$\boxed{\text{MYO 8.26}}$

(a) Laminar flow. $\frac{u}{V_c} = 1 - \left(\frac{r}{R}\right)^2$

Where is $u = \bar{U}$?

$\bar{U} = V_c/2$

$$\frac{V_c/2}{V_c} = 1 - \left(\frac{r}{R}\right)^2$$

$$\frac{1}{2} = 1 - \frac{r^2}{R^2}$$

$$r^2 = \frac{R^2}{2}$$

$$\boxed{r = \frac{R}{\sqrt{2}}} \longleftarrow$$ This is where u happens to equal $\bar{U}$.

(b) Turbulent Flow ($Re = 10,000$)

$$\frac{u}{V_c} = \left[1 - \frac{r}{R}\right]^{1/5}$$

First compute $\bar{U} = \dfrac{\int \rho \vec{V} \cdot \hat{n} \, dA}{\rho A}$ assume incompressible

$$\bar{U} = \frac{1}{A}\int_0^R V_c \left[1 - \frac{r}{R}\right]^{1/5} 2\pi r \, dr$$

$$\bar{U} = \frac{2\pi V_c}{\pi R^2}\int_0^R r\left(1 - \frac{r}{R}\right)^{1/5} dr$$

6.26 continued

substitution, let $\alpha = 1 - \left(\frac{r}{R}\right) \Rightarrow \frac{r}{R} = 1 - \alpha$

$$r = R(1-\alpha)$$

$$dr = -R\,d\alpha$$

$$\bar{U} = \frac{2V_c}{\cancel{R^2}} \int_{\alpha=1}^{\alpha=0} \cancel{R}(1-\alpha)\alpha^{1/5}(-\cancel{R}\,d\alpha)$$

$$\bar{U} = 2V_c \int_{\alpha=0}^{\alpha=1} \left(\alpha^{1/5} - \alpha^{6/5}\right) d\alpha = 2V_c \left[\frac{\alpha^{6/5}}{6/5} - \frac{\alpha^{11/5}}{11/5}\right]_0^1$$

$$\bar{U} = 2V_c \left[\frac{5}{6} - \frac{5}{11}\right] = 10V_c \left[\frac{11-6}{66}\right] = \frac{50}{66} V_c$$

at what r does $u = \bar{U} = \frac{50}{66} V_c$?

$$\frac{50}{66} \cancel{V_c} = \cancel{V_c} \left[1 - \frac{r}{R}\right]^{1/5}$$

$$0.25 = 1 - \frac{r}{R} \qquad \boxed{r = 0.75R}$$