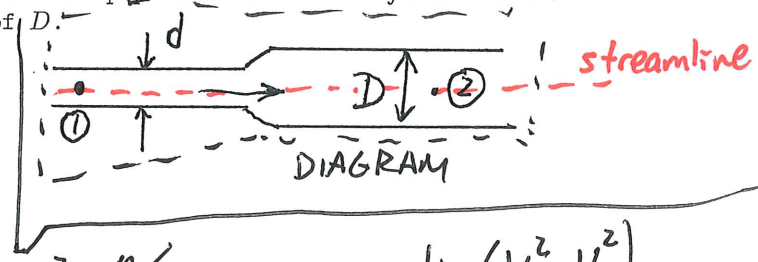
ANNOUNCED QUIZ #4 – ME 2030 –SECTION 001 – Fall 2024

NAME: SOLUTION

This is an open book quiz. A scientific calculator may be used. An unsigned honors pledge will result in a zero. Show all work.

1. Consider water flowing through a horizontal pipe as shown in the figure. Write an explicit equation for D in terms of the variables $(d, \dot{\forall}, p_1, p_2, \rho)$. If $p_1 = 1$ atm (gage), $d = 1.0$cm, $\dot{\forall} = 1000.0$ cm^3/s, and the maximum gage pressure the pipe can withstand anywhere is 1.7 atm, determine the maximum possible value of D.



GIVEN: $P_1, d, \dot{\forall}$, burst pressure

FIND: $D = D(\rho, \dot{\forall}, P_1, P_2, \rho)$; D_{max}

ASSUME: Bernoulli's eqn. applies; $\rho = 1000 \frac{kg}{m^3}$

ANALYSIS: Bernoulli:

$$P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g z_1} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g z_2} \rightarrow P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

$$P_2 - P_1 = \frac{1}{2}\rho\left(\frac{16\dot{\forall}^2}{\pi^2 d^4} - \frac{16\dot{\forall}^2}{\pi^2 D^4}\right) = \frac{1}{2}\rho\frac{16\dot{\forall}^2}{\pi^2}\left(\frac{1}{d^4} - \frac{1}{D^4}\right)$$

$\dot{\forall} = AV$

$V = \frac{\dot{\forall}}{A} = \frac{\dot{\forall}}{\frac{\pi d^2}{4}} = \frac{4\dot{\forall}}{\pi d^2}$

$$\frac{(P_2 - P_1)\pi^2}{8\rho\dot{\forall}^2} = \frac{1}{d^4} - \frac{1}{D^4} \rightarrow \frac{1}{D^4} = \frac{1}{d^4} - \frac{(P_2 - P_1)\pi^2}{8\rho\dot{\forall}^2}$$

$$D = \left[\frac{1}{\frac{1}{d^4} - \frac{(P_2 - P_1)\pi^2}{8\rho\dot{\forall}^2}}\right]^{1/4}$$ ← ANS.

For D_{max}, $P_2 - P_1 = 1.7\,atm - 1\,atm = 0.7\,atm = 70,928$ Pa

$$D_{max} = \left[\frac{1}{\frac{1}{(.01m)^4} - \frac{(70,928Pa)\pi^2}{8(1000\frac{kg}{m^3})(1000\times10^{-6}m^3/s)^2}}\right]^{1/4}$$

$1\,cm = 0.01\,m$

$1\,cm^3 = (0.01m)^3 = 10^{-6}m^3$

$$D_{max} = \left[\frac{1}{100\times10^6 m^{-4} - 87.50\times10^6 m^{-4}}\right]^{1/4}$$

$D_{max} = 0.0168m = 1.68cm$

ANS.

I HAVE NEITHER PROVIDED OR RECEIVED HELP DURING THIS QUIZ.

SIGNATURE