The existence of vortices in the wakes of simulated raindrops

J. R. Saylor[a] and B. K. Jones
Department of Mechanical Engineering, Clemson University, Clemson, South Carolina 29634-0921

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Flow visualizations are presented of the wakes behind water droplets traveling at terminal velocity in a vertically directed airflow. These images, obtained by levitating water drops in a vertical wind tunnel, reveal vortices in the wake behind the drop. It has long been postulated that such wakes trigger transverse oscillations in raindrops. Although such transverse oscillations were not observed here, the images do show a canting of the oblate drop which appears to be connected to the vortex shedding process. These are the first images of simulated raindrops that visualize both the shape of the drop and the vortices in the wake immediately behind the drop. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1874192]

Many investigations have been conducted of flow past a sphere which document the existence and characteristics of vortices in the wake.[1] A raindrop is an example of such a flow, where the drop is traveling at its terminal velocity and a wake exists behind this drop where vortices may form. The dimensionless groups that characterize this flow are the Reynolds number Re, the Weber number We, and the Strouhal number St, defined as

$$\mathrm{Re} = Ud/\nu, \tag{1}$$

$$\mathrm{We} = \rho_a U^2 d/\sigma, \tag{2}$$

$$\mathrm{St} = f_v d/U, \tag{3}$$

where U is the velocity of the air flow which is the terminal velocity of the drop, d is the equivalent drop diameter (the diameter of a sphere having the same volume as the drop), ν and ρ_a are the kinematic viscosity and density of the air, respectively, σ is the surface tension of water, and f_v is the vortex shedding frequency.

It has been postulated since as early as 1949 that vortex shedding may affect the shape of raindrops.[2] Experimental evidence has been accrued to support this idea and to suggest that vortices excite oscillations.[3] However, images of vortex shedding behind a raindrop have not been obtained to show a connection between vortex shedding and raindrop shape. The shape of falling raindrops is important to the measurement of rain rate using dual-polarization radars. This measurement technique requires *a priori* knowledge of the statistical distribution of raindrop shapes, making important an understanding of raindrop oscillations and their causes.[4]

The shape of a raindrop is quantified using the axis ratio

$$\alpha = v/h, \tag{4}$$

where v and h are the maximum vertical and horizontal measures of the drop, respectively. In the absence of oscillations, a drop would exhibit an "equilibrium shape" determined by the aerodynamic, gravitational, and surface tension forces acting on the drop.[5] The α vs d plot for this equilibrium drop shape is referred to here as the "equilibrium plot." Falling raindrops oscillate causing scatter in α. Depending on the type of drop oscillation, this scatter can fall evenly above and below the equilibrium plot (two-sided scatter), or primarily on one side of the equilibrium plot (one-sided scatter). Existing radar measurements,[6] aircraft based raindrop measurements,[7] and drop tower measurements[3] all show that actual raindrops tend to give α values that have one-sided scatter above the equilibrium plot, i.e., the average value of α is vertically offset from the equilibrium plot.

Raindrop oscillations are characterized by the n spherical harmonics that a drop can exhibit. Rayleigh demonstrated that the frequency of these drop oscillations is (for small oscillations)[8]

$$f_d = [2n(n-1)(n+2)\sigma]^{1/2}[\pi^2 \rho d^3]^{-1/2}, \tag{5}$$

where ρ is the density of the drop (see also Landau and Lifshitz[9]). It is generally agreed that shapes displayed by raindrops are determined primarily by the fundamental frequency ($n=2$) and the first harmonic ($n=3$), and that higher order harmonics contribute little to drop shape. For each harmonic there are $m=n+1$ shape modes. Hence, the fundamental frequency exhibits three shape modes and the first harmonic exhibits four shape modes. Of these seven possible shape modes, only two can cause one-sided scatter above the equilibrium plot, and a concomitant vertical offset of the α vs d plot from the equilibrium plot.[10] These two modes are transverse modes and are illustrated in Fig. 1.

Significant research has been conducted to demonstrate that the vertically offset α vs d plots resulting from the transverse modes shown in Fig. 1 are due to vortices shed from the falling raindrop. Moreover, Beard *et al.*[3] obtained measurements of raindrop oscillations showing that a vertical offset and one-sided scatter in the α vs d plot occurs almost exclusively for raindrops having diameters $d=1$–1.5 mm. These authors then showed that the natural oscillation frequencies f_d [Eq. (5)] of water drops in air for this range of diameters are very similar to the vortex shedding frequencies expected for spheres in this diameter range. These vortex shedding frequencies were obtained using the St versus Re

[a]Author to whom correspondence should be addressed. Electronic mail: jrsaylor@ces.clemson.edu

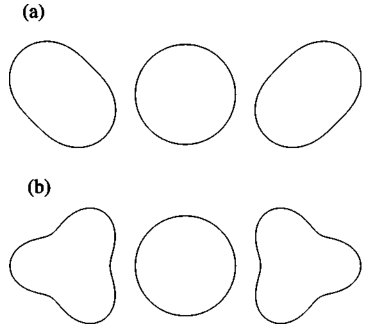

FIG. 1. Drop oscillation modes due to the (a) fundamental harmonic and (b) first harmonic. The modes, m, are numbered from zero. Using the notation (n,m), (a) corresponds to (2, 1) and (b) corresponds to (3,1). The amplitude of oscillation is exaggerated from that typically observed to clearly show the shapes.

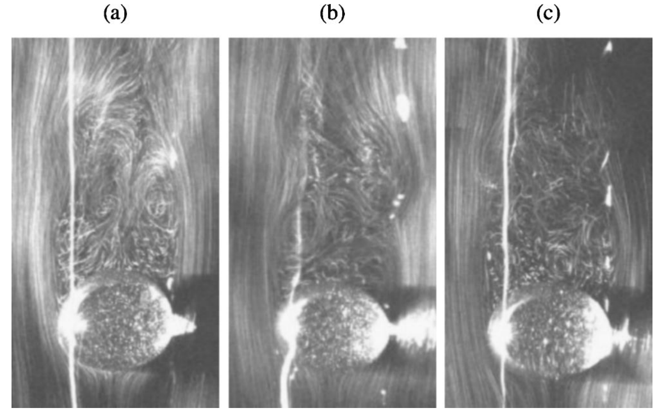

FIG. 3. Three images of drop wakes behind levitated water drops obtained using a high-speed digital camera. The airflow is directed upward, the gravity vector points down, and the laser light sheet enters from the left. Visual anomalies due to laser reflection are described in the text. The dimensions of the drops are (a) $v=2.51$ mm, $h=2.91$ mm, (b) $v=2.20$ mm, $h=2.90$ mm, (c) $v=2.40$ mm, $h=3.10$ mm.

data of other researchers for liquid drops falling in an immiscible liquid[11,12] and solid spheres falling in a liquid.[13] Of course We for a raindrop is different from that for these liquid–liquid data[11,12] and liquid–solid data,[13] however the frequency match suggests a coupling between vortex shedding and drop oscillation at these diameters. Measurements of the rainbow streak caused by a falling drop and images of such drops have been made in drop towers and in actual rain that support the existence of transverse oscillations in this diameter range, further strengthening the idea that transverse oscillations are excited by eddy shedding.[3,10,14,15]

In spite of a large body of work suggesting a role for vortex shedding in drop oscillations, and the importance of these oscillations in precipitation radar measurements, visualizations of such vortices in the wakes of raindrops are lacking. We are aware of only one study, the work of List and Hand,[16] where images of wakes were presented for a water drop traveling near terminal velocity. Moreover, the focus of that investigation was on the effect of the drop in inducing airflow and consequently images of the wake were obtained far from the drop. The drop itself and the separation bubble immediately behind the drop were not presented.

We sought to obtain images of simulated raindrops along with the wake region immediately behind the drop. Drops were levitated in a vertically directed airflow, so that they experienced a terminal velocity flow. The drop levitation tunnel (DLT) used to obtain these images is illustrated in Fig. 2, along with the laser illumination setup. The airflow was made visible by injecting a fog into the region upstream of the test section. A screen was placed at the immediate exit of the tunnel, and just upstream of the drop levitation point to create a velocity minimum in the flow that provided a region of lateral stability for the drop, without which the drop would quickly blow outward from the centerline of the open test section. The screen consisted of eight wires that intersected at the geometric center of the tunnel.

Images obtained in the DLT are presented in Fig. 3 with wakes visualized via the method just described. The illumination source is a sheet of laser light, and hence these images are planar sections through the drop. Imaging drops with laser illumination presents several challenges and creates some optical anomalies that can be seen in these images. First among these is saturation of the digital camera by laser light reflected from the drop surface. This saturation results in a bright spot on the left-hand side of each drop, along with a bright vertical line caused by charge bleeding on the CCD chip. Second, since the drop itself acts as a lens, the laser light sheet is focused in the region just to the right of the drop resulting in a bright cone of light in that area.

In each of the images presented in Fig. 3, the general flow in the high speed regions away from the drop is seen as long streak lines. A stagnation point flow is also seen at the

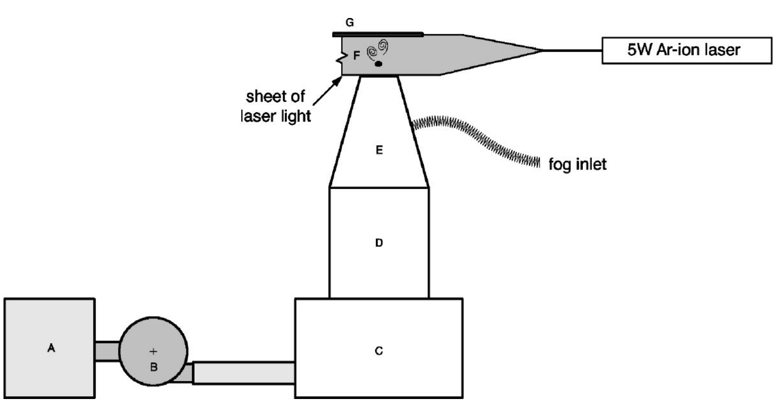

FIG. 2. Experimental setup used to obtain images of drops and wakes. This drop levitation tunnel was based on the designs of Blanchard (Ref. 21) and Kamra *et al.* (Ref. 22). (A) Filtered inlet plenum, (B) blower, (C) settling chamber, (D) flow straightening section, (E) contraction, (F) open test section, and (G) stagnation plate. A drop is shown in the test section, illuminated by a sheet of laser light.

TABLE I. Average, minimum, and maximum of d , α, Re, We, St, f_v, f_{d2}, and f_{d3} for all 95 images. The standard deviation for d is 0.27 mm, and the standard deviation for α is 0.07.

	d (mm)	α	Re	We	St	f_v	f_{d2}	f_{d3}
Average	2.50	0.84	1156	2.2	0.21	617	87	168
Minimum	1.65	0.62	604	0.89	0.205	723	163	313
Maximum	2.91	1.11	1438	2.9	0.215	580	69	134

most upstream portion of the drop. Immediately downstream of the drop, the wake is clearly visualized, and vortical structures can be seen. 95 drop images were recorded using this laser illumination configuration and Figs. 3(a)–3(c) were selected to show the range of wake types that were observed. Figure 3(a) shows two vortices in the wake, symmetrically oriented on the right- and left-hand sides of the drop. These are what would be visualized if a toroidal vortex ring is located immediately downstream of the drop. Figure 3(b) shows two vortices, with the left-hand vortex closer to the drop than the right-hand vortex. This is the type of asymmetric configuration that is expected to trigger transverse oscillations.[3] Finally, Fig. 3(c) shows a wake with a structure that is more complicated than that of a simple vortex.

The flow past these drops has Reynolds numbers on the order of 1000 (see Table I), and the wakes that are observed have structures that are not dissimilar to those recorded in the literature for flow past a solid sphere. For example in the work of Sakamoto and Haniu[17] similar wake structures are observed although the present visualization lacks the resolution necessary to make a detailed comparison of the wake type.

Of the 95 images obtained, 9 (9.5%) had two vortices, 48 (50.5%) had one vortex, and 38 (40%) had no vortices. These numbers are, of course, subjective since there is no way to objectively identify a vortex from images like those presented in Fig. 3. All images showed some sort of separation bubble immediately downstream of the drop.

Table I presents a summary of the values for d, α, Re, We, and St for the data obtained here. In computing Re, We, and St, a value for the velocity is required, as indicated in Eqs. (1)–(3). This velocity is the terminal velocity which was obtained from the equation presented in Atlas *et al.*[18] St in Table I requires f_v, which was not measured. Instead, values of St were obtained using the St versus Re plot presented in Sakamoto and Haniu.[17] The resulting values of f_v are also presented in Table I, as are the values f_{d2} and f_{d3}, the values of f_d obtained from Eq. (5) using $n=2$ and $n=3$, respectively.

Notably absent from the images presented in Fig. 3 is any evidence of the transverse oscillations illustrated in Fig. 1. These transverse oscillations result in a drop whose poles are tilted at an angle of ±45° from the horizontal angle for the (2, 1) mode or ±58.9° from the horizontal for the (3, 1) mode. Many of the images obtained show a slight canting angle. However, of the 95 images acquired, only one approached such large canting angles (39.8°), and only ten drops had a canting angle of 10° or greater. The average canting angle was 4.8°. The lack of transverse oscillations is further supported by an α vs d plot (not shown here) that does not exhibit the vertical deviations from the equilibrium plot that are associated with transverse oscillations. Additionally, the data presented in Table I show that the drop oscillation frequencies f_{d2} and f_{d3} are significantly different from the vortex shedding frequency f_v, indicating that drop oscillations caused by a match with the vortex shedding frequency should not be expected. Indeed, as demonstrated by Beard *et al.*,[3,10] a match between the oscillation frequencies of drops predicted by Eq. (5) and the vortex shedding frequencies should occur only for diameters $d=1$–1.5 mm, significantly smaller than those investigated here. Nevertheless, the drop images obtained do show a tilting or canting of the drop shape, suggesting the possibility that vortices may affect drop shape in ways other than the excitation of transverse oscillations.

To see if vortices played a role in the canting of the drop, a correlation coefficient $\overline{p}$ was computed for drops that had a vortex in the wake. The convention illustrated in Fig. 4 was used to assign values for p. By summing the values for p and dividing by the number of images, an average value $\overline{p}=0.26$ was obtained. This correlation is significant since $\overline{p}$ is bounded by $[-1, 1]$. The fact that $\overline{p}$ is positive in magnitude shows that the presence of a vortex is affiliated with a drop that is canted toward the location of that vortex. Although further investigation is required to determine conclusively if vortices cause drop canting, the present data suggest that this is the case.

Several aspects of the DLT illustrated in Fig. 2 restrict the degree to which these results can be extrapolated to actual raindrops. For example, the velocity well created by the screen just upstream of the test section provides a stable region for the drop, but this stability is marginal, and many drops introduced to this region blew out before images were obtained. Because sideways motion of a falling drop has been attributed to asymmetric vortex shedding,[2] it is likely

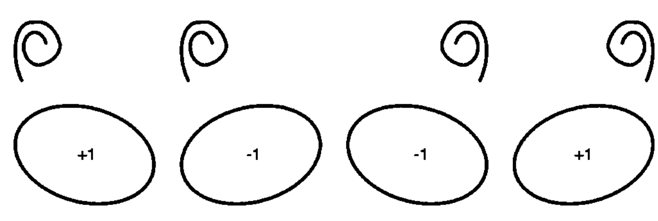

FIG. 4. Convention used to determine the correlation coefficient $\overline{p}$. Drops canted toward the location of the nearest downstream vortex are given a value of $p=+1$. Drops canted away from the nearest downstream vortex are given a value of $p=-1$. If the canting angle was exactly zero, it was assigned a value of $p=0$. (This occurred for only 6 of the 57 images that had a vortex.)

that our tunnel acted as a filter, effectively eliminating these drops from observation. It is, therefore, entirely possible that vortices excite transverse oscillations *and* that these transverse oscillations were absent in all of the images obtained in our tunnel. The data presented here should be interpreted as supporting the existence of vortices in falling raindrops and supporting a second mechanism whereby vortices cause a finite canting angle in the falling raindrop. Further work is needed to provide images showing vortices inducing transverse oscillations in drops. This would require a tunnel better able to stabilize drops undergoing transverse oscillations. It is possible that use of an acoustic standing wave field to help stabilize the drop may be useful in this regard. Such acoustic fields have been used by other researchers to levitate drops and spheres.[19,20]

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