14.8 Consider the reactions

① $CO + H_2O \rightleftharpoons H_2 + CO_2$

② $2CO_2 \rightleftharpoons 2CO + O_2$

③ $2H_2O \rightleftharpoons 2H_2 + O_2$

Show that $K_1 = (K_3/K_2)^{1/2}$

Assume ideal gas behavior

$$\Delta G^\circ = -\bar{R}T \ln K(T)$$

From Eq (14.29a)

$$\Delta G_1^\circ = \bar{g}^\circ_{H_2} + \bar{g}^\circ_{CO_2} - \bar{g}^\circ_{CO} - \bar{g}^\circ_{H_2O} = \bar{g}^\circ_{H_2} + \bar{g}^\circ_{CO_2} - \bar{g}^\circ_{CO} - \bar{g}^\circ_{H_2O}$$

$$\Delta G_2^\circ = 2\bar{g}^\circ_{CO} + \bar{g}^\circ_{O_2} - 2\bar{g}^\circ_{CO_2} = \text{—} \quad -2\bar{g}^\circ_{CO_2} + 2\bar{g}^\circ_{CO} + \bar{g}^\circ_{O_2}$$

$$\Delta G_3^\circ = 2\bar{g}^\circ_{H_2} + \bar{g}^\circ_{O_2} - 2\bar{g}^\circ_{H_2O} = 2\bar{g}^\circ_{H_2} \quad \text{—} \quad \text{—} \quad + \bar{g}^\circ_{O_2} - 2\bar{g}^\circ_{H_2O}$$

$$\Delta G_1^\circ = \frac{1}{2}(\Delta G_3^\circ - \Delta G_2^\circ) = \frac{\Delta G_3^\circ}{2} - \frac{\Delta G_2^\circ}{2}$$

$$\boxed{K = e^{-\frac{\Delta G^\circ}{RT}}}$$

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14.8 (continued)

$$\frac{\Delta G_1^\circ}{RT} = \frac{\Delta G_3^\circ}{2RT} - \frac{\Delta G_2^\circ}{2RT}$$

$$e^{\frac{\Delta G_1^\circ}{RT}} = \frac{e^{\Delta G_3^\circ/2RT}}{e^{\Delta G_2^\circ/2RT}}$$

$$\frac{1}{K_1} = \left(\frac{1/K_3}{1/K_2}\right)^{1/2}$$

$$\boxed{K_1 = \left(K_3/K_2\right)^{1/2}}$$