Robust Geotechnical Design of Drilled Shafts in Sand
- A New Design Perspective

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Abstract: This paper presents a new geotechnical design concept, called robust geotechnical design (RGD). The new design methodology, seeking to achieve a certain level of design robustness, in addition to meeting safety and cost requirements, is complementary to the traditional design methods. Here, a design is considered robust if the variation in the system response is insensitive to the variation of noise factors (mainly uncertain soil parameters). To aid in the selection of the best design, a Pareto Front, which describes a trade-off relationship between cost and robustness at a given safety level, can be established using the RGD methodology. The new design methodology is illustrated with an example of drilled shaft design for axial compression. The significance of the RGD methodology is demonstrated.

Key words: Reliability; Uncertainty; Failure probability; Limit states; Robust design; Drilled shaft; Sand.

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Introduction

In a traditional geotechnical design, multiple candidate designs are first checked against code-specified safety (including strength and serviceability) requirements, and the acceptable designs are then optimized for cost (by means of a mathematical optimization procedure considering all possible designs or simply by selecting the least-cost design from a few possible alternatives) to produce a final design. In this design process, the safety requirements are analyzed by either deterministic methods or probabilistic methods. The deterministic methods use factor of safety ($F_s$) as a measure of safety, while probabilistic methods use reliability index or probability of failure as the measure of safety. With the $F_s$-based approach, the uncertainties in soil parameters and the associated analysis model are not considered explicitly in the analysis but their effect is considered in the design by adopting a threshold $F_s$ value. With the probabilistic (or reliability-based) approach, these uncertainties are included explicitly in the analysis based on ultimate limit state (ULS) or serviceability limit state (SLS), and the design is considered acceptable if the reliability index or failure probability requirement is satisfied. Finally, cost optimization among the acceptable designs is performed to yield the final design.

Regardless of whether the $F_s$-based approach or the reliability-based approach is employed, the traditional design focuses mainly on safety and cost; design “robustness” is not explicitly considered. Robust Design, which originated in the field of Industrial Engineering (Taguchi 1986; Tsui 1992; Chen et al. 1996) aims to make the product of a design insensitive to (or robust against) “hard-to-control” input parameters (called “noise factors”) by adjusting
“easy-to-control” input parameters (called “design parameters”). The essence of this design approach is to consider robustness explicitly in the design process along with safety and economic requirements. The focus of this paper is to turn this robust design concept into a Robust Geotechnical Design (RGD) methodology.

The traditional design approach that does not consider robustness against noise factors (such as soil parameters variability and/or construction variation) may have two drawbacks. First, the lowest-cost design may no longer satisfy the safety requirements if the actual variations of the noise factors are underestimated. Here, the design requirements may be violated because of the high variation of the system response due to the underestimated variation of noise factors. Second, facing high variability of the system response, the designer may choose an overly conservative design that guarantees safety; as a result, the design may become inefficient and costly. This dilemma between the over-design for safety and the under-design for cost-savings is, of course, not a new problem in geotechnical engineering. By reducing the variation of the system response to ensure the design robustness against noise factors, the RGD approach can ease such dilemma in the decision making process. Of course, the variation of the system response may also be reduced by reducing the variation in soil parameters. However, in many geotechnical projects the ability to reduce soil variability is restricted by the nature of soil deposit (i.e., inherent soil variability) and/or the number of soil test data that is available. In this regard, it is important to note that the RGD methodology seeks the reduction in the variation of system responses by adjusting only the “easy-to-control”
design parameters, and not the “hard-to-control” noise factors.

It should be noted that adjusting the design parameters through the concept of robust design is just one option to meet the design requirements. It may be feasible to achieve similar goal by improving soil parameter characterization. A balanced approach is to adopt a suitable site characterization and testing program, followed by a robust design with the estimated parameter uncertainty.

RGD is not a design methodology to compete with the traditional $F_S$-based approach or the reliability-based approach; rather, it is a design strategy to complement the traditional design methods. With the RGD approach, the focus is to satisfy three design objectives, namely safety, cost, and robustness (against the variation in system response caused by noise factors). As with many multi-objective engineering problems, it is possible that no single best solution exists that satisfies all three objectives. In such situations, a detailed study of the trade-offs among these design objectives can lead to a more informed design decision.

In this paper, robustness is first considered within the framework of a reliability-based design. Specifically, a reliability-based RGD procedure is proposed herein and illustrated with an example of a drilled shaft in sand for axial load. In the sections that follow, a brief review of a reliability-based model for axial capacity of drilled shaft (Phoon et al. 1995; Wang et al. 2011a) is first provided. Then, the reliability-based RGD methodology is presented, followed by an illustrative example to demonstrate the significance of design robustness and the effectiveness of this methodology for selection of the “best” design based on multiple objectives. Finally, to
demonstrate the applicability of the RGD methodology in a deterministic approach, the robust design concept is integrated with Load and Resistance Factor Design (LRFD) procedure.

**Reliability-Based Design of Drilled Shafts**

A summary of a reliability-based design of drilled shafts in sand presented by Phoon et al. (1995) is provided herein. The schematic diagram of a drilled shaft in loose sand subjected to an axial load under drained condition is shown in Figure 1. In this example, the water table is set at the ground surface. The diameter and depth (length) of the shaft are denoted as $B$ and $D$, respectively. Other design parameters regarding soil and structure properties are listed in Table 1. In the reliability-based design framework, $B$ and $D$ are selected to meet the target reliability index through a trial-and-error process.

The requirements of both ultimate limit state (ULS) and serviceability limit state (SLS) have to be satisfied in a reliability-based design. For either ULS or SLS requirement, the drilled shaft is considered failed if the compression load exceeds the shaft compression capacities. In this study, the axial compression load $F$ is set as the 50-year return period load $F_{50}$ for both ULS and SLS design ($F_{50} = 800$ kN in this example). The ULS compression capacity (denoted as $Q_{ULS}$) is determined with the following equation (Kulhawy 1991):

$$Q_{ULS} = Q_{side} + Q_{tip} - W$$

(1)

where $Q_{side}$, $Q_{tip}$, and $W$ are side resistance, tip resistance, and effective shaft weight,
respectively. Considering that the cohesion term is neglected in the design of drilled shafts in sand, the $Q_{\text{side}}$ and $Q_{\text{tip}}$ can be computed as (Kulhawy 1991):

$$Q_{\text{side}} = \pi BD \left( \frac{K}{K_0} \right)_n K_0 \sigma'_{vm} \tan \phi'$$  \hspace{1cm} (2)

$$Q_{\text{tip}} = 0.25 \pi B^3 \left[ 0.5B \left( \gamma - \gamma_w \right) N_q \zeta_{yr} \zeta_{yd} \zeta_{yr} + \left( \gamma - \gamma_w \right) DN_q \zeta_{qr} \zeta_{qr} \zeta_{qr} \right]$$  \hspace{1cm} (3)

where $(K/K_0)_n = \text{nominal operative in-situ horizontal stress coefficient ratio}$; $\sigma'_{vm} = \text{mean vertical effective stress along the shaft depth}$; $\phi' = \text{soil effective stress friction angle}$; and $N_q$, $N_{\gamma}$, $N_q$, $N_{\gamma}$ = bearing capacity factors defined as (Vesić 1975):

$$N_q = \tan^2 \left( 45^o + \phi' / 2 \right) \exp(\pi \tan \phi')$$  \hspace{1cm} (4)

$$N_{\gamma} = 2(N_q + 1) \tan \phi'$$  \hspace{1cm} (5)

And $\zeta_{yr}$ and $\zeta_{qr}$ = shape correction factors; $\zeta_{yd}$ and $\zeta_{qr}$ = depth correction factors; and $\zeta_{yr}$ and $\zeta_{qr}$ = rigidity correction factors for respective bearing capacity factors. Detailed methods for computing the bearing capacity factors and correction factors are documented in Kulhawy (1991). Then, the SLS compression capacity (denoted as $Q_{\text{SLS}}$) is determined with the following equation (Phoon et al. 1995; Wang et al. 2011a):

$$Q_{\text{SLS}} = 0.625a \left( \frac{y_a}{B} \right)^b Q_{\text{ULS}}$$  \hspace{1cm} (6)

where $a = 4.0$ and $b = 0.4$ are the curve-fitted parameters for the load-displacement model, and
$y_a = \text{allowable displacement, which is 25mm for this problem.}$

The probability of ULS failure ($p_f^{ULS}$) and the probability of SLS failure ($p_f^{SLS}$) are defined as $P_r(Q_{ULS} < F_{50})$ and $P_r(Q_{SLS} < F_{50})$, respectively. The reliability-based design can be realized by meeting the target failure probability requirements, namely, $p_f^{SLS} < p_f^{SLS}$ and $p_f^{ULS} < p_f^{ULS}$, where $p_f^{SLS}$ and $p_f^{ULS}$ are the target failure probabilities based on the serviceability limit state and the ultimate limit state, respectively.

Methodology for Reliability-Based Robust Geotechnical Design

In a reliability-based RGD, the design robustness is considered explicitly in the reliability-based design framework. Although the robustness may be interpreted differently (e.g., Taguchi 1986; Chen et al. 1996; Doltsinis and Kang 2005; Park et al. 2006; Ait Brik et al. 2007; Papadopoulos and Lagaros 2009), in this paper a geotechnical design is considered \textit{robust} if the performance measure (i.e., failure probability $p_f^{ULS}$ or $p_f^{SLS}$) is insensitive to the variation of noise factors (i.e., uncertain soil parameters). Note that the probability of failure is usually determined using Monte Carlo simulation (MCS) or reliability-based methods that require knowledge of the variation of soil parameters. If the actual variations of soil parameters are greater than the estimated variations that are used in the reliability-based analysis, the probability of failure may be underestimated. Thus, the generally accepted reliability-based designs that do not consider robustness in the analysis could violate the safety requirements ($p_f^{ULS} < p_f^{ULS}$ and $p_f^{SLS} < p_f^{SLS}$) if the variations of soil parameters are underestimated. The
chance for this violation may be greatly reduced if the variation of the failure probability, which
is considered as the system response, can be minimized by adjusting design parameters.

Thus, in a reliability-based RGD, the goal is to achieve design robustness by adjusting
design parameters (such as B and D in the drilled shaft design) to minimize the variation of the
probability of failure. In many cases, however, greater robustness can only be achieved at a
high cost and thus, a trade-off exists, which can best be investigated through multi-objective
optimization.

**Estimation of the coefficients of variation of soil parameters**

As pointed out by Phoon et al. (1995), drained friction angle $\phi'$ and coefficient of
earth pressure at rest $K_0$ are the two random variables that should be considered for the
reliability-based design of drilled shaft in loose sand.

In geotechnical practice, soil parameters are often determined from a limited number of
test data, thus, the statistical parameters derived from a small sample may be subjected to error.

In general, the “population” mean can be adequately estimated from the “sample” mean even
with a small sample (Wu et al. 1989). However, the estimation of standard deviation of the
population based on a sample is often not as accurate, especially with a smaller sample. Of
course, the measurement error and the model with which soil parameters are derived (e.g.,
estimation of $\phi'$ based on SPT or other means) could also contribute to the variation of the
derived parameters.

Duncan (2000) suggested that the standard deviation of a random variable might be
obtained by (1) direct calculation from data, (2) use of published coefficient of variation (COV), or (3) estimate based on the “three-sigma rule.” The evaluation of parameter uncertainty for a specific problem is the duty of the engineer in charge. In this paper, the published COVs are adopted for illustration of robustness concept in a geotechnical design. The COV of $\phi'$ of loose sand, denoted as $COV[\phi']$, typically ranges from 0.05 to 0.10 (Amundaray 1994), and the COV of $K_0$, denoted as $COV[K_0]$, typically ranges from 0.20 to 0.90 (Phoon et al. 1995). For a typical reliability-based design, it is reasonable to take the mean value of the range of COV of a given parameter as its coefficient of variation. Thus, $COV[\phi'] \approx 0.07$ and $COV[K_0] \approx 0.50$ may be used in a reliability-based design of drilled shafts in sand if there is no additional data.

The outcome of a reliability-based design is affected by the accuracy of the estimated COVs of soil parameters. Because of the uncertainty of the estimated COVs, there will be uncertainty regarding the outcome of the design (e.g., we are not sure whether the design really meets the target reliability index requirement if the COVs are underestimated). In this paper, we incorporate the concept of robustness to ensure that the design will meet the target reliability index requirement in the face of uncertainty on the estimated COVs.

The uncertainty of the COV of a given soil parameter may be characterized with a range. In fact, when COV is expressed as a range, the uncertainty is readily characterized. For example, if we consider $COV[\phi']$ to vary from 0.05 to 0.10 based on Amundaray (1994), then the uncertainty about the value of $COV[\phi']$ to be used in the reliability design is readily
characterized, as the mean and standard deviation of $COV[\phi']$, denoted as $\mu_{COV[\phi']}$ and $\sigma_{COV[\phi']}$, respectively, can be readily determined based on the three-sigma rule. Thus, for the COV of loose sand varying in the range of 0.05 and 0.10, $\mu_{COV[\phi']} \approx 0.07$ and $\sigma_{COV[\phi']} \approx (0.10-0.05)/4 = 0.0125$. It should be noted that when applying the so-called three-sigma rule to geotechnical problems, Duncan (2001) recommended use of a divisor of 4. For all practical purposes, the uncertainty of the estimated $COV[\phi']$ is mainly reflected in the standard deviation, $\sigma_{COV[\phi']}$. The above discussion indicates that the uncertainty of the estimated COV of a given parameter may be estimated from a range of COV published in the literature. According to Duncan (2001), the range can also be defined with the highest and the lowest conceivable values based on site condition and engineering judgment. Furthermore, when limited test data are available, the bootstrapping method (Amundaray 1994; Luo et al. 2012) may be used to compute the mean and standard deviation of COV in a statistically rigorous manner. Thus, characterization of the uncertainty of the estimated $COV[\phi']$ is within the means of a geotechnical engineer.

Similarly, the mean and standard deviation of $COV[K_0]$, denoted as $\mu_{COV[K_0]}$ and $\sigma_{COV[K_0]}$, respectively, can be determined based on the typical range (0.20 to 0.90) reported in the literature (Phoon et al. 1995). Following the three-sigma rule, $\mu_{COV[K_0]} \approx 0.50$ and $\sigma_{COV[K_0]} \approx (0.9-0.2)/4 = 0.175$.

Finally, $\phi'$ and $K_0$ of loose, normally consolidated sands are negatively correlated.
According to Mayne and Kulhawy (1982), and personal communications with Mayne (2012) and Phoon (2012), the correlation coefficient $\rho_{\phi', K_0}$ is estimated to be in the range of −0.6 to −0.9. Following the three-sigma rule, the mean and standard deviation of $\rho_{\phi', K_0}$, denoted as $\mu_{\rho_{\phi', K_0}}$ and $\sigma_{\rho_{\phi', K_0}}$, are estimated to be −0.75 and 0.075, respectively. Furthermore, for illustration purpose, both $\phi'$ and $K_0$ are assumed to follow lognormal distribution.

As is shown later, the robustness of a reliability-based design is achieved if the system response (in terms of the probability of failure) is insensitive to the variation of the estimated COVs of $\phi'$ and $K_0$ and their correlation.

**Reliability-based robust geotechnical design approach**

A framework for reliability-based robust geotechnical design is presented below using design of drilled shaft in loose sand as an example. This framework is a modification of the authors’ recent work (Juang et al. 2012; Juang and Wang 2013; Wang et al. 2013). In reference to Figure 2, the RGD approach is summarized in the following steps (presented with rationale):

**Step 1:** Select design parameters and noise factors and identify the design space. For the design of drilled shaft in sand, the diameter ($B$) and depth ($D$) of the drilled shaft are considered as the design parameters, and the soil parameters $\phi'$ and $K_0$ are considered as the noise factors. The statistics of the noise factors are estimated based on available data and guided by experience, as discussed previously. The choice of diameter $B$ is usually limited to equipment and local practice, and for illustration purpose in this paper, only three discrete values ($B = 0.9$ m, 1.2 m, and 1.5 m) are considered here. The depth $D$ is often computed for a
given $B$ that satisfies ULS or SLS requirements, and is typically rounded to the nearest 0.2 m (Wang et al. 2011a). Thus, design parameters $B$ and $D$ can be conveniently modeled in the discrete domain and the design space will consist of finite number of designs (say, $M$ designs). For example, $M$ will be equal to 93 if $D$ is selected from the likely range of 2 m to 8 m (for the drilled shaft shown in Figure 1 subjected to an axial compression load $F_{50} = 800$ kN) for each of the three discrete $B$ values.

**Step 2:** Evaluate the variation of the system response as a measure of robustness of a given design. For each possible design in the design space, the probability of failure can be computed based on either ultimate limit state (ULS) or serviceability limit state (SLS). Here, the probability of failure is treated as a system response (or more precisely, an effect of the system response), and the variation of the system response as a result of the variation of the sample statistics of the noise factors is adopted as a measure of robustness. In this paper, the modified point estimate method (PEM) by Zhao and Ono (2000) is used for evaluating the mean and standard deviation of the failure probability. The PEM approach requires evaluation of the failure probability at each of a set of $N$ “estimating” points (or sampling points) of the input noise factors, as reflected by the inner loop shown in Figure 2. In each repetition, statistics of each of the noise factors at each PEM estimating point must be assigned, and then the failure probability is computed using the First Order Reliability Method (FORM; see Ang and Tang 1984; Phoon 2004). The resulting $N$ failure probabilities are then used to compute the mean and standard deviation of the failure probability.
Step 3: Repeat Step 2 for each of the M designs in the design space. For each design, the mean and standard deviation of the failure probability are determined. This step is represented by the outer loop shown in Figure 2.

Step 4: Perform a fast elitist non-dominated sorting to establish a Pareto Front. For multi-objective optimization, Non-dominated Sorting Genetic Algorithm version II (NSGA-II) by Deb et al. (2002) is widely used. The sorting technique of NSGA-II is adopted herein.

Note that in single-objective optimization, one tries to get a design that is superior to all other designs. For example, in a reliability-based optimization, one may seek to find the least-cost design using reliability as a constraint. Such a scheme tends to result in a design with the least cost but barely meet the reliability requirement. However, this design may not be the “best” solution for stakeholders who are willing to pay more for less risk.

When multiple objectives are enforced, it is likely that no single best design exits that is superior to all other designs in all objectives. However, a set of designs may exist that are superior to all other designs in all objectives; but within the set, none of them is superior or inferior to each other in all objectives. These designs constitute a Pareto Front. Figure 3 shows a conceptual illustration of a Pareto Front in a bi-objective setting (Gencturk and Elnashai 2011). Each point on the Pareto Front is optimal in the sense that no improvement can be achieved in one objective without worsening in at least one other objective. When the optimization process yields a Pareto Front, a trade-off situation is implied. For example, if the cost and the robustness are two objectives in the trade-off relationship, the designer can
approach it in two ways. If an acceptable cost range of the design is pre-defined, the most robust design within the cost range will be the best design. On the other hand, if certain level of robustness is required and specified, the least cost design that meets the robustness requirement will be the best design.

Finally, it should be noted that the procedure described above (in reference to Figure 2) is only one possible implementation of the RGD methodology. Other implementations may be equally effective. For example, FORM as a means to compute the failure probability for a given design with a set of known statistics of each of the noise factors may be replaced by MCS. Similarly, PEM as a means to compute the variation of the system response (i.e., the failure probability) may also be replaced by MCS or other means. Since only finite, and relatively small, number of designs are considered in this illustrative example (M = 93), only the sorting part of the NSGA-II algorithm is employed for selecting “points” (or designs) for the Pareto Front. However, if M becomes much larger, the full algorithm of NSGA-II may be employed for the multi-objective optimization.

Reliability-Based Design without Robustness Consideration

To provide a reference for reliability-based RGD, reliability-based design of a drilled shaft without considering robustness is first presented. For a drilled shaft shown in Figure 1 with soil parameters described in Table 1 (in particular, $COV[\phi'] = 0.07$, $COV[K_0] = 0.50$, and $\rho_{\phi',K_0} = -0.75$), the probability of SLS and ULS failure for various designs for a given axial
load of $F_{50} = 800$ kN is analyzed using FORM. The results are shown in Figure 4. The results indicate that the SLS requirement controls the design of drilled shaft under axial compression load, which is consistent with those reported by other investigators (e.g., Wang et al. 2011a). In fact, in all analyses performed in this study, the SLS requirement always controls the design of drilled shafts in sand for axial compression. Thus, in the subsequent analysis only the SLS failure probability is considered.

In a reliability-based design, the reliability requirement is generally used as a constraint (i.e., the actual reliability index must be greater than the target value or the corresponding failure probability must be less than the target value) to screen for the acceptable designs, and then the optimal design is obtained by minimizing the cost (Zhang et al. 2011). For a comprehensive design, the total life-cycle cost of the structure may be considered (Frangopol and Maute 2003). For simplicity, only the initial cost of a drilled shaft is considered in this paper so that we can focus on the subject of design robustness. The initial cost generally refers to the cost for completing a drilled shaft construction, including both material and labor cost, which can be estimated from published, annually updated literature, such as Means Building Construction Cost Data (R.S. Means Co. 2007). The U.S. national average unit costs for constructing drilled shafts with respective diameters of 0.9 m, 1.2 m and 1.5 m are summarized in Table 2. The costs for constructing a unit depth (0.3 m) are USD 77.5, 116 and 157, respectively for the three diameters (Wang et al. 2011a). If the “best” design is to be chosen based on least cost subjected to the constraint that the SLS failure probability is less than a
target value (say, 0.0047), the design with $B = 0.9$ m and $D = 5.6$ m will be selected.

To demonstrate the effect of the variation in the estimated statistics of soil parameters, two series of the analysis are performed. One is to determine the least cost designs for various assumed COV and correlation coefficient values (see Table 3), and the other seeks to determine the failure probability of a given design under various levels of COV and correlation coefficient (see Table 4).

Table 3 shows that the least cost designs are different for different assumed COV and correlation coefficient values. The implication is that the determination of least cost design in a reliability-based design is meaningful only if the statistics of soil parameters ($COV[\phi']$, $COV[K_0]$, and $\rho_{\phi',K_0}$) are fixed values. Thus, if the COV and correlation coefficient values are underestimated or overestimated by a certain margin, then there is a chance (significant probability) that an acceptable design (a design that satisfies ULS and SLS constraints based on fixed statistics values) will no longer satisfactory. This inference is demonstrated with results shown in Table 4, where the performance of an acceptable design ($B = 0.9$ m and $D = 5.6$ m based on an assumption of fixed statistics values, $COV[\phi'] = 0.07$, $COV[K_0] = 0.50$, $\rho_{\phi',K_0} = -0.75$, and a target failure probability of $p^{SLS}_{T} = 0.0047$) is reanalyzed with various levels of variation in soil parameters. As can be seen from Table 4, if this design is implemented in a sand site with $COV[\phi'] = 0.10$, the SLS failure requirement will no longer be satisfied, as the failure probability will be greater than the target probability of failure of $p^{SLS}_{T} = 0.0047$. 

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Reliability-Based Design Considering Robustness

To investigate the issue of design robustness, the same drilled shaft problem (see Figure 1 and Table 1) is analyzed with additional knowledge of the variation of $\text{COV}[\phi']$, $\text{COV}[K_0]$ and $\rho_{\phi,K_0}$. Specially, the reliability-based design is based on the following additional data:

\[
\begin{align*}
\mu_{\text{COV}[\phi']} &= 0.07, & \sigma_{\text{COV}[\phi']} &= 0.125, & \mu_{\text{COV}[K_0]} &= 0.50, & \sigma_{\text{COV}[K_0]} &= 0.175, & \mu_{\rho_{\phi,K_0}} &= -0.75, & \sigma_{\rho_{\phi,K_0}} &= 0.075.
\end{align*}
\]

It should be noted that these values are just used as an example to illustrate the concept of robustness in the design, although they are deemed appropriate based on an assessment of these parameters presented previously.

Following the procedure described previously (in reference to Figure 2), the mean and standard deviation of the probability of SLS failure $p_{f}^{\text{SLS}}$, namely $\mu_{p}$ and $\sigma_{p}$, can be obtained for various designs (i.e., various pairs of $B$ and $D$). Figure 5 shows the mean SLS failure probability ($\mu_{p}$) for various designs. As can be seen, many designs have a mean failure probability greater than the target failure probability ($p_{T}^{\text{SLS}} = 0.0047$). Figure 6 shows the standard deviation of the SLS failure probability ($\sigma_{p}$). Note that in this figure, only acceptable designs (those that have a $\mu_{p}$ less than the target failure probability) are plotted.

It is noted that the robustness of a reliability-based design, in which acceptance of the design is based on the requirement that $p_{f}^{\text{SLS}} < p_{T}^{\text{SLS}}$, may be measured in terms of the standard deviation of the SLS failure probability ($\sigma_{p}$). Thus, a design is said to have a greater robustness if $\sigma_{p}$ caused by the uncertainty of $\text{COV}[\phi']$, $\text{COV}[K_0]$ and $\rho_{\phi,K_0}$ is smaller. In Figure 6, all designs are acceptable based on the traditional reliability-based design concept. If
the cost is the only design objective after satisfying the SLS failure requirement, then the
design of $B = 0.9$ m and $D = 6.0$ m will be selected, which has the least cost of 1550USD.
However, this design has the greatest standard deviation of the SLS failure probability,
indicating that the design has the lowest level of robustness or is most sensitive to the
uncertainty in the estimated statistics of soil parameters. The implication is that such design,
albeit most economical from the traditional reliability-based design viewpoint, is likely to fail
the SLS failure requirement if the uncertainty of soil parameters statistics is ignored. On the
other hand, if a design with a smaller $\sigma_p$ is chosen, it will be more robust, albeit at a higher
cost.

To further illustrate the trade-off between cost and robustness, all acceptable designs
($\mu_p < P_{F_{SLS}} = 0.0047$) are computed for their costs, and these costs are plotted against robustness
(in terms of standard deviation of the SLS failure probability, $\sigma_p$). The results are shown in
Figure 7. Three designs, as indicated in Figure 7, are used as an example for further discussion
of the trade-off between cost and robustness. Design 1 ($B = 0.9$ m, $D = 6.0$ m) has the least cost
among all acceptable designs, while Design 2 ($B = 0.9$ m, $D = 6.4$ m) and Design 3 ($B = 0.9$ m,
$D = 7.0$ m) cost more but have a smaller $\sigma_p$ value (meaning that the designs are more robust).
For each of these three designs, the probability of SLS failure is reanalyzed for various COV
and correlation levels for the two soil parameters, $\phi'$ and $K_0$. The results of these additional
analyses are shown in Table 5. It is noted that the uncertainty levels of 27 cases for loose sand
shown in Table 5 can be roughly divided into three categories, Low variation ($COV[\phi'] = 0.05$
and $COV[K_0] = 0.2$), Medium variation ($COV[\phi'] = 0.07$ and $COV[K_0] = 0.5$), and High variation ($COV[\phi'] = 0.10$ and $COV[K_0] = 0.9$).

Based on the results shown in Table 5, Design 1 will perform satisfactory (in terms of meeting the requirement that $P_I^{SLS} < P_{II}^{SLS} = 0.0047$) if the variation of soil parameters for this sand is Low or Medium. However, Design 1 does not meet the SLS failure probability requirement if it is implemented in a site with high variation in $\phi'$ and $K_0$. Thus, for this design, the SLS failure probability is sensitive to the level of uncertainty of soil parameters.

On the contrary, with Design 3, the SLS failure probabilities under all uncertainty scenarios meet the requirement. Thus, for this design, the SLS failure probability is insensitive to the uncertainty in the COV levels of soil parameters. However, the cost is 1808USD for Design 3, compared to the cost of 1550USD for Design 1. Finally, Design 2 is a compromise between Design 1 and Design 3, in which the increase in cost is not as prohibitive (only 1653USD) but it still has some chance of violating the SLS failure probability requirement.

The results presented above suggest that design aids are needed for making more informed engineering decisions. In the sections that follow, the concept of Pareto Front is presented to explain the trade-off relationship between cost and robustness, followed by a procedure for selecting better designs.

Two-Objective Non-dominated Sorting for Pareto Front

Pareto-Front consists of designs that are not dominated by other designs with respect to
all design objectives. Design A is \textit{dominated} by Design B if A is inferior or equal to B in \textit{every} objective measure, except one scenario that the performance of A is equal to the performance of B in all objectives (Cheng and Li 1997). If a design is not dominated by any other designs, it belongs to the Pareto Front.

A non-dominant sorting technique of the Non-dominated Sorting Genetic Algorithm II (NSGA-II), developed by Deb et al. (2002), is used in this paper to select the non-dominated designs with multiple objectives, which are points on the Pareto Front. For the geotechnical design of drilled shaft in sand, this multi-objective optimization may be set up as follows:

\textbf{Find} \quad \mathbf{d} = [B, D]

\textbf{Subject to:} \quad B \in \{0.9m, 1.2m, 1.5m\} \quad \text{and} \quad D \in \{2m, 2.2m, 2.4m, \ldots, 8m\}

\begin{align*}
\mu_p^{\text{ULS}} &\leq p_T^{\text{ULS}} = 0.00069 \\
\mu_p^{\text{SLS}} &\leq p_T^{\text{SLS}} = 0.0047
\end{align*}

\textbf{Objectives:} \quad \text{Minimizing the standard deviation SLS failure probability (} \sigma_p \text{)}

\quad \text{Minimizing the cost for drilled shaft.}

As reflected in the optimization set-up stated above, the design is to be selected from a finite set of B and D pairs. The target probabilities of failure based on SLS and ULS requirements can both be specified but generally, the design of drilled shafts in sand is controlled by the SLS failure probability requirement. It should be noted that the ULS requirement \( \mu_p^{\text{ULS}} < p_T^{\text{ULS}} = 0.00069 \) is based on a reliability index of 3.2, and the SLS
requirement $\mu_{p}^{SLS} < P_{f}^{SLS} = 0.0047$ is based on a reliability index of 2.6 (Wang et al. 2011a).

Although the multi-objective optimization as prescribed above can easily be carried out using NSGA-II, the number of possible designs in the design space in this drilled shaft example is finite and relatively small ($M = 93$). Thus, only the non-dominated sorting technique of NSGA-II is applied herein. Among the 93 designs, 56 are found acceptable based on the SLS and ULS failure probability requirements. With the non-dominated sorting, 27 of the 56 acceptable designs are selected into the Pareto Front (with two objectives, cost and robustness in terms of $\sigma_p$), as shown in Figure 8. It should be noted that the non-dominated sorting is generally more efficient if a larger number of acceptable designs is to be sorted for Pareto Front, especially when more design parameters are involved and/or the interactions between the design parameters and noise factors are much more complex.

While the traditional reliability-based design approach often selects the best design based solely on cost, after satisfying the failure probability requirements, the reliability-based robust design considers robustness in addition to cost. In the case of drilled shaft design, optimization of both cost and robustness yields a Pareto Front, which enables the engineer to make informed decision based on a well-defined trade-off relationship between cost and robustness against the possible soil parameters variability. If a certain maximum cost is desired (i.e., the cost must be less than some desired amount), then the design with greatest robustness will be the best choice. If a certain minimum level of robustness is desired, then the design with least cost will be the best choice.
Selection of the best design based on feasibility robustness

Although the Pareto Front provides a well-defined trade-off relationship between cost and robustness, it is desirable to take the process further to ease decision making. Here, the concept of “feasibility robustness” (Parkinson et al. 1993) is further adopted. The design with feasibility robustness is the design that can remain “feasible” (i.e., acceptable in terms of satisfying the safety and serviceability requirements) in a pre-defined constraint for certain probability even when it undergoes variations. In this paper, the feasibility robustness is the robustness against the SLS failure requirement, \( p_{f}^{SLS} < p_{f}^{SLS} = 0.0047 \). Because of the uncertainty in the estimated sample statistics, \( COV[\phi'] \), \( COV[K_0] \) and \( \rho_{\phi,K_0} \), the SLS failure probability \( p_{f}^{SLS} \) may be treated as a random variable. Parkinson et al. (1993) suggest that feasibility robustness can be expressed with the following constraint:

\[
Pr[(p_{f}^{SLS} - 0.0047) < 0] \geq P_0
\]  
(7)

where \( Pr[(p_{f}^{SLS} - 0.0047) < 0] \) is the probability that the SLS failure requirement can be satisfied (and thus, the system is still feasible), and \( P_0 \) is an acceptable level of this probability selected by the designer. The probability \( Pr[(p_{f}^{SLS} - 0.0047) < 0] \) is referred to herein as the feasibility probability.

Determination of the probability \( Pr[(p_{f}^{SLS} - 0.0047) < 0] \) requires the knowledge of distribution type of \( p_{f}^{SLS} \), which is generally difficult to ascertain. Simulations of a given design (for example, \( B = 0.9 \text{ m}, D = 6.8 \text{ m} \)) show that the resulting histogram of \( \beta^{SLS} \) can be
approximated well with a lognormal distribution, as depicted in Figure 9. Thus, an equivalent
counterpart in terms of \( \Pr[(\beta^{SLS} - \beta_T^{SLS}) > 0] \), where \( \beta_T^{SLS} = 2.6 \) (corresponding to
\( p_T^{SLS} = 0.0047 \), may be used to assess the feasibility robustness.

The mean and standard deviation of \( \beta^{SLS} \), denoted as \( \mu_\beta \) and \( \sigma_\beta \), respectively, can
be determined using FORM within the framework of PEM. When \( \beta^{SLS} \) is assumed to follow
lognormal distribution, the feasibility probability can be computed using simplified procedure
such as first order second moment (FOSM) method as follows (Juang and Wang 2013):

\[
\Pr[(\beta^{SLS} - 2.6) > 0] = \Phi(\beta_\beta)
\]

where \( \Phi \) is the cumulative standard normal distribution function, and \( \beta_\beta \) is defined as:

\[
\beta_\beta = \frac{\ln \left[ \frac{\mu_\beta}{\sqrt{1 + \left( \frac{\sigma_\beta}{\mu_\beta} \right)^2}} \right] - \ln(2.6)}{\sqrt{\ln \left[ 1 + \left( \frac{\sigma_\beta}{\mu_\beta} \right)^2 \right]}}
\]

If the acceptable level of the feasibility probability is specified as \( P_0 = 97.72\% \), then
the required \( \beta_\beta \) value will be 2. In other words, if the \( \beta_\beta \) computed based on \( \mu_\beta \) and \( \sigma_\beta \)
is equal to 2, then there is a feasibility probability of 97.72\% that the SLS failure requirement
\( (p_f^{SLS} < p_T^{SLS} = 0.0047 \) or equivalently \( \beta^{SLS} > \beta_T^{SLS} = 2.6 \)) is satisfied.

Thus, the \( \beta_\beta \) value may be used as an index for feasibility robustness. Figure 10
shows the \( \beta_\beta \) values computed for all 27 points on the Pareto Front versus the corresponding
costs. As expected, the results show that a design with higher feasibility robustness costs more.
By selecting a desired feasibility robustness level (in terms of $\beta_{\beta}$), the least-cost design among those on the Pareto Front can readily be determined. Table 6 shows final designs selected from the Pareto Front for various specified feasibility robustness levels. As a reference, it is observed that the final design obtained for the feasibility robustness level of $\beta_{\beta} = 2$, namely $B = 0.9$ m and $D = 6.8$ m, is approximately the same as the threshold acceptable design that was obtained by the traditional reliability-based design under the higher-end level of soil variability that was examined in Table 3. The developed Pareto Front, especially with the computed feasibility robustness, makes it easier to select the best design to meet the designer’s objectives.

### Integration of RGD with Load and Resistance Factor Design (LRFD)

In the above sections, we have shown how the robust design concept can be integrated with rigorous reliability methods such as FORM. To this end, the reliability-based RGD method is proposed. In the current geotechnical practice, however, reliability-based design is often carried out using the LRFD approach. From the user’s perspective, the LRFD approach is a deterministic approach, although the parameter and model uncertainties may have been considered in the calibration of resistance factors using a database of load test results. The LRFD approach is often regarded as an approximate way to achieve the objective of a reliability-based design, which is to maintain an acceptable and consistent safety level throughout the design of all components. Because it is a deterministic approach and does not require statistical characterization of soil parameters on the part of the user, the LRFD approach is easier to use and more practical. However, the desire for simplicity and practicality naturally comes at the expense of the flexibility and versatility that are possible with the reliability-based design. For example, as noted by Wang et al. (2011b), LRFD codes are only calibrated for some
predefined value of target failure probability, which limits designer’s flexibility in selecting a proper target failure probability for a particular project. Furthermore, the adopted LRFD code may not address the effect of site-specific soil variability and the dependency of the bias factor on the design parameters (Kulhawy et al. 2012). Previous studies by Phoon et al. (2003) and Paikowsky et al. (2004) also recognized the shortcoming of the LRFD code in dealing with these issues.

Development of “robust” resistance factors for LRFD so that the result (i.e., final design) is insensitive to (or robust against) the “noises” such as site-specific soil variability and the dependency of the bias factor on the design parameters is one possible solution to dilemma discussed previously. In this paper, however, a simpler approach is taken, which utilizes the current LRFD approach with fixed resistance factors. In the remaining of this section, we demonstrate how robust design can be integrated into the current LRFD approach.

The general form of a LRFD method (Phoon et al. 1995; Phoon et al. 2003; Paikowsky et al. 2004; AASHTO 2007; FHWA 2010; Roberts et al. 2011; Kulhawy et al. 2012; Basu and Salgado 2012) may be expressed as follows:

\[ \eta F_n \leq \Psi Q_n \]  

(10)

where \( \eta \) is load factor (≥1); \( \Psi \) is resistance factor (≤1); \( F_n \) is the nominal load; \( Q_n \) is the nominal resistance. The load factor \( \eta \) from structural engineering is generally adopted in foundation design to maintain design consistency (Ellingwood et al. 1982a,b).

The LRFD method has been successfully used in structural engineering, where the variation of input parameters and nominal resistance is typically low. However, the variation for soil parameters can be much higher, which demands a smaller resistance factor to compensate for the effect of larger variation in the nominal resistance. As a user of a particular
LRFD code or method, however, the resistance factor is fixed. Thus, in a site where the variation of soil parameters is high, it would be a challenging task to select nominal parameter values for design regardless of whether the LRFD or the traditional factor-of-safety-based approach is taken.

The focus of this section is to investigate how the proposed RGD methodology can be integrated with LRFD method. In particular, we seek an answer to the question, “how can the design of drilled shafts by means of the LRFD method be “robust” in the face of uncertainty?” In other words, as a user, how do we ensure that the design requirement as specified in Eq. (10) is satisfied with the estimated nominal parameter values?

To answer the above question, let us consider the design example of drilled shaft in sand subjected to drain compression loading (see Figure 1). The LRFD formulation developed with generalized reliability theory (Phoon et al. 1995) is adopted as the deterministic model. Specifically, the ULS and SLS design equations for drilled shaft under drained compression is written as:

\[ F_{50} = \Psi_s Q_{sN} + \Psi_t Q_{tN} - \Psi_w W \]  
\[ F_{50} = \Psi_c Q_{cN} \]

where \( F_{50} \) is the factored load based on 50-year return period, which is the left-hand-side of Eq. (10); \( \Psi_s \) is the resistance factor for side resistance; \( Q_{sN} \) is the nominal side resistance; \( \Psi_t \) is the tip resistance factor; \( Q_{tN} \) is the nominal tip resistance; \( \Psi_w \) is the weight resistance factor; \( W \) is the foundation weight; \( \Psi_c \) is the deformation factor; \( Q_{cN} \) is the nominal allowable compression capacity. The resistance factors are given by Phoon et al. (1995) as follows:

\( \Psi_s = 0.42, \quad \Psi_t = 0.39, \quad \Psi_w = 0, \) and \( \Psi_c = 0.56. \) The nominal resistances \( Q_{sN}, Q_{tN}, \) and \( Q_{cN} \) are
calculated based on nominal values of soil parameters, including $\phi'$ and $K_0$. The reader is referred to Phoon et al. (1995) for detailed equations for $Q_{sN}$, $Q_{tN}$, and $Q_{cN}$.

As discussed previously, the determination of nominal values for $\phi'$ and $K_0$ involves certain degree of uncertainty. For illustration purpose, let us assume site investigation and testing program yields statistics of these soil parameters as those shown in Table 1. These sample statistics are considered *fixed* values. With the recognition of the variation of these soil parameters as shown in Table 1, the LRFD requirements may be re-written as limit states:

$$g_{ULS}() = \Psi_s Q_{sN} + \Psi_t Q_{tN} - \Psi_w W - F_{50}$$

$$g_{SLS}() = \Psi_c Q_{cN} - F_{50}$$

(13)

(14)

In a deterministic approach, a design is said to satisfy the LRFD requirements if $g_{ULS} > 0$ and $g_{SLS} > 0$. Because of the uncertainty of the soil parameters involved, however, the predicted values of $Q_{sN}$, $Q_{tN}$, and $Q_{cN}$, and thus $g_{ULS}$ and $g_{SLS}$, are no longer fixed numbers. Instead of checking whether the LRFD requirements of $g_{ULS} > 0$ and $g_{SLS} > 0$ are satisfied in a yes-or-no manner, new criterion is needed. To this end, the concept of “feasibility robustness” described previously is again employed. Here, robust design is aimed at finding a design (i.e., a pair of design parameters $B$ and $D$) so that a certain confidence level (i.e., a probability) can be achieved that the LRFD requirement will be satisfied in the face of uncertainty. In other words, the design can remain “feasible” in terms of satisfying the LRFD requirement at a prescribed probability level.

For the drilled shaft example analyzed, the probability of satisfying or exceeding ULS-based LRFD requirement, denoted as $Pr[g_{ULS}() > 0]$, is always greater than the probability
of satisfying or exceeding SLS-based LRFD requirement, denoted as $\Pr[g_{SLS} > 0]$, for all
designs in the design space (illustrated later in Figure 11). Thus, the SLS-based LRFD
requirement controls the design in this example, and only this requirement is focused in the
subsequent discussion. The feasibility robustness requirement is defined as:

$$
\Pr[g_{SLS} > 0] = \Pr[(\Psi, Q_{SN} - F_{S0}) > 0] \geq P_0
$$

where $\Pr[(\Psi, Q_{SN} - F_{S0}) > 0]$ is the probability of satisfying or exceeding the SLS-based LRFD
requirement, which is a measure of feasibility robustness; and $P_0$ is a target feasibility
probability. A requirement of $P_0 = 0.5$ is approximately equivalent to the deterministic
SLS-based LRFD requirement assessed with the mean nominal parameter values. A
requirement of $P_0 > 0.5$ is desirable and needed to assure a higher chance of satisfying or
exceeding the LRFD requirement in the face of parameter uncertainty.

For convenience of presentation, the probability of satisfying or exceeding the LRFD
requirement, $\Pr[(\Psi, Q_{SN} - F_{S0}) > 0]$, is referred to hereinafter as the probability of exceedance
($P_E$). Figure 11 shows the probability of satisfying or exceeding the ULS- and SLS-based
LRFD requirements, denoted as $P_E^{ULS}$ and $P_E^{SLS}$, respectively. Designs with $P_E > 0.5$ are
desired as noted previously.

Assuring a higher probability of exceedance in the face of uncertainty will likely cost
more. Thus, in the face of uncertainty, the essence of robust design is to find a design (i.e., a
pair of design parameters $B$ and $D$) that satisfies the deterministic LRFD requirement,
maximizes the probability of exceedance, and minimizes the cost. As was discussed previously,
a single best design that satisfies all requirements usually does not exist. Rather, a Pareto Front
may exist that offers a set of non-dominated designs. Thus, a multi-objective optimization can be set up to find the Pareto Front:

Find \( \mathbf{d} = [B, D] \)

Subject to: \( B \in \{0.9\text{m}, 1.2\text{m}, 1.5\text{m}\} \) and \( D \in \{2\text{m}, 2.2\text{m}, 2.4\text{m}, \ldots, 8\text{m}\} \)

\[ \mu_g^{ULS} > 0 \text{ and } \mu_g^{SLS} > 0 \]

Objectives: Maximizing the probability of exceedance

Minimizing the cost for drilled shaft.

It is noted that the deterministic LRFD requirement is satisfied by the constraints of \( \mu_g^{ULS} > 0 \) and \( \mu_g^{SLS} > 0 \), where \( \mu_g^{ULS} \) and \( \mu_g^{SLS} \) are the mean values of \( g_{ULS} \) and \( g_{SLS} \), respectively. These mean values may be determined with sample statistics shown in Table 1 using PEM method (Zhao and Ono 2000). Alternatively, they may be approximately determined using the deterministic approach with mean input parameter values. It should be noted that the constraints of \( \mu_g^{ULS} > 0 \) and \( \mu_g^{SLS} > 0 \) are equivalent to the constraints based on the probability of exceedance \( P_E^{ULS} > 0.5 \) and \( P_E^{SLS} > 0.5 \).

The probability of exceedance \( P_E \) (or more specifically, \( P_E^{SLS} \), since it controls the design of drilled shaft in sand) may be obtained through a reliability analysis using FORM. Greater probability of exceedance signals a more robust design in the face of uncertainty. Alternatively, the mean and standard deviation of \( g_{SLS} \), denoted as \( \mu_g^{SLS} \) and \( \sigma_g^{SLS} \), respectively, may be determined using PEM (Zhao and Ono 2000), from which the probability of exceedance can be determined. Furthermore, from the perspective of robust design, we want...
to maximize $\mu_{g}^{SLS}$ (for safety) and minimize $\sigma_{g}^{SLS}$ (for robustness). Thus, the ratio of $\mu_{g}^{SLS}$ over $\sigma_{g}^{SLS}$ can also serve as a robustness measure. In fact, it is analog to the signal-to-noise ratio (SNR) commonly used in the fields of Industrial Engineering and Electrical Engineering (Taguchi 1986). For convenience, this robustness measure is referred to herein as Robustness Index ($R_I$). It should be noted that this Robustness Index ($R_I$) is analog to the reliability index using FORM that adopts Eq. (14) as limit state.

Figure 12 shows the Pareto Front obtained from non-dominant sorting procedure of NSGA-II, which shows a more robust design (greater Robustness Index) generally costs more in this case. Here, selected levels of probability of exceedance are also plotted as a reference to the Robustness Index. Note that each point on the Pareto Front represents a non-dominated design, a unique set of $B$ and $D$ in this drilled shaft example. The least cost design on this Pareto Front is $B = 0.9$ m and $D = 6.4$ m, which has a cost of 1653USD and a Robustness Index of $R_I = 0.05$ (corresponding to a probability of exceedance of $P_E = 0.522$ or 52.2%). If the probability of exceedance of $P_E = 0.7$ (or 70%) is desired, the least cost design on the Pareto Front would be $B = 0.9$ m and $D = 7.0$ m with a cost of 1808USD. Table 7 shows examples of final designs (least cost designs) selected from the Pareto Front for various target Robustness Index values. Thus, the developed Pareto Front makes it easier to select the best design to meet the designer’s objectives.

Further Discussions

The results presented previously clearly illustrated the need for, and the significance and solution of, robust design to handle the uncertainty in the noise factors. Although the robust
geotechnical design (RGD) methodology presented is far from perfect, and indeed several outstanding issues are still being examined in an ongoing study, this paper is considered a first step, and an important step, in developing the RGD methodology. A brief description of the issues that are being investigated is provided below.

First, the advantages of Pareto Front for identifying the best designs of drilled shaft as presented in this paper are not fully realized, as the number of possible designs in the design space is finite and relatively small in the example presented. In this case, the robustness and cost of each possible design can be calculated, as there are only a limited number of combinations of \( B \) and \( D \). The advantages of using Pareto Front will become more obvious when more design parameters are involved, more selections of discrete design parameters are implemented (so that the discrete variables are getting closer to being continuous random variables), and/or the interactions between the design parameters and noise factors are more complex. For example, in an ongoing study of robust design of a braced excavation system, the advantages of Pareto Front for identifying the best designs through multi-objectives optimization become more obvious.

Second, robust design concept can be implemented to a deterministic (i.e., factor of safety-based or LRFD) approach or a probabilistic (i.e., reliability-based) approach. Robustness concept may be implemented in different ways to adapt to the domain problem and/or the solution approach (deterministic or probabilistic approach). In either approach, the presented RGD methodology can be adjusted slightly to adapt to the domain problem.

Third, although the robustness concept has been demonstrated in this paper, further studies to consider robustness against other sources of uncertainty are warranted. In particular, design robustness against the following uncertainties may also be considered: (1) the
distribution type of the input random variables (noise factors), (2) the effect of spatial
correlation distance, (3) the loading complexity, and (4) the effect of construction noise.

**Summary and Concluding Remarks**

Robustness as one of the design objectives has been illustrated in this paper. In fact, the
concept of robustness is incorporated into the reliability-based design to deal with the
uncertainty in the estimated sample statistics of soil parameters, which is often a major problem
in a reliability-base design. In the context of robust geotechnical design of drilled shafts for
axial load in sand, B (diameter) and D (depth or length) are considered as the design parameters
(denoted as \( d \)), and the soil parameters \( \phi' \) and \( K_0 \) are considered as the noise factors (denoted
as \( z \)). In the reliability-based design, the safety and serviceability requirements are satisfied by
meeting the constraint, \( p_f^{SLS}(d, z) \leq p_f^{SLS} = 0.0047 \). It is noted that probability of the SLS failure
\( p_f^{SLS}(d, z) \) is a random variable, the value of which depends on both design parameters \( d \) and
noise factors \( z \). The essence of robustness design is to minimize the variation of \( p_f^{SLS}(d, z) \)
caused by the uncertainty in the estimated sample statistics of soil parameters by adjusting the
design parameters.

To consider the robustness of the design against the uncertainty in the estimated sample
statistics of soil parameters, the standard deviation of the SLS failure probability \( p_f^{SLS}(d, z) \) is
adopted as a measure of robustness. It is considered along with cost as the design objectives,
and as a result, a Pareto Front is established through non-dominated sorting. This Pareto Front
gives a trade-off relationship between cost and robustness. To improve the decision making process further, the concept of feasibility robustness is adopted. Through an implementation of feasibility robustness, the best design can be selected from the Pareto Front based on the designer’s objectives.

The Robust Geotechnical Design (RGD) methodology is demonstrated through an application to design of drilled shafts in sand. Although the advantages of this RGD methodology have not been fully realized because the problem is fairly simple (since the design space consists of finite pairs of two design parameters $B$ and $D$), the methodology has been shown effective in addressing the issue of design robustness. Furthermore, the RGD methodology yields a Pareto Front that describes a trade-off relationship between cost and robustness. Finally, the index for feasibility robustness is shown to be an effective design aid that can be used to select the best design from a Pareto Front.

The RGD methodology can also be applied to a deterministic design approach such as LRFD method. Robustness Index defined in this paper is shown effective as a robustness measure for implementing robust design concept in the drilled shaft design using LRFD. As in the reliability-based RGD, the dilemma of the parameter uncertainty at a given site is overcome with the Pareto Front developed by implementing robustness in the LRFD approach.

It should be noted that the RGD is not a methodology to compete with the traditional design approaches; rather, it is a complementary design strategy. With the RGD approach, the focus is to satisfy three design objectives, namely safety (including strength and serviceability
requirements), cost, and robustness. Robustness, which is often not considered explicitly in geotechnical design, may have to be achieved at a higher cost, and thus, development of Pareto Front as a design aid through multi-objective optimization is often required for trade-off consideration in the design. This paper is a first step in developing the RGD methodology. Further investigation is warranted to advance this design methodology.

ACKNOWLEDGMENTS

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Table 6. Selected reliability-based RGD designs at various feasibility robustness levels

Table 7. Selected LRFD-based robust designs at various Robustness Index levels
Table 1. Sample statistics of soil parameters

<table>
<thead>
<tr>
<th>Soil Parameter</th>
<th>Type of Distribution</th>
<th>Mean</th>
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<tr>
<td>Effective friction angle, $\phi'$</td>
<td>Lognormal</td>
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<td>Coefficient of earth pressure at rest, $K_0$</td>
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Note: The correlation coefficient between $\phi'$ and $K_0$ is -0.75.
Table 2. Summary of drilled shaft unit construction cost (data from R.S. Means Co. 2007)

<table>
<thead>
<tr>
<th>Drilled shaft diameters, B (m)</th>
<th>National average unit construction cost (USD) for shaft depth D = 0.3m</th>
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Table 3. Least-cost designs under various COV and correlation assumptions for soil parameters

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Table 4. SLS failure probability of a given design (B= 0.9m, D= 5.6m) under various COV and correlation assumptions

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Table 5. Comparison of SLS failure probability for three designs under various COV and correlation assumptions for soil parameters

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SLS failure probability, $p^{SLS}$

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Table 6. Selected reliability-based RGD designs at various feasibility robustness levels

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Table 7. Selected LRFD-based robust designs at various Robustness Index levels

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Lists of Figures

Figure 1. An example drilled shaft under drained compression (data from Wang et al. 2011a)

Figure 2. Flowchart illustrating robust geotechnical design of drilled shaft

Figure 3. Conceptual illustration of a Pareto Front in a bi-objective space

Figure 4. Probability of failure obtained using FORM with $COV[\phi'] = 7\%$ $COV[K_o] = 50\%$ $\rho_{\phi',K_o} = -0.75$ : (a) SLS failure; (b) ULS failure

Figure 5. Mean of the SLS failure probability using the PEM procedure

Figure 6. Standard deviation of the SLS failure probability for all acceptable designs using the PEM procedure

Figure 7. Relationship between cost and standard deviation of the SLS failure probability (all acceptable designs are shown, including three arbitrarily selected designs)

Figure 8. Pareto Front based on two-objective non-dominated sorting

Figure 9. Distribution of reliability index for a given design ($B = 0.9$ m, $D = 6.8$ m)

Figure 10. Cost versus feasibility robustness for all designs on Pareto Front

Figure 11. Probability of exceeding ULS- and SLS-based LRFD requirements

Figure 12. Cost versus Robustness Index for all designs on Pareto Front