NAME: _____ SOLUTION _____

INSTRUCTIONS: This is a closed book/closed notes exam. You may use a 4 function calculator. Zero credit will be earned if the honors pledge is not signed.

1. [10 points] Consider a nozzle with an inlet diameter of 10.00 cm and an outlet diameter of 2.00 cm. Compute the volumetric flowrate of the liquid jet emanating from the nozzle exit into the atmosphere if the inlet pressure to the nozzle is 2.00 atm (gage pressure), and the nozzle is horizontally oriented. The density of liquid water is 1000 kg/m³. Report your answer in m³/s.

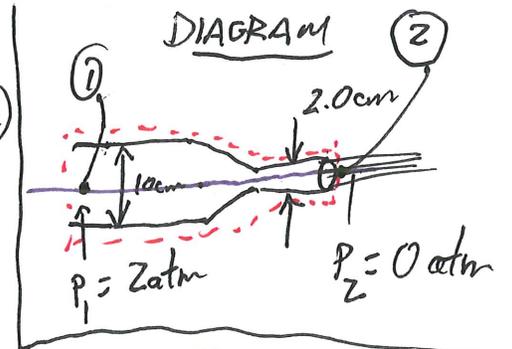GIVEN: $d_1, d_2, P_1, P_2, \rho\ H_2O$, horizontal

FIND: $\dot{\forall}$ of jet

ASSUME: Bernoulli's equation applies (inviscid, S.S., incompressible)

ANALYSIS: $P_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \leftarrow$ Bernoulli

gage pressure is zero $z_1 = z_2$

DIAGRAM

$P_1 = 2\,atm$ 2.0cm   10cm $P_2 = 0\,atm$

$$\boxed{V_2^2 = \frac{2P_1}{\rho} + V_1^2}$$

Conservation of mass: $\rho V_1 A_1 = \rho V_2 A_2 \rightarrow V_1 \frac{\pi d_1^2}{4} = V_2 \frac{\pi d_2^2}{4}$ $\boxed{V_1 = \left(\frac{d_2}{d_1}\right)^2 V_2}$

$V_2^2 = \frac{2P_1}{\rho} + \left(\frac{d_2}{d_1}\right)^4 V_2^2 \rightarrow V_2^2\left(1 - \left(\frac{d_2}{d_1}\right)^4\right) = \frac{2P_1}{\rho}$

$V_2 = \sqrt{\dfrac{2P_1/\rho}{\left(1 - \left(\frac{d_2}{d_1}\right)^4\right)}} = \sqrt{\dfrac{2(2\cdot101325\,Pa)/1000\frac{kg}{m^3}}{\left(1 - \left(\frac{.02m}{0.1m}\right)^4\right)}}$

$\boxed{V_2 = 20.1\ m/s}$

$V_1 = 0.804\ m/s$

Check units:
$\left(\frac{Pa\,m^3}{kg}\right)^{1/2} = \left(\frac{N}{m^2}\frac{m^3}{kg}\right)^{1/2}$

$\downarrow$

$\left(\frac{kg\,m}{s^2}\frac{m}{kg}\right)^{1/2}$

$= \frac{m}{s}\ \checkmark$

$\dot{\forall} = A_2 V_2 = \dfrac{\pi(.02m)^2}{4} \cdot 20.1\ m/s = 0.00631\ m^3/s$

$$\boxed{\dot{\forall} = 0.00631\ m^3/s} \leftarrow ANS.$$

2. [10 points] Consider a heat pump with a coefficient of performance of 6.2. When the unit is on, the transfer of heat to the heated space is 0.75kW. If electricity costs 20.00 cents per kW·hr, compute how many hours per month the unit is turned on if the monthly electric bill is $12, and the heater is the only electrical appliance in the house.

GIVEN: Heater, $\gamma = 6.2$, $\dot{Q}_H = 0.75\,kW$, electric cost

FIND: Hours that heat pump is on

ASSUME: S.S. operation

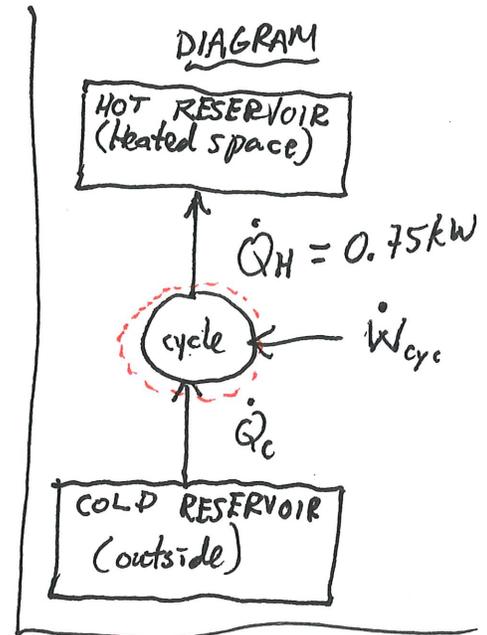ANALYSIS: $\gamma = \dfrac{Q_{out}}{W_{cyc}} = \dfrac{\dot{Q}_H}{\dot{W}_{cyc}} = \dfrac{\dot{Q}_H}{\dot{W}_{cyc}}$

$\gamma = 6.2 = \dfrac{0.75\,kW}{\dot{W}_{cyc}} \rightarrow \boxed{\dot{W}_{cyc} = 0.121\,kW}$

So,

$$cost = (\dot{W})(t\ hrs)\cdot(\$/kW\cdot hr)$$

$$\$12 = (0.121\,kW)(t\ hrs)(\$0.2/kW\cdot hr)$$

$$\boxed{t = 496\ hrs}$$

DIAGRAM

HOT RESERVOIR (Heated space)

$\dot{Q}_H = 0.75\,kW$

cycle $\leftarrow \dot{W}_{cyc}$

$\dot{Q}_c$

COLD RESERVOIR (outside)

3. [10 points] Consider 1.8 kg of water in a rigid box. The initial quality and pressure are 61.98% and 1.5 bar, respectively. Heat is added until the water becomes a saturated vapor. Determine the heat added to the water in kJ.

GIVEN: m, rigid box, H_2O, $X_1 = 0.6198$, $P_1 = 1.5$ bar, $X_2 = 1.0$

FIND: $Q = ?\,kJ$

ASSUME: No KE or PE effects

ANALYSIS: $\Delta E = Q - W \rightarrow \Delta U = Q - W \rightarrow \Delta U = Q$

b/c no KE or PE effects ↑

0 b/c $W = 0$ when $\Delta V = 0$

DIAGRAM

$Q \rightarrow$ ① $X_1 = 0.6198$

② sat. vapor

$Q = m(U_2 - U_1)$

At 1.5 bar, $v_f = 1.0528 \times 10^{-3} \frac{m^3}{kg}$ $\quad v_g = 1.159 \frac{m^3}{kg}$

$u_f = 466.94\,kJ/kg$ $\quad u_g = 2519.7\,kJ/kg$

$u_1 = (1 - X_1)u_{f_1} + X_1 u_{g_1} = (1 - 0.6198)\left(466.94 \frac{kJ}{kg}\right) + (0.6198)\left(2519.7 \frac{kJ}{kg}\right)$

$\boxed{u_1 = 1739.2\,kJ/kg}$

$v_1 = (1 - 0.6198)\left(1.0528 \times 10^{-3} \frac{m^3}{kg}\right) + 0.6198\left(1.159 \frac{m^3}{kg}\right)$
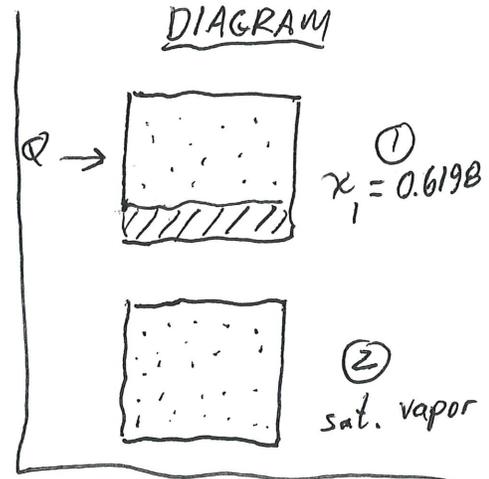
$\boxed{v_1 = 0.7187\,m^3/kg}$

Since tank is rigid, $v_2 = v_1$. And, $v_2 = v_g$. $v_g = 0.7187$ @ 2.5 bar

At 2.5 bar, $\boxed{u_2 = u_g = 2537.2\,kJ/kg}$

$Q = m(U_2 - U_1)$

$Q = (1.8\,kg)\left(2537.2 \frac{kJ}{kg} - 1739.2 \frac{kJ}{kg}\right)$

$\boxed{Q = 1436.4\,kJ/kg}$ ← ANS.

I HAVE NEITHER PROVIDED OR RECEIVED HELP DURING THIS EXAM.

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SIGNATURE