

$$11.55 \quad \beta = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_p$$

$$\kappa = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

(a) Ideal gas  $pv = RT$

$$v = \frac{RT}{p} \Rightarrow \left( \frac{\partial v}{\partial T} \right)_p = \frac{R}{p} ; \quad \left( \frac{\partial v}{\partial p} \right)_T = -\frac{RT}{p^2}$$

$$\beta = \frac{1}{\cancel{RT}} \frac{\cancel{R}}{\cancel{p}} \quad \boxed{\beta = \frac{1}{T}}$$

$$\kappa = -\frac{\cancel{p}}{\cancel{RT}} \left( \frac{-\cancel{RT}}{p^2} \right) \quad \boxed{\kappa = \frac{1}{p}}$$

(b)  $p(v-b) = RT$

$$p = \frac{RT}{v-b}$$

$$v-b = \frac{RT}{p} \Rightarrow v = \frac{RT}{p} + b$$

$$\left( \frac{\partial v}{\partial T} \right)_p = \frac{R}{p}$$

$$\left( \frac{\partial v}{\partial p} \right)_T = -\frac{RT}{p^2}$$

$$\beta = \left( \frac{1}{\frac{RT}{p} + b} \right) \frac{R}{p} = \left( \frac{1}{\frac{RT}{p} + b} \right) \frac{\cancel{R}}{\cancel{RT}/v-b} = \frac{v-b}{T} \left( \frac{1}{\frac{RT}{p} + b} \right)$$

simpler to write

$$\boxed{\beta = \frac{v-b}{vT}}$$

$$\kappa = -\frac{1}{v} \left( \frac{-RT}{p^2} \right) = \frac{1}{v} \frac{RT(v-b)^2}{(RT)^2} = \frac{v-b}{v p}$$

$$\boxed{\kappa = \frac{v-b}{v p}}$$