

4.72

4.72 The wind blows across a field with an approximate velocity profile as shown in Fig. P4.72. Use Eq. 4.16 with the parameter b equal to the velocity to determine the momentum flowrate across the vertical surface $A-B$, which is of unit depth into the paper.

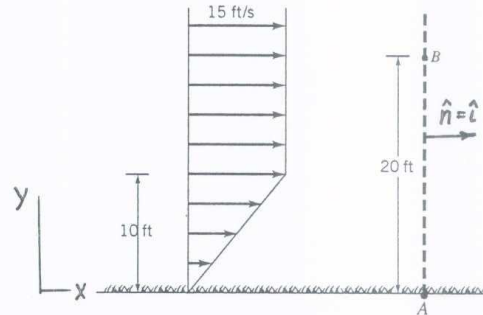


FIGURE P4.72

$$\begin{aligned}\vec{B}_{AB} &= \int_{AB} \rho \vec{b} \vec{V} \cdot \hat{n} \, dA = \int_{AB} \rho \vec{V} \vec{V} \cdot \hat{n} \, dA = \rho \int_{y=0}^{y=20 \text{ ft}} (V \hat{i}) [(V \hat{i}) \cdot \hat{i}] (1 \text{ ft}) \, dy \\ &= \rho \hat{i} \int_0^{20} V^2 \, dy\end{aligned}$$

But, $V = \frac{15}{10} y \frac{\text{ft}}{\text{s}}$ for $0 \leq y \leq 10 \text{ ft}$ (i.e., $V = 0$ at $y = 0$; $V = 15 \frac{\text{ft}}{\text{s}}$ at $y = 10$)
 and $V = 15 \frac{\text{ft}}{\text{s}}$ for $y \geq 10 \text{ ft}$

Thus,

$$\begin{aligned}\vec{B}_{AB} &= \rho \hat{i} \left[\int_0^{10} \left(\frac{15}{10} y\right)^2 \, dy + \int_{10}^{20} (15)^2 \, dy \right] = \rho \hat{i} \left[2.25 \frac{y^3}{3} \Big|_0^{10} + 225 y \Big|_{10}^{20} \right] \\ &= 0.00238 \frac{\text{slug}}{\text{ft}^3} \left[750 \frac{\text{ft}^4}{\text{s}^2} + 2250 \frac{\text{ft}^4}{\text{s}^2} \right] \hat{i} \\ &= \underline{\underline{7.14 \hat{i} \frac{\text{slug ft}}{\text{s}^2}}} = \underline{\underline{7.14 \hat{i} \text{ lb}}}\end{aligned}$$