PROBLEM 4.22

KNOWN: Data are provided for a cylindrical tank being drained of water.

FIND: Determine the time, in min., when the tank holds 900 kg of water.

SCHEMATIC & GIVEN DATA:

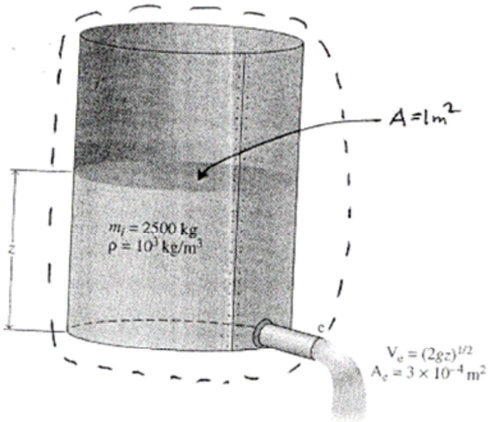

Fig. P4.22

ENGR. MODEL:

1. As shown in the sketch, a control volume enclosing the tank is being considered.
2. $g = 9.81\ m/s^2$
3. The density of water is $10^3\ kg/m^3$.

ANALYSIS: Mass rate balance: $\frac{dm_{cv}}{dt} = -\dot{m}_e$, where $\dot{m}_e$ is the mass flow rate at the exit. That is, $\dot{m}_e = \rho_e A_e V_e$ (Eq. 4.4a). Accordingly

constant at ρ

$$\frac{dm_{cv}}{dt} = -\rho A_e V_e \quad , \text{ where } V_e = (2gz)^{1/2}$$

$$= -\rho A_e (2gz)^{1/2} \qquad (1)$$

Also, note that $m_{cv}(t) = \rho A z$, where A is the area of the water free surface. Thus, $z = m_{cv}(t)/\rho A$ and Eq. (1) becomes

$$\frac{dm_{cv}}{dt} = -\rho A_e \left[\frac{2g\, m_{cv}(t)}{\rho A}\right]^{1/2} = -\left[\frac{2g\rho A_e^2}{A}\right]^{\frac{1}{2}} (m_{cv}(t))^{1/2}$$

Thus,

$$\frac{1}{(m_{cv})^{1/2}} \frac{dm_{cv}}{dt} = -\left[\frac{2g\rho A_e^2}{A}\right]^{1/2}$$

Integrating from $t = 0$ to t_f, we get $2(m_{cv})^{1/2}\Big]_0^{t_f} = -\left[\frac{2g\rho A_e^2}{A}\right]^{1/2} t_f$. Or

$$t_f = -\left[\frac{2A}{g\rho A_e^2}\right]^{1/2}\left[(m_{cv}(t_f))^{1/2} - (m_{cv}(0))^{1/2}\right] \qquad (2)$$

Inserting $A = 1\ m^2$, $g = 9.81\ m/s^2$, $\rho = 10^3\ kg/m^3$, $A_e = 3\times10^{-4}\ m^2$, $m_{cv}(0) = 2500\ kg$, and $m_{cv}(t_f) = 900\ kg$, we get

$$t_f = -\left[\frac{2(1\ m^2)}{(9.81\ \frac{m}{s^2})(10^3 \frac{kg}{m^3})(3\times10^{-4}\ m^2)^2}\right]^{1/2}\left[(900\ kg)^{1/2} - (2500\ kg)^{1/2}\right]\left|\frac{1\ min}{60\ s}\right|$$

$$= 15.86\ min$$