5.24 Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.24. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also U. If the x direction velocity profile at section (2) is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at x where the boundary layer thickness is δ.
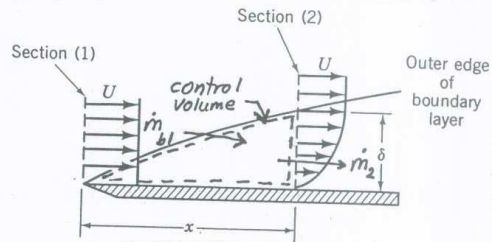

FIGURE P5.24

From the conservation of mass principle applied to the flow through the control volume shown in the figure we have

$$\dot{m}_{bl} = \dot{m}_2 = \int_{A_2} \rho \, \vec{V} \cdot \hat{n} \, dA$$

For incompressible, flow

$$\rho Q_{bl} = \rho U \ell \delta \int_0^1 \left(\frac{y}{\delta}\right)^{\frac{1}{7}} d\left(\frac{y}{\delta}\right)$$

where

$$\ell = \text{width of the plate}$$

and thus

$$Q_{bl} = \frac{7}{8} U \ell \delta$$