1.60

1.60 Two flat plates are oriented parallel above a fixed lower plate as shown in Fig. P1.60. The top plate, located a distance b above the fixed plate, is pulled along with speed V. The other thin plate is located a distance cb, where $0 < c < 1$, above the fixed plate. This plate moves with speed V_1, which is determined by the viscous shear forces imposed on it by the fluids on its top and bottom. The fluid on the top is twice as viscous as that on the bottom. Plot the ratio V_1/V as a function of c for $0 < c < 1$.

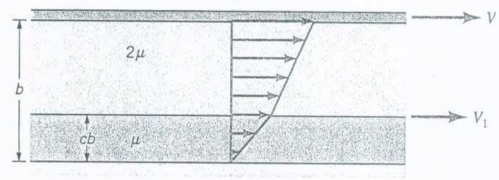

FIGURE P1.60

For constant speed, V_1, of the middle plate, the net force on the plate is 0. Hence, $F_{top} = F_{bottom}$, where $F = \tau A$. Thus, the shear stress on the top and bottom of the plate must be equal.

$$\tau_{top} = \tau_{bottom} \quad \text{where} \quad \tau = \mu \frac{du}{dy} \qquad (1)$$

For the bottom fluid $\frac{du}{dy} = \frac{V_1}{cb}$, while for the top fluid $\frac{du}{dy} = \frac{(V-V_1)}{b-cb}$

Hence, from Eqn. (1),

$$(2\mu)\frac{(V-V_1)}{b(1-c)} = (\mu)\frac{V_1}{cb}, \text{ which can be written as:}$$

$$2cV - 2cV_1 = V_1 - cV_1$$

or

$$\underline{\underline{\frac{V_1}{V} = \frac{2c}{c+1}}}$$

Note: If $c = 0$, $\frac{V_1}{V} = 0$

If $c = \frac{1}{2}$, $\frac{V_1}{V} = \frac{2}{3}$

If $c = 1$, $\frac{V_1}{V} = 1$