*3.58

*3.58 As shown in Fig. P3.58, water from a large reservoir flows without viscous effects through a siphon of diameter D and into a tank. It exits from a hole in the bottom of the tank as a stream of diameter d. The surface of the reservoir remains H above the bottom of the tank. For steady-state conditions, the water depth in the tank, h, is constant. Plot a graph of the depth ratio h/H as a function of the diameter ratio d/D.

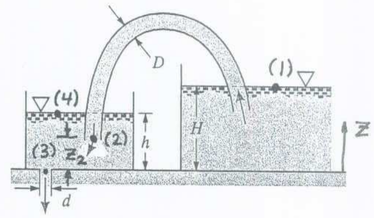

FIGURE P3.58

From the Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = V_1 = 0$, $z_1 = H$, and at the "free jet" end of the siphon,

$$p_2 = \gamma(h - z_2).$$

Thus, Eq. (1) becomes

$$H = (h - z_2) + \frac{V_2^2}{2g} + z_2 = h + \frac{V_2^2}{2g}$$

or

(1) $$V_2 = \sqrt{2g(H-h)}$$

Also,

$$\frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3,$$ where $p_4 = V_4 = p_3 = z_3 = 0$ and $z_4 = h$

Thus,

$$h = \frac{V_3^2}{2g}$$ or

(2) $$V_3 = \sqrt{2gh}$$

Also, for constant liquid levels in the tanks, $Q_2 = Q_3$

or

$$A_2 V_2 = A_3 V_3$$

so that

(3) $$\frac{\pi}{4} D^2 V_2 = \frac{\pi}{4} d^2 V_3$$

From Eqs. (1), (2), and (3):

$$D^2 \sqrt{2g(H-h)} = d^2 \sqrt{2gh} \quad \text{or} \quad H - h = \left(\frac{d}{D}\right)^4 h$$

Thus,

$$\underline{\underline{\frac{h}{H} = \frac{1}{1 + (d/D)^4}}}$$

This result is plotted on the next page.

(con't)

3.58 (con't)