EXAM #1 – ME 2030 – Section 001 – Fall 2024
October 3, 2024

NAME: SOLUTION

INSTRUCTIONS: This is a closed book exam. You may use a 4 function calculator. Zero credit will be earned if the honors pledge is not signed. The equation sheet can be found at the back of the exam.

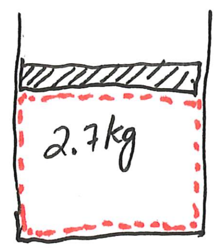
1. [10 points] The pressure of 2.7 kg of an ideal gas in a piston-cylinder assembly increases from 2 bar to 5 bar in an isothermal process at 300K. What must the heat transfer be in kJ in order for the internal energy of the gas to not change during this process? Note that $\ln(2/5) = -0.9163$, and $\ln(5/2) = 0.9163$. The molecular weight of the gas is 44.01 kg/kmol.

GIVEN: $m = 2.7\,kg$; $P_1 = 2\,bar$; $P_2 = 5\,bar$; isothermal; $T = 300K$; $\Delta U = 0$

FIND: $Q = ?\,kJ$

ASSUME: No KE or PE effects.

DIAGRAM



ANALYSIS: C.O.E. for a closed system $\boxed{\Delta E = Q - W}$

$\Delta E = \Delta U$ b/c no KE or PE effects. $\Delta U = 0$,

So $\boxed{Q = W}$

$$W = \int p\,d\forall \qquad p\forall = mRT \qquad p = \frac{mRT}{\forall}$$

$$W = \int_{\forall_1}^{\forall_2} \frac{mRT}{\forall} d\forall = mRT \int_{\forall_1}^{\forall_2} \frac{d\forall}{\forall} = \boxed{mRT \ln\left(\frac{\forall_2}{\forall_1}\right)}$$

$$\boxed{\forall = \frac{mRT}{P}} \quad \text{so} \quad W = mRT\ln\left(\frac{\cancel{mRT}/P_2}{\cancel{mRT}/P_1}\right) = \boxed{mRT\ln\left(\frac{P_1}{P_2}\right)}$$

$$W = mRT\ln\left(\frac{2\,bar}{5\,bar}\right) = (2.7kg)R(300k)(-0.9163)$$

$$R = \frac{\bar{R}}{MW} = \frac{8.314\,kJ/\cancel{kmole}\cdot K}{44.01\,kg/\cancel{kmole}} = 0.1889\frac{kJ}{kg\cdot K}$$

$$W = (2.7\cancel{kg})(0.1889\frac{kJ}{\cancel{kg}\cdot\cancel{K}})(300\cancel{K})(-0.9163)$$

$$W = -140.2\,kJ$$

$$\boxed{Q = -140.2\,kJ}$$

2. [10 points] Use your knowledge of hydrostatics to develop an equation for the pressure at a location z above the Earth's surface assuming that the air behaves as an ideal gas and that the temperature varies with z according to the equation:

$$T = T_a - \beta z \tag{1}$$

where β is a constant (called the lapse rate) and T_a is the temperature at $z = 0$. Present you equation in terms of p_a, β, T_a, z, g and R, where p_a is the pressure at $z = 0$. Present your equation in simplest possible form.

GIVEN: Ideal gas; $T = f(z)$

FIND: $p = p(z, p_a, \beta, T_a, g, R)$

ASSUME: g is constant

ANALYSIS:

$$\frac{dp}{dz} = -\gamma = -\rho g$$

$$p = \rho R T \rightarrow \rho = \frac{p}{RT}$$

$$T = T_a - \beta z$$

So, $$\rho = \frac{p}{R(T_a - \beta z)}$$

So, $$\boxed{\frac{dp}{dz} = -\frac{p g}{R(T_a - \beta z)}}$$

$$\frac{dp}{p} = \frac{-g}{R}\frac{dz}{(T_a - \beta z)}$$

$$\left.\begin{aligned} u &= T_a - \beta z \\ du &= -\beta\, dz \end{aligned}\right\} \int_{p_a}^{p} \frac{dp}{p} = \frac{+g}{R}\frac{1}{\beta}\int_0^z \frac{-\beta\, dz}{(T_a - \beta z)}$$

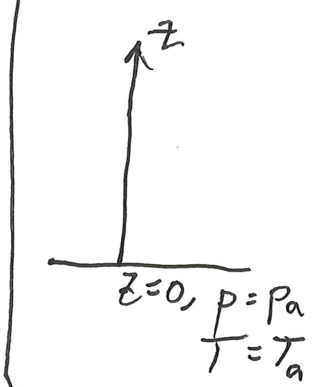
$$\ln\left(\frac{p}{p_a}\right) = \frac{g}{R\beta}\ln(T_a - \beta z)\Big|_0^z = \frac{g}{R\beta}\ln\left(\frac{T_a - \beta z}{T_a}\right)$$

$$\frac{p}{p_a} = \left(\frac{T_a - \beta z}{T_a}\right)^{g/R\beta} \rightarrow \boxed{p = p_a\left(\frac{T_a - \beta z}{T_a}\right)^{g/R\beta}}$$

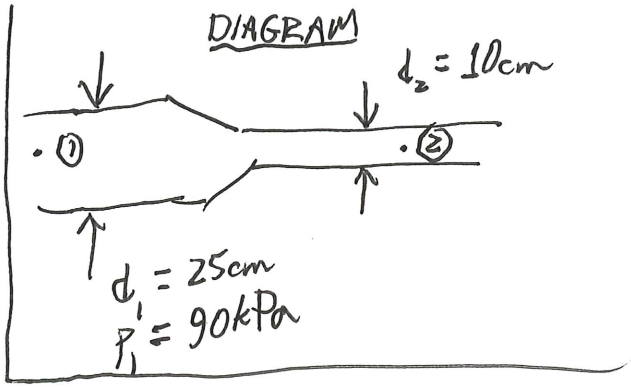
OR

$$\boxed{p = p_a\left(1 - \frac{\beta z}{T_a}\right)^{g/R\beta}}$$

DIAGRAM



3. [10 points] A hose exhibits a change in diameter from 25 cm to 10 cm in the flow direction. If the collapse pressure for the tubing material is −10,000 Pa, what is the maximum flow rate the hose can sustain without collapse if the inlet pressure is 90kPa? Express your answer in m^3/s. All of the above pressures are gauge pressures. The density of water is 1000 kg/m^3.



GIVEN: d_1, d_2, P_1, collapspressure, ρ

FIND: $\dot{\forall}_{max}$

ASSUME: S.S., inviscid flow, tube is horizontal

ANALYSIS: We set P_2 to −10,000 Pa because the larger the pressure drop, the larger $\dot{\forall}$ and $\dot{\forall}_{max}$ will occur just as the tube collapses at 2

Bernoulli $P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\gamma z_1} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\gamma z_2}$ b/c tube is horizontal

$$\dot{\forall} = AV = \frac{\pi d^2}{4} V \rightarrow V = \frac{4\dot{\forall}}{\pi d^2} \qquad \boxed{V^2 = \frac{16\dot{\forall}^2}{\pi^2 d^4}}$$

So, $$\boxed{P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)} = \frac{1}{2}\rho\left(\frac{16\dot{\forall}^2}{\pi^2 d_2^4} - \frac{16\dot{\forall}^2}{\pi^2 d_1^4}\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho \frac{16\dot{\forall}^2}{\pi^2}\left(\frac{1}{d_2^4} - \frac{1}{d_1^4}\right) = \frac{8\rho\dot{\forall}^2}{\pi^2}\left(\frac{1}{d_2^4} - \frac{1}{d_1^4}\right)$$

$$\dot{\forall}^2 = \frac{(P_1 - P_2)\pi^2}{\rho\, 8} \Big/ \left(\frac{1}{d_2^4} - \frac{1}{d_1^4}\right) = \frac{\dfrac{(90{,}000\text{ Pa} - -10{,}000\text{ Pa})\pi^2}{8(1000\text{ kg/m}^3)}}{\left(\dfrac{1}{(0.1\text{m})^4} - \dfrac{1}{(0.25\text{m})^4}\right)}$$

$$\dot{\forall}^2 = \frac{123.4\ \text{Pa/kg/m}^3}{(10{,}000 - 256)\text{m}^{-4}}$$

$$\dot{\forall}^2 = 0.0127 \left(\frac{\text{m}^3}{\text{s}}\right)^2$$

$$\boxed{\dot{\forall} = 0.113\ \text{m}^3/\text{s}}$$

Check units

$$\frac{\text{Pa}}{\text{kg/m}^3\ \text{m}^{-4}} = \frac{\text{Pa}\cdot\text{m}^4\cdot\text{m}^3}{\text{kg}}$$

$$\frac{\text{N}}{\text{m}^2}\frac{\text{m}^7}{\text{kg}} = \frac{\cancel{\text{kg}}\,\text{m}}{\text{s}^2}\frac{\text{m}^5}{\cancel{\text{kg}}}$$

$$= \frac{\text{m}^6}{\text{s}^2} \checkmark$$

$$\sqrt{\frac{\text{m}^6}{\text{s}^2}} = \frac{\text{m}^3}{\text{s}} \checkmark$$

I HAVE NEITHER PROVIDED OR RECEIVED HELP DURING THIS EXAM.

SIGNATURE

EQUATION SHEET EXAM #1 - ME 2030

Miscellaneous

$$v = \frac{1}{\rho} \qquad \gamma = \rho g \qquad SG = \frac{\rho}{\rho_{H_2O}} \qquad p_{gage} = p_{absolute} - p_{atm} \tag{2}$$

Quality (using v as the relevant property)

$$v = (1-x)v_f + xv_g = v_f + x(v_g - v_f) \tag{3}$$

Ideal Gas Law

$$p = \rho RT \qquad pv = RT \qquad p\overline{v} = \overline{R}T \qquad pV = mRT \qquad pV = n\overline{R}T \tag{4}$$

Fluid Statics

$$-\nabla p - \gamma \hat{\mathbf{k}} = \rho \mathbf{a} \tag{5}$$

With $\mathbf{a} = 0$

$$\nabla p + \gamma \hat{\mathbf{k}} = 0 \tag{6}$$

in component form

$$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\gamma \tag{7}$$

therefore

$$\frac{dp}{dz} = -\gamma \tag{8}$$

Inviscid, Incompressible, Steady Flow

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant} \quad \text{along} \quad \text{streamline} \tag{9}$$

$$p + \rho \int \frac{V^2}{\mathbf{R}} dn + \gamma z = \text{constant} \quad \text{across} \quad \text{streamline} \tag{10}$$

where $\mathbf{R}$ is the radius of curvature.

Conservation of Energy - First Law of Thermodynamics

$$E = KE + PE + U \qquad \Delta E = Q - W \qquad \frac{dE}{dt} = \dot{Q} - \dot{W} \tag{11}$$

$$\eta = \frac{W_{cycle}}{Q_{in}} \text{(power cycle)} \qquad \beta = \frac{Q_{in}}{W_{cycle}} \text{(refrigeration cycle)} \qquad \gamma = \frac{Q_{out}}{W_{cycle}} \text{(heat pump cycle)} \tag{12}$$
