorem 3.1: The least-squares update law, as described by (16) and (17), ensures $\tilde{\theta}(t) \to 0$ as $t \to \infty$, provided that four sufficient conditions are met.

1) The plant of estimation is strictly proper.
2) The input is piecewise continuous and bounded.
3) The output of the plant of estimation is bounded.
4) The following persistence of excitation condition [12] holds:

$$\alpha_1 I_2 \leq \int_{t_0}^{t+\delta} W^T(\tau) W(\tau) d\tau \leq \alpha_2 I_2 \tag{20}$$

where α_1, α_2, and $\delta \in \mathbb{R}^+$ are constants, and $W(\cdot)$ is defined by (3) and (4).

Proof: To prove $\tilde{\theta}(t) \to 0$ as $t \to \infty$, [12, Th. 2.5.3] is directly followed. To prove that sufficient condition 1 is valid, the plant of estimation, as described in (8), can be written as follows:

$$\frac{a_f(s)}{a(s)} = \frac{\beta_o}{s + \beta_o} \tag{21}$$

where the vehicle's longitudinal acceleration $a(t)$ is the input of the plant of estimation. From (21), the plant is shown to be strictly proper. To prove that sufficient condition 2 is valid, from (1) and Assumption 1, it can be proven that $a(t)$ is piecewise continuous and bounded. To prove that sufficient condition 3 is valid, (8) and standard linear analysis tools are utilized to show that $a_f(t)$, $\dot{a}_f(t) \in \mathcal{L}_\infty$, therefore proving that the output of the plant of estimation is bounded. The rest follows the proof in [12].

Remark: For Theorem 3.1 to be valid, the unknown parameters θ_1 and θ_2 are assumed to be constant or slowly time varying; hence, $\dot{\theta}(t) \approx 0$. Based on the system dynamics defined in (1) and the definitions made in (5) and (6), it is clear that $\theta_1 = 1/M$ (hence, $\dot{\theta}_1(t) = 0$) and that $\theta_2(t) = \sin(\beta(t) + \beta_\mu)$. Under normal highway conditions, it is apparent that the condition will occur when $\dot{\theta}_2(t) \neq 0$. Under these circumstances, the least-squares estimation strategy will not completely work; hence, $\hat{\theta}(t) \neq \theta$. For these cases, a subsequently developed nonlinear estimator is designed to estimate the time-varying road grade. As shown in subsequent experimental results, the least-squares estimation strategy does provide a good estimation for the vehicle's mass; hence, this strategy is not abandoned. With this in mind, the authors propose a two-stage approach to estimate mass and road grade.

B. Nonlinear Estimator Development

For the nonlinear estimator development, Assumption 4, as defined in Section II, is redefined as follows:

Assumption 4: Road grade $\beta(t)$ is assumed to be an unknown time-varying bounded function, where $\beta(t), \dot{\beta}(t), \ddot{\beta}(t) \in \mathcal{L}_\infty$. The vehicle mass is assumed to be a known constant. To facilitate the design of the nonlinear estimator, the system model from (2) can be rewritten as

$$a = W_1\theta_1 + f \tag{22}$$

where $W_1(t) \in \mathbb{R}$ is the first element of the regression vector, as defined in (3); θ_1 is the first element of the parameter vector, as defined in (5); and $f(t)$ is the remaining terms from the dynamic model, as defined in (1), and is written as

$$f = \frac{-g}{\cos(\beta_\mu)} \sin(\beta + \beta_\mu). \tag{23}$$

Velocity estimator error signal $e(t)$ is defined as

$$e \triangleq v - \hat{v} \tag{24}$$

where $\hat{v}(t)$ denotes the following velocity estimate:

$$\hat{v} = \int_{t_0}^{t} W_1(\sigma)\theta_1(\sigma)d\sigma + \int_{t_0}^{t} \hat{f}(\sigma)d\sigma \tag{25}$$

where $\hat{f}(t)$ is the subsequently designed nonlinear estimator. In an effort to complete the design, the time derivative of (24) is taken and can be written as follows:

$$\dot{e} = \dot{v} - \dot{\hat{v}}. \tag{26}$$

Upon utilizing (25), an expression for $\dot{\hat{v}}(t)$ can be written as

$$\dot{\hat{v}} = W_1\theta_1 + \hat{f}. \tag{27}$$

Substituting (22) and (27) into (26), the simplified expression can be written as

$$\dot{e} = \tilde{f} \tag{28}$$

where estimator error $\tilde{f}(t)$ is

$$\tilde{f} \triangleq f - \hat{f}. \tag{29}$$

From the structure of (28) and the subsequent stability analysis, a proportional–integrallike nonlinear estimator is designed to identify $f(t)$, which generates a road grade estimate $\hat{\beta}(t)$. This estimator is formulated as

$$\hat{f} \triangleq (k_1 + 1)\left[e(t) - e(T_{\rm ls}) + \int_{t_0}^{t} e(\sigma)d\sigma\right] + \int_{t_0}^{t} k_2 \mathrm{sgn}(e(\sigma))\, d\sigma \tag{30}$$

where $k_1, k_2 \in \mathbb{R}^+$ are constant gains, and $\mathrm{sgn}(\cdot)$ is the signum function. To facilitate the subsequent analysis, an auxiliary error signal $s(t)$ is defined as

$$s \triangleq \dot{e} + e. \tag{31}$$

Taking the time derivative of (31) provides the following dynamic expression for $s(t)$:

$$\dot{s} = \dot{f} - (k_1 + 1)s - k_2\mathrm{sgn}(e) + \dot{e} \tag{32}$$

where the time derivatives of (28)–(30) were all utilized.

Remark: The subsequent theorem proves that the nonlinear estimator $\hat{f}(t)$ defined in (30) converges to estimate signal $f(t)$. Based on the definition of $f(t)$ in (23), road grade estimate $\hat{\beta}(t)$ can be written as

$$\hat{\beta} = \sin^{-1}\left(-\frac{\hat{f}}{g}\cos(\beta_\mu)\right) - \beta_\mu. \tag{33}$$

Theorem 3.2: The road grade estimator given in (30) ensures that

$$\hat{f}(t) \to f(t) \quad \text{as} \quad t \to \infty \tag{34}$$

provided that estimator gain k_2 is selected to meet the following sufficient condition:

$$k_2 > \left|\dot{f}(t)\right| + \left|\ddot{f}(t)\right|. \tag{35}$$

Based on Assumption 1 and (23), an upper bound can be shown to exist for $\dot{f}(t)$ and $\ddot{f}(t)$.

Proof: To prove that $\hat{f}(t) \to f(t)$ as $t \to \infty$, a nonnegative function $V(t) \in \mathbb{R}$ is defined as

$$V = \frac{1}{2}e^2 + \frac{1}{2}s^2 \tag{36}$$

where $e(t)$ and $s(t)$ were defined in (24) and (31), respectively. After taking the time derivative of (36) and using (31) and (32), the following expression can be obtained:

$$\dot{V}(t) = e(s - e) + s\left(\dot{f} - (k_1 + 1)s - k_2\mathrm{sgn}(e) + \dot{e}\right). \tag{37}$$

The expression can be simplified by utilizing (31)

$$\dot{V}(t) = -e^2 - k_1 s^2 + \dot{e}\dot{f} + e\dot{f} - k_2(\dot{e} + e)\mathrm{sgn}(e). \tag{38}$$

The integral of (38) from t_0 to t can be expressed as

$$V(t) \leq V(t_0) - \int_{t_0}^{t} |e(\sigma)|^2 \, d\sigma - k_1 \int_{t_0}^{t} |s(\sigma)|^2 \, d\sigma + \int_{t_0}^{t} \dot{e}(\sigma)\dot{f}(\sigma) d\sigma - k_2 \int_{t_0}^{t} \dot{e}(\sigma)\text{sgn}\,(e(\sigma))\, d\sigma + \int_{t_0}^{t} e(\sigma)\left(\dot{f}(\sigma) - k_2\text{sgn}\,(e(\sigma))\right) d\sigma. \quad (39)$$

After integrating the fourth term on the right-hand side of (39) by parts and integrating the fifth term on the right-hand side of (39) with respect to time, the following expression is obtained for $V(t)$:

$$V(t) \leq V(t_0) - \int_{t_0}^{t} |e(\sigma)|^2 \, d\sigma + e(t)\dot{f}(t) - k_1 \int_{t_0}^{t} |s(\sigma)|^2 \, d\sigma - e(t_0)\dot{f}(t_0) + k_2 e(t_0)\text{sgn}\,(e(t_0)) - k_2 e(t)\text{sgn}(e(t)) + \int_{t_0}^{t} e(\sigma)\left(\dot{f}(\sigma) - \ddot{f}(\sigma) - k_2\text{sgn}(e(\sigma))\right) d\sigma.$$

Provided that k_2 is selected according to (35), $V(t)$ can be further upper bounded as follows:

$$V(t) \leq -k_0 \int_{t_0}^{t} |s(\sigma)|^2 \, d\sigma - \int_{t_0}^{t} |e(\sigma)|^2 \, d\sigma + C \quad (40)$$

where $C \in \mathbb{R}$ represents the following positive bounding constant:

$$C \triangleq V(t_0) - e(t_0)\left(\dot{f}(t_0) - k_2\text{sgn}\,(e(t_0))\right). \quad (41)$$

From the structure of (40) and the definition in (41), it is proven that $V(t) \in \mathcal{L}_\infty$; hence, $s(t)$, $e(t) \in \mathcal{L}_\infty$. Since $s(t)$, $e(t) \in \mathcal{L}_\infty$, (31) can be used to prove that $\dot{e}(t) \in \mathcal{L}_\infty$. From Assumption 1 and the definition in (23), it is possible to demonstrate that $\dot{f}(t) \in \mathcal{L}_\infty$. From the fact that $\dot{f}(t)$, $s(t)$, $e(t)$, and $\dot{e}(t) \in \mathcal{L}_\infty$, it is clear from (32) that $\dot{s}(t) \in \mathcal{L}_\infty$. The inequality defined by (40) can be used to prove that $s(t), e(t) \in \mathcal{L}_2$. Since $s(t), e(t), \dot{s}(t), \dot{e}(t) \in \mathcal{L}_\infty$ and $s(t)$, $e(t) \in \mathcal{L}_2$, then Barbalat's lemma can be used to prove that $|s(t)|$ and $|e(t)| \to 0$ as $t \to \infty$, and based on (28) and (29), it is clear that $\hat{f}(t) \to f(t)$ as $t \to \infty$, thus completing the proof of Theorem 3.2.

IV. Results With Simulated Data

The two-stage estimator proposed in Sections III-A and B was first tested using simulated data. Two simulated data sets were created: one with step changes in road grade and one with sinusoidal grade variation. A constant vehicle mass of $M = 20\,000$ kg was assumed for both data sets. Persistent excitation is required for the accurate estimation of parameters, which was enforced by choosing a sufficiently varying fueling profile. The engine torque signal was calculated based on this fueling command; in addition, a random noise signal was added to the torque signal to reflect a more realistic situation. It was assumed that brakes are not applied and that there is no gearshift during simulation.

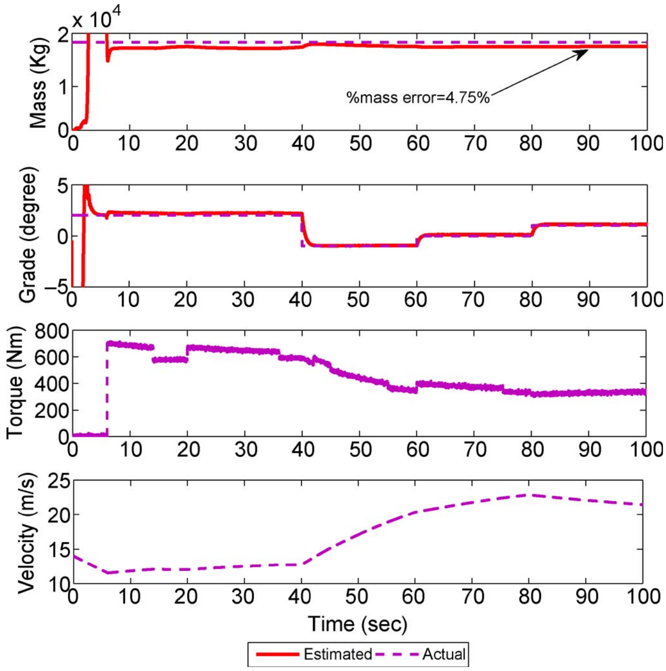

Fig. 2. Estimation results for simulated data when the grade is varied in steps.

Fig. 2 shows the estimation results when the road grade changes in steps. The mass estimate reaches within 10% of its actual value within 7 seconds. After about 7 seconds, as the mass estimate nears its actual value, the estimate of the grade also accurately tracks the actual grade. The maximum root-mean-square (RMS) error for the estimate of the grade is 0.2. Fig. 3 shows the same for sinusoidal variation of the grade profile, and the maximum RMS error for the estimate of the grade is 0.4.

V. Results With Experimental Data

With the two-stage estimation strategy proven effective with simulated data, the algorithm was next tested with several experimental data sets obtained during road tests at Eaton. In implementing this strategy, we deviated from the theoretical development of Section III-B, which requires vehicle mass M to be known *a priori*. Instead, the vehicle mass is estimated using the adaptive least-square method in Section III-A and utilized in place of the known mass M in the nonlinear grade estimator development for the experiment. The following experimental results demonstrate the proof of concept of this theoretical deviation. To implement this approach, the following changes are required. Velocity estimate $v(t)$, as defined in (25), is modified as follows:

$$\hat{v} = \int_{t_0}^{t} W_1(\sigma)\hat{\theta}_1(\sigma) d\sigma + \int_{t_0}^{t} \hat{f}(\sigma) d\sigma \quad (42)$$

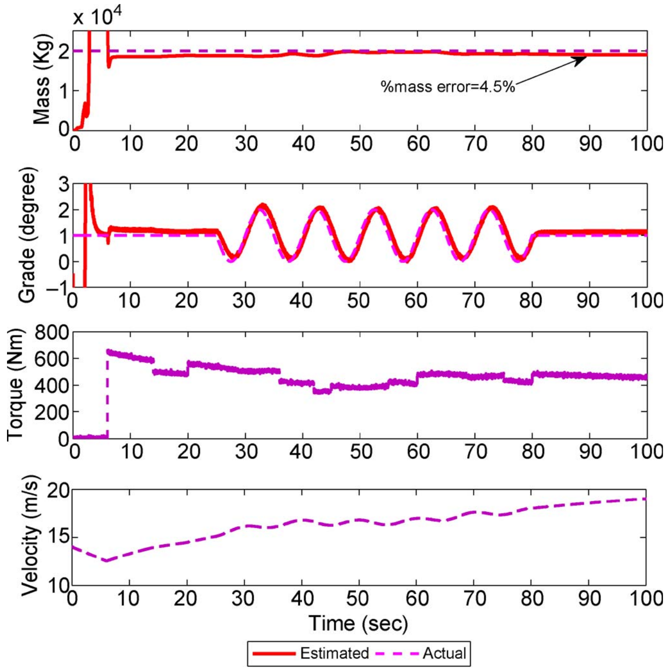

Fig. 3. Estimation results for simulated data when the grade is sinusoidally varied.

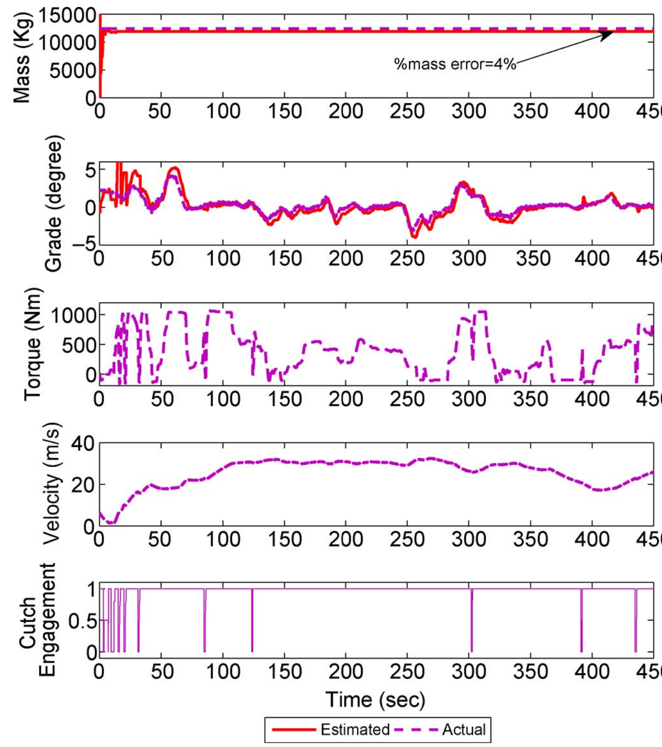

Fig. 4. Estimation results for data set 1.

where $\hat{\theta}_1(t)$ is generated by the least-squares estimator, as defined in (16). The expression. as defined by (43), is also modified as follows:

$$\dot{\hat{v}} = W_1\hat{\theta}_1 + \hat{f}. \tag{43}$$

Based on the new definitions of (42) and (43), these adjustments affect $e(t)$, $\dot{e}(t)$, and $s(t)$, although their definitions remain unchanged. The definition of the nonlinear estimator, as defined in (30), also remains unchanged.

An accurate model of the drivetrain was not available, and the brake pressure values were also not available, resulting in spikes in the road grade estimate whenever the clutch was not fully engaged or the brake was applied. To remedy this problem, during clutch disengagement and the braking period, the grade estimate is taken as its latest value calculated just before clutch disengagement or the braking period. However, even after full engagement of the clutch, we observed the spikes in the grade estimate. The drivetrain vibration subsequent to clutch engagement may be the cause of this problem. To reduce the spikes in the grade estimate, the estimate of the road grade is discarded for 40 additional sampling times (0.4 s) after clutch engagement. Instead, the latest estimate obtained before the clutch disengagement is used. The same strategy is used during the braking periods.

The experimental data sets were obtained at Eaton from an HDV with automated manual transmission. The data were obtained from the Engine Control Module through J1939, the driveline control module, and the clutch control module. The truck was driven on three different roads, which were referred to data sets 1–3. Each experimental run was selected to demonstrate varying road and load conditions. At each experimental run, the total vehicle mass was modified by changing the payload and was known *a priori*.

For all experimental runs, the least-squares estimator gains were chosen to be

$$\begin{aligned} \beta_0 &= 5 \quad \gamma = 5 \\ K_{\rm ls} &= \mathrm{diag}\{69, 40\} \\ P_{\rm ls}(t_0) &= \mathrm{diag}\{1, 1\} \end{aligned} \tag{44}$$

where $\mathrm{diag}\{\cdot\}$ represents the diagonal elements of a 2×2 matrix. The nonlinear estimator had the following gain values:

$$k_1 = 7 \quad \text{and} \quad k_2 = 10.$$

A. Experiment 1

For this experiment, we selected 450 seconds of data from the data set referred to as data set 1. The total vehicle mass was approximately $M = 12\,400$ kg. Fig. 4 shows the results of the least-squares mass estimator, as defined in Section III-A, along with the results of the nonlinear road grade estimator. From the result of the mass estimate, it is clear that this strategy provides a mass estimate $\hat{M}(t)$ that is very close to the actual vehicle's mass when $t \geq 20$ s. The maximum percent error in mass after 20 s is less than 5%. Once the estimate of mass converges to the actual mass, the nonlinear road grade estimator also very closely tracks the actual grade, and the RMS error in the estimate of road grade is 0.55.

B. Experiment 2

For this experiment, we selected 200 seconds of data from the data set referred to as data set 2, for which the total vehicle mass is $M = 14\,000$ kg. Fig. 5 shows the results of the least-squares

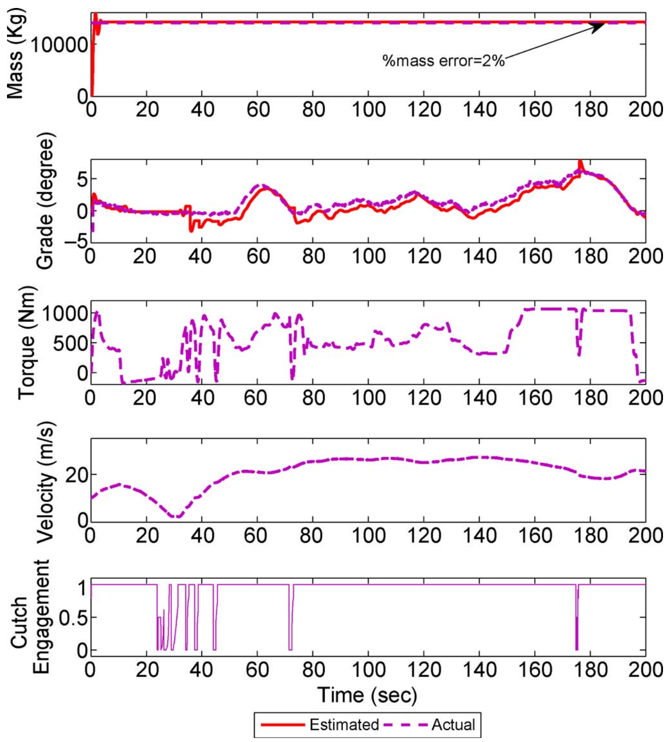

Fig. 5. Estimation results for data set 2.

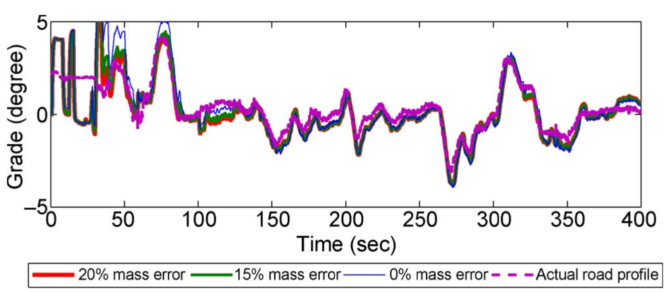

Fig. 6. Demonstration of the sensitivity of the grade estimator to errors in mass estimate.

mass estimator, as defined in (16), and the nonlinear road grade estimator, as defined in (30). From Fig. 5, it is clear that this strategy provides an accurate mass estimate $\hat{M}(t)$ of the actual vehicle mass when $t \geq 10$ s. The percent mass error for $t \geq 10$ s is less than 3%, and the RMS error in grade is about 0.6.

C. Experiment 3

For this simulated experiment, we exploit a potential weakness in our estimation scheme. Due to the fact that our two-stage approach requires an estimation of the mass to subsequently estimate the road grade, we wanted to see how sensitive the road grade estimation was to potential estimation mass error. With this in mind, we modified our simulation test bed to artificially inject a mass estimation error and reran the data set from Experiment 1. We introduced 15% and 20% mass estimation errors into the simulation testbed. Fig. 6 shows the effect that the individual mass estimation error signals has on road grade estimation $\hat{\beta}(t)$ when applied to the data set from Experiment 1. From Fig. 6, it is clear that the injected mass estimation error does not significantly influence the estimate of road grade $\hat{\beta}(t)$. Table I shows the RMS error in the estimate of road grade for different mass errors. We have neglected the initial 50 s of data to calculate the RMS error.

TABLE I
RMS ERROR IN GRADE FOR DIFFERENT PERCENT MASS ERRORS FOR DATA SET 1

% Mass error	RMS error in grade
0%	0.55
15%	0.49
20%	0.49

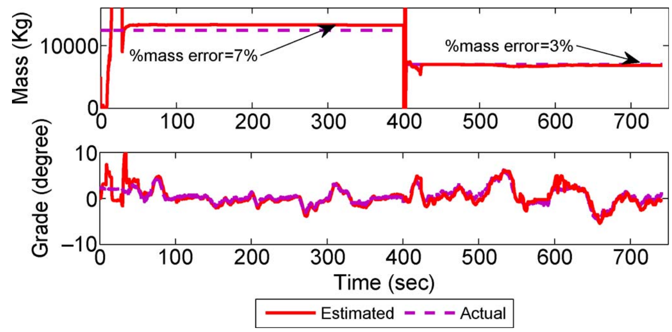

Fig. 7. Estimation results when data sets 1 and 3 are ran in series.

D. Experiment 4

To check the robustness of the two-stage algorithm against changes in vehicle loading, data set 1 was run, followed by data set 3. This is another "simulation" case of putting two experimental data sets together, which can represent an actual operation of heavy trucks, i.e., loaded and then unloaded during a business trip. The mass of the truck for data set 1 is $M = 12\,400$ kg, and that for data set 3 is $M = 7000$ kg. Fig. 7 shows the result for the estimated mass and grade, compared with the actual mass and grade profile. The spikes in the estimate of mass when there is change in mass at about $t = 400$ s are due to the resetting of the covariance matrix when the vehicle stops and before it starts a new run; this allows the estimate of mass to converge to the actual mass. The estimate of grade also very closely follows the actual grade.

VI. CONCLUSION

This paper has developed a model-based two-stage estimation strategy to determine an HDV's mass and road grade. The first stage of this approach utilized an adaptive least-squares estimation strategy to determine the vehicle's mass. Due to the time-varying nature of the road grade, a nonlinear estimator that can be utilized to obtain accurate road grade estimation was developed. The estimation results using simulated data and experimental data sets show excellent tracking for both estimated parameters. The estimation results follow the actual parameters with good accuracy, even during the braking phase or when the clutch is not fully engaged. The algorithm presented here estimates the vehicle mass and road grade, which can be utilized in advanced vehicle-control systems. An estimation analysis proved for both stages that, under a set of qualifying conditions, both the mass and road grade can be estimated. Experimental results were presented, illustrating the feasibility and robustness of the two-stage estimation strategy.

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Michael L. McIntyre (M'06) received the B.S. and M.Eng. degrees from the University of Louisville, Louisville, KY, in 1997 and 2000, respectively, and the Ph.D. degree from Clemson University, Clemson, SC, in 2006.

From 1995 to 1998, he was with Phillip Morris USA, Louisville, KY, as a Process Control Engineer. From 1998 to 2003, he was with General Electric as a Design Engineer. He is currently an Assistant Professor with the Department of Engineering, Western Kentucky University, Bowling Green. His research interests include nonlinear control techniques for mechatronic systems, control of electrical energy systems, and engineering education.

Tejas J. Ghotikar received the Bachelors of Engineering degree from Maharaja Sayajirao University of Baroda, Baroda, India, in 2005 and the Master's degree from Clemson University, Clemson, SC, in 2008, both in mechanical engineering.

From 2007 to 2008, he was a Research Assistant with the Department of Mechanical Engineering, Clemson University. His research interests include the control and simulation of automotive systems.

Ardalan Vahidi received the B.S. and M.S. degrees in civil engineering from Sharif University, Tehran, Iran, in 1996 and 1998, respectively, the M.S. degree in transportation safety from George Washington University, Washington, DC, in 2002, and the Ph.D. degree in mechanical engineering from the University of Michigan, Ann Arbor, in 2005.

He is currently an Assistant Professor of mechanical engineering with the Department of Mechanical Engineering, Clemson University, Clemson, SC. His current research interests include optimization-based control methods and the control of vehicular and energy systems.

Xubin Song (M'03) received the B.S. degree in control engineering from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 1986, the M.S. degree in control engineering from the China Academy of Launch Vehicles, Beijing, China, in 1988, the M.S. degree in mechanical engineering from North Carolina A&T State University, Raleigh, in 1996, and the Ph.D. degree in mechanical engineering from Virginia Polytechnic Institute and State University, Blacksburg, in 1999.

From 1988 to 1994, he was an Engineer with the China Academy of Launch Vehicles. From 1999 to 2000, he was a Control Engineer with MSX International, where he worked on the iBOT human transporter. From 2000 to 2004, he was a Technical Professional with Visteon Corporation, where he developed vehicle-suspension control systems and traction controls. Since 2004, he has been a Principal Engineer with Eaton Corporation, Southfield, MI, working on advanced powertrain controls for commercial vehicles. He is the holder of six U.S. patents and one European patent. He has published more than 40 papers. His research interests include vehicle-related nonlinear and nonsmooth dynamics and control, particularly advanced powertrain and chassis systems for passenger and commercial vehicles.

Dr. Song is a member of the American Society of Mechanical Engineers (ASME) and the Society of Automotive Engineers (SAE), serving the ASME Vehicle Design Committee and the SAE Driveline Standard Committee. He is currently the Chair of the SAE Commercial Vehicle Committee of the Powertrain, Driveline, and Transmission Committee.

Darren M. Dawson (SM'94) was born in Macon, GA, in 1962. He received the Associate degree in mathematics from Macon Junior College in 1982 and the B.S. and Ph.D. degrees in electrical engineering from the Georgia Institute of Technology, Atlanta, in 1984 and 1990, respectively.

From 1985 to 1987, he was with Westinghouse, as a Control Engineer. In July 1990, he joined the Department of Electrical and Computer Engineering, Clemson University, Clemson, SC, where he is currently the McQueen Quattlebaum Professor.

Dr. Dawson is a Senior Member of Sigma Xi.