#### **RESEARCH ARTICLE**



# Scaling analysis of the Plateau–Rayleigh instability in thin film flow down a fiber

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## Abstract

A thin viscous film flowing down a fiber is subject to Plateau–Rayleigh instability leading to temporal beading patterns, characterized by the bead spacing  $S_b$ , bead velocity  $V_b$ , and bead diameter  $D_b$ . In this study, experiments are performed to document the beading pattern as it depends upon the flow rate Q, nozzle diameter  $D_n$ , fiber diameter  $D_f$  and liquid viscosity  $\mu$ . The large experimental data set collapses upon scaling with the Nusselt solution for gravity-driven thin film flow with viscous time scale showing the dimensionless bead diameter scales with the capillary number. The dimensionless bead frequency and bead spacing are similarly related and characterize the dispersion relationship.

## **Graphic abstract**



## **1** Introduction

A thin film flowing down a fiber will destabilize into a unique pattern that defines the bead-on-fiber morphology. Such flows experience a number of interfacial instabilities such as Plateau–Rayleigh breakup, Kapitza waves, isolated bead formation, and convective instabilities. These thin film flow instabilities play a critical role in microfluidics (Gilet et al. (2009)), heat and mass transfer applications

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<sup>1</sup> Department of Mechanical Engineering, Clemson University, Clemson, SC 29631, USA (Sadeghpour et al. (2019), Zeng et al. (2018, 2017), Chinju et al. (2000), Grünig et al. (2012), Hosseini et al. (2014)), and coating processes (Mashayek and Ashgriz (1995), Tu and Ellen (1986), Lin and Liu (1975)). Previous experimental investigations of thin film flows down fibers used intricate nozzle-fiber systems in order to keep the fiber concentric to the nozzle. These complex setups require the flow to be fed through the nozzle via an overhead tank, requiring the resulting flow rate to be measured during the experiment. In this work, we perform an experimental investigation of thin film flow down a fiber using a nozzlefiber system which provides a simpler, more robust setup where the flow rate can be set manually and is suitable for research applications and classroom demonstrations.

We focus our investigation on liquid beads formed via the Plateau-Rayleigh instability (PRI) (Plateau (1873), Rayleigh (1878)), a surface tension-driven phenomenon. The PRI is observed in numerous applications and continues to be heavily explored with recent studies evaluating the role of viscosity on breakup time (Javadi et al. (2013)), the stability of liquid bridges (Lowry and Steen (1995), Bostwick and Steen (2015)), and the role of liquid/solid contact (Bostwick and Steen (2010, 2018)). The PRI is also observed when a liquid film coats a fiber. Several investigations have documented the emergence of the PRI and the resulting bead profiles in this situation (Carroll (1984, 1986), McHale et al. (1999, 2001), McHale and Newton (2002)). The addition of a non-trivial base flow results in a unique variation of PRI where convective instabilities emerge, resulting in both steady and unsteady temporal beading patterns. Kliakhandler et al. (2001) documented three unique flow regimes based on the beading pattern: i) isolated, ii) Plateau-Rayleigh, and iii) convective. Analyzing the influence of slip between the liquid and fiber, Haefner et al. (2015) showed the hydrodynamic boundary conditions at the solid-liquid interface do not affect the dominate wavelength but do affect the growth rate of undulations. Alternatively, a thin fiber can be withdrawn from a liquid bath, as is common in many coating applications. Quéré (1999) and Weinstein and Ruschak (2004) provide extensive reviews on this alternative problem and note the dependency of the film thickness on the withdrawal speed and fiber radius.

Assuming a small fiber radius and ignoring gravitational effects, Goren (1962) conducted a linear stability analysis on the Navier-Stokes equations for the limiting cases of Stokes flow and inviscid flow and found analytical expressions for the growth rate of perturbations as a function of the wavelength. The inclusion of gravity allows the axial flow to affect the capillary instability, a scenario initially investigated by Goucher and Ward (1922) and Kapitza and Kapitza (1964). Their work on the interaction between gravity-induced flow and capillary instability was followed by experimental work by Quéré (1990) who determined a critical film thickness for surface undulations to appear. Kalliadasis and Chang (1994) adopted the scaling arguments presented by Frenkel (1992) and determined a critical thickness  $h_c$  for beads to develop and defined a dimensionless saturation number  $\beta$  for which they showed that when  $\beta > \beta_c = 1.41$  unbounded growth occurs corresponding to experiments by Quéré (1990). Duprat et al. (2007) expand upon this by exploring the transition between convective and absolute instability in the inertialess limit and showed this transition occurs at a critical value  $\beta^* = 1.507$  where  $\beta^*$ is a generalization of the saturation number  $\beta$ . Both  $\beta$  and  $\beta^*$  are a function of only the aspect ratio  $\tilde{\alpha} = h_N/(D_f/2)$  and the Goucher number  $Go = (D_f/2)/l_c$  where  $h_N$  is the Nusselt flat film solution for the film thickness,  $D_f$  is the fiber diameter, and  $l_c$  is the capillary length defined by  $l_c = \sqrt{\sigma/(\rho g)}$ .

Furthermore, Duprat et al. (2009) identified two distinct, liquid-dependent regimes in the  $\tilde{\alpha} - Go$  parameter space, the drag-gravity and drag-inertia regime, where the behavior of the traveling beads is expected to be governed by gravity and inertia, respectively. Ruyer-Quil and Kalliadasis (2012) later identified a soliton-like regime corresponding to the balance of the nonlinearities with the dispersion induced by second-order viscous effects. This additional regime may also be presented in the  $\tilde{\alpha} - Go$  space, resulting in four total instability regimes. We focus our work in the drop-like regime dominated by the PRI.

Several experimental investigations have been conducted to uncover the dependence of the bead properties on the relevant experimental parameters. Smolka et al. (2008) performed experiments with three different liquids and documented how their properties affect the resulting bead properties. Nozzle diameter effects have been explored by both Sadeghpour et al. (2017) and Ji et al. (2020) where they show how the nozzle diameter is critical in determining the observed regime. Interestingly, their work shows that any of the three regimes is observable simply by altering the nozzle diameter. In each of these investigations, the nozzle-fiber system is constructed such that the fiber is concentric with the nozzle allowing the flow to start uniformly wrapped around the fiber but requiring the mass flow rate to be measured by collecting and weighing the fluid flowing down the fiber. Herein, we present a simple experimental setup that does not require such complex nozzle-fiber system and allows the flow rate to be set prior to the experiment, ensuring a constant flow rate throughout the duration of each test. We use this setup to perform a large experimental investigation in which we report how the i) bead diameter, ii) bead spacing, and iii) bead velocity, depend upon the nozzle diameter, fiber diameter, viscosity, and flow rate. Trends with the experimental parameters are reported, and we show that our experimental data collapse upon scaling with the Nusselt flat film solution with viscous time scale.

## 2 Experiment

The experimental setup is shown in Fig. 1a. A NE-1000 syringe pump is used to pump liquid at a flow rate Q through a stainless steel nozzle of diameter  $D_n$  onto a nylon fiber of diameter  $D_f$ . The nozzle is oriented perpendicular to the fiber allowing the exiting fluid to naturally wrap around the fiber which is set in a vertical orientation using a small weight and then secured into position using two pinning devices located at the top and bottom of the testing apparatus. The length of the fiber is 550 mm, which is sufficient for the beading patterns to become fully developed. We note that, despite applying the liquid asymmetrically, the thin film wrapped symmetrically around the fiber within the first 50

mm and this gave rise to the symmetric beading patterns studied here. Recent observations by Gabbard and Bostwick (2021) have shown that an asymmetric instability emerges for liquids with larger surface tension (glycerol/water mixtures) and large fiber diameters. This is outside the scope of the current study which focuses on symmetric beading patterns.

The experimental variables we investigate include the flow rate Q, nozzle diameter  $D_n$ , fiber diameter  $D_f$ , and fluid properties: viscosity  $\mu$  and surface tension  $\sigma$ . The range of Q was 5-300 mL/hr,  $D_n$  was 0.4-3.3 mm, and  $D_f$  was 0.101-0.508 mm. Silicone oils and mineral oil were used as the working liquid and provided a density range  $\rho$  of 848-974 kg/m<sup>3</sup>, surface tension range  $\sigma$  of 22-25 mN/m and viscosity range  $\mu$  of 9.4-974 mPa · s. The density  $\rho$ , viscosity  $\mu$  and surface tension  $\sigma$  of each liquid used were measured using an Anton Paar DMA 35 density meter, Anton Paar MCR 302 rheometer and Kruss K100 surface tensiometer with Wilhelmy plate, respectively. Experiments are recorded using a digital camera at 960 frames per second. During a typical experiment, a beading pattern develops like that shown in Fig. 1a. The pattern is defined by the bead spacing  $S_b$ , bead velocity  $V_b$ , and bead diameter  $D_b$ , as shown in Fig. 1b. We also define the bead frequency as  $f = V_b/S_b$ .

During experiments, the liquid is allowed ample time to flow onto and down the fiber prior to image recording to avoid any transient effects. The flow rate is held constant. and five videos are recorded for each experiment. The average of each bead property from the five videos is determined and used as the final value for that experiment. Three regimes are observed: i) isolated, ii) Plateau-Rayleigh, and iii) convective, consistent with prior results (Kliakhandler et al. (2001)). The isolated and Plateau-Rayleigh regimes both exhibit a regular bead pattern with primary beads moving with constant velocity and spacing. However, the isolated regime experiences a secondary breakup of the thin liquid column between the primary beads leading to smaller, secondary beads. The convective regime is characterized by irregular bead patterns that result in random coalescence events between primary beads. In the convective regime, bead patterns have properties that vary significantly from moment-to-moment due to coalescence events occurring at irregular distances down the fiber. Thus, we are focused on the bead properties whenever the bead morphology falls into the isolated or Plateau-Rayleigh regime, where the beading patterns are steady and repeatable.

#### 2.1 Image analysis

The bead velocity  $V_b$ , spacing  $S_b$ , and diameter  $D_b$  are obtained from video recordings of each experiment using image analysis. We use MATLAB to process and analyze the



Fig. 1 a Experimental set-up and **b** sketch of the bead properties showing the bead spacing  $S_b$ , bead velocity  $V_b$ , bead diameter  $D_b$ , and fiber diameter  $D_f$ 

data. Custom lighting, a sturdy camera mount, and manually focusing the camera for each experiment are steps taken to optimize the image recordings and highlight the bead profile. We place an object of known size in place of the fiber before each set of experiments to allow for pixel-to-length calibration. The testing apparatus and camera mount are fixed to each other such that any change in the camera position keeps the vertical fiber orthogonal to the direction of imaging, a critical aspect to ensure the pixel-to-length calibration is valid for all locations in the image.

Five video recordings are taken per experiment with each recording consisting of 150 individual images. The images are analyzed in a three step process shown in Fig. 2a. First, the code extracts the individual images from the video. Second, each individual image is converted into a binary image with the fiber and liquid bead represented by white pixels and the background by black pixels. This step often does not fully convert the interior of the liquid bead into white pixels, as seen in the middle image of Fig. 2a. Thus, a final step fills in the remaining portions of the bead using a series of built-in MATLAB functions. The resulting image consists of a binary matrix with 1's representing white pixels and 0's representing black pixels. Summing the number of 1's across each row in the matrix allows for the thickness of the fluid at that position to be calculated. Performing this operation for all rows produces a vector containing the width of the fluid at each location along the fiber in terms of pixels. We plot the total pixel count (light intensity) at each location along the fiber and fit a smooth spline to the data where each peak in the spline is used to define the location of a liquid bead. The resulting curve and peaks are shown in Fig. 2b. We calibrate the light intensity from pixels to millimeters and define the bead diameter  $D_b$  as the magnitude of the spline peaks. The bead velocity  $V_b$  is determined by tracking the peak from image-to-image, and the bead spacing  $S_h$  is simply the average distance between each peak. In the isolated regime, small secondary beads are also present and thus we set a minimum pixel intensity for any location to be considered a peak to avoid tracking the peak of a secondary bead.

# **3 Results**

#### 3.1 Regime transition

Kliakhandler et al. (2001) categorized beading patterns that emerge from thin film flow on a fiber into three primary regimes: isolated, Plateau–Rayleigh, and convective, and we also observe these three regimes, as shown in Fig. 3a. The isolated and Plateau–Rayleigh regimes are classified as absolute instabilities and differ from the convective instability seen in the convective regime. We focus on categorizing the absolute instability and for that reason are interested in



**Fig. 2 a** The original (left), black and white (middle), and final image (right) from image processing. **b** The pixel intensity is fitted to a spline function from which the bead spacing is obtained

the boundary between absolute and convective instability. Figure 3b shows a phase diagram in the  $\tilde{\alpha} - Go$  space which separates regions of absolute and convective instability. Here, the Goucher number  $Go = (D_f/2)/l_c$  is defined as the ratio of the azimuthal curvature set by the fiber radius  $D_f/2$  and the vertical curvature, set by the capillary length  $l_c$ . The aspect ratio  $\tilde{\alpha} = h_N/(D_f/2)$  is defined using the Nusselt thickness  $h_N$  and can be altered by varying the flow rate Q without affecting Go. The Nusselt solution for the film thickness  $h_N$  and mean velocity  $u_N$  arise as the solution to an ideal film flow scenario where the role of inertia and surface tension are suppressed. Thus, the flow is governed by a balance of viscosity and gravity, and the resulting film thickness and mean velocity are only functions of the flow rate, Reynolds number, and liquid properties,

$$h_N = \left(\frac{3\mu^2}{g\rho^2} \operatorname{Re}\right) = l_\mu (3\operatorname{Re})^{1/3} \tag{1}$$

$$u_N = \frac{gh_N^2 \rho}{3\mu} = \frac{l_\mu}{3t_\mu} (3\text{Re})^{2/3}$$
(2)

with the viscous-gravity length scale  $l_{\mu} = (\mu^2 / \rho^2 g)^{1/3}$  and time scale  $t_{\mu} = (\mu / \rho g^2)^{1/3}$  defined in the scaling arguments presented by Shkadov (1967). The Reynolds number  $Re = 4\rho Q D_f / \pi D_n^2 \mu$  is defined using the flow rate Q with the fiber diameter  $D_f$  as the characteristic length. Note that the nozzle diameter appears implicitly in  $h_N$ ,  $u_N$  through the Reynolds number Re.

A dimensionless parameter  $\beta^*$  arises from a linear stability analysis of the Nusselt solution in the inertialess limit and compares the time scale of growth for linear perturbations



**Fig.3 a** Bead morphology exhibit isolated (left), Plateau–Rayleigh (middle), and convective (right) regimes. **b** Phase diagram in the  $\tilde{\alpha}$ -Go parameter space separates regions where convective instabilities occur. The loci for  $\beta^*=1$  and  $\beta^*=1.507$  are shown as the solid and dashed curves, respectively. Red crosses denote experiments

 $\tau_g$  to the advection time scale  $\tau_a$  (Ruyer-Quil and Kalliadasis (2012)),

$$\beta^* = \left(\frac{\tau_a}{\tau_g}\right)^{2/3} = \left(\frac{\tilde{\alpha}}{\operatorname{Go}^2 c_k(\tilde{\alpha})(1+\tilde{\alpha})^4}\right)^{2/3}$$
(3)

where  $c_k$  is the kinematic wave speed defined using  $b = 1 + \tilde{\alpha}$ ,

$$c_k = \frac{8(b-1)(2\log(b)b^2 - b^2 + 1)}{3(4\log(b)b^4 - 3b^4 + 4b^2 - 1)}.$$
(4)

The dimensionless parameter  $\beta^*$  is thus a saturation number and is a generalization of the parameter  $\beta = \tilde{\alpha}^{2/3} (1/Go)^{4/3}$ developed by Kalliadasis and Chang (1994) to explain the saturation of the Plateau-Rayleigh instability by the flow advection. The loci of the saturation number  $\beta^* = 1$  and  $\beta^* =$ 1.507 are plotted in Fig. 3b as a solid and dashed curve, respectively. Here, the solid curve  $\beta^* = 1$  separates the phase diagram into two regions. To the left of this curve, the PRI mechanism dominates over the flow advection, whereas the right side represents the dominance of the flow advection. The dashed curve  $\beta^* = 1.507$  represents the convective to absolute transition in the inertialess limit as determined by Duprat et al. (2007). Thus, the light green region of the phase diagram represents flows with an absolute instability dominated by the PRI mechanism. The light blue region represents flows with a convective instability dominated by the PRI mechanism, and the white region represents flows that are convectively unstable as advection saturates the emergence of the PRI. The theoretical phase diagram can be further refined by including the drag-inertia, drag-gravity, and soliton-like regions, but we omit these further distinctions. Our experiments fall within the region of absolute instability and vary over a large range of  $\tilde{\alpha}$  and Go.

Previous investigations have shown a unique relationship between the nozzle diameter  $D_n$  and the resulting bead morphology (Sadeghpour et al. (2017), Ji et al. (2020)). We observe a similar relationship in our experiments as shown in a phase plot in the  $D_n - Q$  space in Fig. 4. We note that the nozzle diameter is a critical parameter in determining the flow regime, and we can observe all three of the primary regimes by merely changing the nozzle diameter for a fixed flow rate. When the nozzle diameter  $D_n$  is less than or approximately equal to the capillary length  $l_c = \sqrt{\sigma/\rho g}$  all three primary regimes are observable by varying the flow rate. Alternatively, when the nozzle diameter is much larger than the capillary length  $D_n > l_c$  the Plateau–Rayleigh regime vanishes and only the isolated and convective regimes are observed irrespective of the flow rate. A less complex relationship between the other experimental parameters is observed. For example, decreasing the fiber diameter D<sub>f</sub> results in a transition from

the isolated to Plateau–Rayleigh to convective regime. This monotonic transition is also observed as the flow rate Q is increased or as the viscosity  $\mu$  is increased.

## 3.2 Bead properties

We analyze liquid bead properties whenever the beading pattern falls into the isolated or Plateau-Rayleigh regime. For each subsequent plot, the data point with the largest flow rate represents the transition point before entering the convective regime and thus signifies the transition from an absolute to convective instability. Figure 5 plots the critical bead properties, the bead velocity  $V_b$ , diameter  $D_b$ , spacing  $S_b$ , and frequency f, as they depend upon the nozzle diameter  $D_n$ for fixed fiber diameter  $D_f$ , viscosity  $\mu$ , and surface tension  $\sigma$ . Figure 5a shows a monotonic increase in bead velocity with increasing nozzle diameter. For each nozzle diameter, the bead velocity increases with flow rate, achieves a maximum value, and then decreases until the convective regime is reached. Figure 5b shows the bead spacing decreases with flow rate and increases with nozzle diameter. The bead diameter remains approximately constant with respect to flow rate but does decrease as the convective regime is approached, as well as increase with increasing nozzle diameter, as shown in Fig. 5c. The behavior of the bead diameter as the flow rate is increased closely resembles those observed for the bead velocity in Fig. 5a where a dip in the bead diameter is observed as the flow rate causing transition to the convective regime is approached. This implies that either the drop shape is changing as the convective regime is approached,



**Fig. 4** Phase diagram of the beading regime, as it depends upon the nozzle diameter  $D_n$  and flow rate Q for a fixed fiber diameter  $D_f$  =0.101 mm, viscosity  $\mu$ =48 mPa s, surface tension  $\sigma$ =22 mN/m, and density  $\rho$  =967 kg/m<sup>3</sup>. Experiments are represented by solid shapes with shaded colors illustrating the approximate regions where the isolated (blue), Plateau–Rayleigh (green) and convective regime (red) are observed. The capillary length for all experiments is  $l_c$  =1.52 mm

since the diameter is decreasing yet the volumetric flow rate is increasing, or the thin film between the primary beads is absorbing a higher portion of the liquid volume than the beads as the convective regime is approached. Lastly, Fig. 5d plots the bead frequency f against flow rate Q showing an increase in nozzle diameter decreases the bead frequency. Interestingly, the frequency trend for the smallest nozzle diameter  $D_n = 0.41$  mm follows a drastically different trend than the linear trend followed by all other nozzle diameters suggesting new dynamics may emerge whenever the nozzle diameter is comparable to the fiber diameter. This should be explored further in future works.

Figure 6 illustrates how the fiber diameter  $D_f$  affects the bead properties. In Fig. 6a, the bead velocity  $V_h$  is plotted against the flow rate Q and we observe an increase in the bead velocity as the fiber diameter is decreased. This follows intuition as a thinner fiber reduces the amount of viscous dissipation between the fluid and fiber. In Fig. 6b, we plot the bead spacing  $S_h$  against the flow rate Q. For fixed flow rate, an increase in the fiber diameter leads to a decrease in the bead spacing. Here we see that while the fiber diameter can alter the bead spacing, the same bead spacing trend is followed as the flow rate is increased, irrespective of the fiber diameter. Figure 6c plots the bead diameter  $D_h$  against the flow rate Q, and we observe that increasing the fiber diameter leads to a decrease in the bead diameter. Lastly, in Fig 6d we plot the bead frequency f against flow rate Qand observe that for a fixed flow rate, the bead frequency increases as the fiber diameter increases with the exception of our smallest fibers which overlay one another indicating the presence of the fiber is diminished.

Figure 7 plots the bead properties against flow rate and highlights the role of viscosity. Viscosity can greatly increase or decrease the bead dynamics, and thus, we present our data on a logarithmic scale to allow the data to be compared in a meaningful way. Note that the bead diameter does not change with viscosity so we omit it from the bead properties being presented. In Fig. 7a the bead velocity  $V_{h}$ is plotted against flow rate Q for seven different viscosities. When the flow rate is fixed, the bead velocity increases as the viscosity decreases. Note the slight reduction in the bead velocity for the farthest right data point where the convective flow regime is being approached. Figure 7b plots the bead spacing  $S_b$  against the flow rate Q where we observe an increase in the bead spacing as the viscosity is decreased for a fixed flow rate. The bead frequency f against flow rate Qis reported in Fig. 7c. Interestingly, the frequency trends for each viscosity fall along a single trendline signaling a f - Qrelationship that is independent of the viscosity. In heat and mass transfer applications, the bead frequency is a critical parameter for optimizing the system and having a f - Qrelationship that does not depend upon the viscosity greatly



Fig. 5 a Bead velocity  $V_b$ , b bead spacing  $S_b$ , c bead diameter  $D_b$ , and d frequency against flow rate Q as it depends upon nozzle diameter  $D_n$  for a fixed fiber diameter  $D_f$ =0.101 mm, viscosity  $\mu$ =48 mPa·s, and surface tension  $\sigma$ =22 mN/m

simplifies predicting how a fluids viscosity will affect its performance (Sadeghpour et al. (2019), Zeng et al. (2018)).

## 3.3 Scaling the data

The monotonic trends with experimental parameters previously discussed suggest that our data will collapse upon scaling, thus illuminating the physics associated with the bead dynamics. Revisiting the Nusselt flat film solution previously discussed, we define the dimensionless bead diameter  $\tilde{D}_b = D_b/h_N$  and dimensionless bead spacing  $\tilde{S}_b = S_b/h_N$ where  $h_N$  is the Nusselt film thickness. Recall that the fiber diameter appears implicitly in the Nusselt solution through the Reynolds number *Re*. The capillary number *Ca* is defined using the Nusselt mean velocity  $u_N$  as  $Ca = \mu u_N/\sigma$  and compares the viscous and capillary forces. Lastly, we define our dimensionless frequency  $\tilde{f} = fT_{\mu}$  where  $T_{\mu} = \mu D_f / \sigma$  is the characteristic time of growth of the Plateau–Rayleigh instability in the viscous limit (Gallaire and Brun (2017)). The time scale  $T_{\mu}$  results from the suppression of surface tension due to viscous dissipation, a critical consideration for experimental data collected over a wide range of viscosity and fiber diameter.

Figure 8a plots the dimensionless bead diameter  $\tilde{D}_b$ against the capillary number *Ca* for all of our data. We observe a collapse of the data with power law exponent n = -0.535. The Nusselt thickness  $h_N$  represents the minimum film thickness observed when the PRI is not present; thus, we see that  $\tilde{D}_b > 1$  for all flows. Furthermore, the collapse of all dimensionless bead diameters  $\tilde{D}_b$  with the capillary number *Ca* signifies that as the viscous dissipation



**Fig. 6** a Bead velocity  $V_b$ , **b** bead spacing  $S_b$ , **c** bead diameter  $D_b$ , and **d** frequency against flow rate Q as it depends upon fiber diameter  $D_f$  for a fixed nozzle diameter  $D_n=1.2$  mm, viscosity  $\mu=48$  mPa<sup>•</sup> s, and surface tension  $\sigma=22$  mN/m

becomes more significant, it suppresses the capillarydriven growth of the PRI. Figure 8b plots the dimensionless frequency  $\tilde{f}$  against the dimensionless bead spacing  $\tilde{S}_{h}$  for all of our data. We again observe a collapse of the data with power law exponent n = -1.257. The dimensionless frequency  $\tilde{f}$  is scaled with a viscous time scale  $T_{\mu}$  that reflects the suppression of the surface tension, the driving mechanism for the PRI, due to the viscous dissipation. Thus, an increasing  $\tilde{f}$  signals the viscous dissipation becoming more prominent in the flow, and we show this results in a decreasing dimensionless bead spacing  $\tilde{S}_{h}$ . The negative correlation between the dimensionless frequency  $\tilde{f}$  and dimensionless bead spacing  $\tilde{S}_h$  illustrates a decreasing spacing between the beads as the viscous dissipation slows down the flow, allowing beads to produce at a higher frequency despite the decreased significance of surface tension. Consequently, while the Plateau–Rayleigh instability is suppressed by increasing viscous dissipation, the resulting decrease in bead velocity is greater, leading to increasing bead frequency. This is also predicted by the model of Ruyer-Quil and Kalliadasis (2012). These correlations provide insight into how bead dynamics of thin film flow down a fiber, where the dominant instability mechanism is the PRI, are affected by the suppression of surface tension as viscous dissipation is increased.





**Fig. 7 a** Bead velocity  $V_b$ , **b** bead spacing  $S_b$ , and **c** frequency against flow rate Q as it depends upon viscosity  $\mu$  for a fixed fiber diameter  $D_f$ =0.2032 mm, nozzle diameter  $D_n$ =1.2 mm, and surface tension  $\sigma$  =22 mN/m

## **4** Conclusions

An experimental investigation into the bead dynamics of thin film flow down a fiber using a simplified nozzle-fiber system was conducted. The simplified experimental setup has numerous advantages, namely the ability to set the flow rate, ease of alignment, and could be used as a classroom demonstration on hydrodynamic instabilities. We focus on absolute PRI instabilities, as determined by the dimensionless saturation number  $\beta^*$  which is the ratio of the advection time scale to the linear instability time scale (cf. Figure 3b). We observe all three primary regimes reported in previous work (Kliakhandler et al. (2001)) showing a relationship between the viscosity  $\mu$ , fiber diameter  $D_f$ , and flow rate Q with the observed regime. We highlight a more unique and complex relationship between the nozzle diameter and the observed regime for our nozzle-fiber system, mirroring the complex relationships observed for other nozzle-fiber systems (Sadeghpour et al. (2017), Ji et al. (2020)). We report a large experimental data set for the bead velocity  $V_b$ , spacing  $S_b$ , diameter  $D_b$ , and frequency f as it depends upon the flow rate Q, fiber diameter  $D_f$ , nozzle diameter  $D_n$ , and viscosity  $\mu$ . We observe unique relationships between the bead properties and independent variables not captured in previous experiments. Lastly, we scale our data using the Nusselt flat film solution for the film thickness  $h_N$  with mean velocity  $u_N$ using a viscous timescale  $T_{\mu}$ . We observe a collapse of the data with the dimensionless bead diameter  $\hat{D}_{h}$  scaling with the capillary number Ca and the dimensionless frequency  $\tilde{f}$ scaling with the dimensionless bead spacing  $\tilde{S}_{h}$ .

Our comprehensive experiments document the bead properties over a large range of experimental parameters to an extent not previously documented. The viscosity, previously only investigated over a small range, is explored for a range spanning two decades allowing a comprehensive understanding of how viscosity affects the bead properties. The negative correlation between the dimensionless bead diameter  $\tilde{D}_{h}$  and capillary number Ca illustrates the role that viscous dissipation plays in suppressing the surface tension-driven PRI and thus the bead diameter. The negative correlation between the dimensionless frequency  $\tilde{f}$  and dimensionless bead spacing  $\tilde{S}_b$  illustrates a decreasing spacing between the beads as the viscous dissipation slows down the flow, allowing beads to produce at a higher frequency despite the decreased significance of surface tension. Lastly, while we limit this work to flows destabilized via the PRI mechanism, future work to investigate each flow regime will



Fig. 8 a Dimensionless bead diameter  $\tilde{D}_b$  against capillary number Ca and **b** dimensionless frequency  $\tilde{f}$  against dimensionless bead spacing  $\tilde{S}_b$  for all data

prove critical for fully understanding how the dominating instability mechanism shapes the physics governing liquid beads traveling down a fiber.

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